Zoran Škoda: Geometry of connections and integrability
(advanced graduate course, 60 hours)

Description: In differential geometry, a connection is a structure allowing comparing infinitesimal quantities at different points enabling differentiation. In physics, the connections may be dynamical *gauge fields*. A connection has an invariant, its curvature. When the curvature is non-zero its value leads to cohomological invariants called characteristic classes. Curvature is zero iff **Maurer-Cartan equation (MCE)** holds. This condition is flatness or integrability appears widely in modern mathematics. I geometry, integrability of a structure means that it appears as infinitesimal expression of a global object, e.g. integral curves of a vector field, Lie group for a Lie algebra, solution of a system of differential equations.

1. Connections on vector bundles (and Lie algebroids) in differential geometry: several descriptions; examples of integrability in terms of MCE

2. Extend connections from vector bundles to sheaves. For flat connections, this essentially amounts to D-modules. Sheaves allow singularities. We follow Grothendieck to extend intrinsically differential calculus to singular situations: introduce structured spaces and quasicoherent modules; filtration of infinitesimal neighborhoods and dual differential filtrations on bimodules, leading to differential operators. D-modules appear in the form of the related descent data, **crystals**, or equivalently a flat "Grothendieck" connection. Main local properties are defined in similar terms: smoothness in terms of the lifting of maps to infinitesimal neighborhoods. Allowing all orders of infinitesimals leads to formal schemes. Time permiting, we here insert formal deformation theory (in terms of MCE).

3. Main examples, Chern-Weil theory of characteristic classes and the integration of Lie algebr(oid)s to Lie group(oid)s, show the importance of characteristic 0 and finite dimensionality. Under these conditions, Lie algebras have Koszul duals (Chevalley-Eilenberg dg-algebras), which can be, using ideas from rational homotopy theory, turned into topological spaces, underlying integrating object.

Texts Future lectures http://ncatlab.org/zoranskoda/edit/hom11connections

S. Morita, *Geometry of characteristic classes*, AMS 2001

J. L. Dupont, *Fibre bundles and Chern-Weil theory*, Aarhus 2003, pdf

J. Lurie, *Crystals and D-modules*, pdf

D. Sullivan, *Infinitesimal computations in topology*, Publ. IHES 47 (1977), p. 269-331. (pdf); E. Getzler, *Lie theory for nilpotent L_∞ algebras*, Annals of Mathematics **170** (2009), 271–301, math.AT/0404003

R. Bott, L. W. Tu, *Differential forms in algebraic topology*, Springer

M. Crainic, R. L. Fernandes, *Lectures on integrability of Lie brackets*, math.DG/0611259