Description f LF in TS style

Vladimir Voevodsky

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1 Expressions and terms of LF

Definition 1.1 [**lfexp**] *The following labels are permitted in expressions of LF: names of t-constants, names of o-constants, names of o-variables, Type,* $[\prod_k; x]$, $[\prod_t; x]$, $[\lambda_t; x]$, $[ev_t]$, $[\lambda_o; x]$ *and* $[ev_o]$.

Definition 1.2 [**lfclassesofexp**] *We distinguish three classes of expressions:*

1. *K-expressions are the ones with the root node* $[Type]$ *or* $[\prod_k; x]$,
2. *T-expressions are the ones with the root node* $[X]$ *where* X *is the name of a t-constant,* $[\prod_t; x]$, $[\lambda_t; x]$ *or* $[ev_t]$,
3. *O-expressions are the ones with the root node* $[x]$ *where* x *is the name of an o-constant or an o-variable,* $[\lambda_o; x]$ *and* $[ev_o]$.

Definition 1.3 [**lfterms**] *An LF-term is an expression of LF which satisfies the following conditions:*

1. *any node of the form* $[Type]$ *has valency* 0,
2. *any node of the form* $[\prod_k; x]$ *has valency 2, its first branch is a t-expression which does not depend on* x *and its second branch is a k-expression,*
3. *any node of the form* $[\prod_t; x]$ *has valency 2, its first branch is a t-expression which does not depend on* x *and its second branch is a t-expression,*
4. *any node of the form* $[\lambda_t; x]$ *has valency 2, its first branch is a t-expression which does not depend on* x *and its second branch is a t-expression,*
5. *any node of the form* $[ev_t]$ *has valency 2 and both its branches are t-expressions,*
6. *any node of the form* $[\lambda_o; x]$ *has valency 2, its first branch is a t-expression which does not depend on* x *and its second branch is a o-expression,*
7. *any node of the form* $[ev_o]$ *has valency 2 and both its branches are o-expressions.*

2 Derivation rules for LF

The derivation (inference) rules for LF are as follows:

1. $$\frac{}{\triangleright}$$

2. for each $i \in \mathbf{N}$ $$\frac{\Gamma, x : T, \Gamma' \triangleright \quad \mathit{where}\ l(\Gamma) = i}{\Gamma, x : T, \Gamma' \vdash x : T}$$

3. $$\frac{\Gamma, x : T \triangleright \quad \Gamma, x : T' \triangleright}{\Gamma \vdash T \stackrel{d}{=} T}$$

4. $$\frac{\Gamma \vdash T_1 \stackrel{d}{=} T_2}{\Gamma \vdash T_2 \stackrel{d}{=} T_1}$$

5. $$\frac{\Gamma \vdash T_1 \stackrel{d}{=} T_2 \quad \Gamma \vdash T_2 \stackrel{d}{=} T_3}{\Gamma \vdash T_1 \stackrel{d}{=} T_3}$$

6. $$\frac{\Gamma \vdash o : T \quad \Gamma \vdash o' : T \quad o \sim_A o'}{\Gamma \vdash o \stackrel{d}{=} o' : T}$$

7. $$\frac{\Gamma \vdash o_1 \stackrel{d}{=} o_2 : T}{\Gamma \vdash o_2 \stackrel{d}{=} o_1 : T}$$

8. $$\frac{\Gamma \vdash o_1 \stackrel{d}{=} o_2 : T \quad \Gamma \vdash o_2 \stackrel{d}{=} o_3 : T}{\Gamma \vdash o_1 \stackrel{d}{=} o_3 : T}$$

9. $$\frac{\Gamma \vdash o : T \quad \Gamma \vdash T \stackrel{d}{=} T'}{\Gamma \vdash o : T'}$$

10. $$\frac{\Gamma \vdash o \stackrel{d}{=} o' : T \quad \Gamma \vdash T \stackrel{d}{=} T'}{\Gamma \vdash o \stackrel{d}{=} o' : T'}$$

11. if A is a t-constant name unused in Γ then $$\frac{\Gamma \triangleright}{\Gamma, A : \mathit{Type} \triangleright}$$

12. If A is a t-expression and K is a k-expression then

$$\frac{\Gamma, x : A, y : K \rhd}{\Gamma, z : [\prod_k; x](A, K) \rhd}$$

13. If A, A' are t-expressions and K, K' are k-expressions then

$$\frac{\Gamma \vdash A \stackrel{d}{=} A' \quad \Gamma, x : A \vdash K \stackrel{d}{=} K'}{\Gamma \vdash [\prod_k; x](A, K) \stackrel{d}{=} [\prod_k; x](A', K')}$$

14. If A, B are t-expressions then

$$\frac{\Gamma, x : A, y : B \rhd}{\Gamma, z : [\prod_t; x](A, B) \rhd}$$

15. If A, A', B, B' are t-expressions then

$$\frac{\Gamma \vdash A \stackrel{d}{=} A' \quad \Gamma, x : A \vdash B \stackrel{d}{=} B'}{\Gamma \vdash [\prod_t; x](A, B) \stackrel{d}{=} [\prod_t; x](A', B')}$$

16. If A is a t-expression and K is a k-expression then

$$\frac{\Gamma, x : A \vdash B : K}{\Gamma \vdash [\lambda_t; x](A, B) : [\prod_k; x](A, K)}$$

17. If A, A' are t-expressions and K is a k-expression then

$$\frac{\Gamma \vdash A \stackrel{d}{=} A'' \quad \Gamma, x : A \vdash B \stackrel{d}{=} B' : K}{\Gamma \vdash [\lambda_t; x](A, B) \stackrel{d}{=} [\lambda_t; x](A', B') : [\prod_k; x](A, K)}$$

18.

$$\frac{\Gamma \vdash F : [\prod_k; x](A, K) \quad \Gamma \vdash a : A}{\Gamma \vdash [ev_t](F, a) : K[a/x]}$$

19.

$$\frac{\Gamma \vdash F \stackrel{d}{=} F' : [\prod_k; x](A, K) \quad \Gamma \vdash a \stackrel{d}{=} a' : A}{\Gamma \vdash [ev_t](F, a) \stackrel{d}{=} [ev_t](F', a') : K[a/x]}$$

20. If A and B are t-expressions then

$$\frac{\Gamma, x : A \vdash o : B}{\Gamma \vdash [\lambda; x](A, o) : [\prod_t; x](A, B)}$$

21. If A, A' and B are t-expressions then

$$\frac{\Gamma \vdash A \stackrel{d}{=} A' \quad \Gamma, x : A \vdash o \stackrel{d}{=} o' : B}{\Gamma \vdash [\lambda; x](A, o) \stackrel{d}{=} [\lambda; x](A', o') : [\prod; x](A, B)}$$

22.
$$\frac{\Gamma \vdash f : [\prod_t; x](A, B) \quad \Gamma \vdash o : A}{\Gamma \vdash [ev_o](f, o) : B[o/x]}$$

23.
$$\frac{\Gamma \vdash f \stackrel{d}{=} f' : [\prod_t; x](A, B) \quad \Gamma \vdash o \stackrel{d}{=} o' : B}{\Gamma \vdash [ev_o](f, o) \stackrel{d}{=} [ev_o](f', o') : B[o/x]}$$

24. If A, B are t-expressions then
$$\frac{\Gamma \vdash o : A \quad \Gamma, o : A \vdash o' : B}{\Gamma \vdash [ev_o]([\lambda_o; x](A, o), o') \stackrel{d}{=} o'[o/x] : B[o/x]}$$

25. If A, B are t-expressions then
$$\frac{\Gamma \vdash o : A \quad \Gamma, x : A \vdash B : K}{\Gamma \vdash [ev_t]([\lambda_t; x](A, B), o) \stackrel{d}{=} B[o/x]}$$

26.
$$\frac{\Gamma \vdash F : [\prod_k; x](A, K)}{\Gamma \vdash [\lambda_t; x](A, [ev_t](F, x)) \stackrel{d}{=} F : [\prod_k; x](A, K)}$$

27.
$$\frac{\Gamma \vdash f : [\prod_t; x](A, B)}{\Gamma \vdash [\lambda_o; x](A, [ev_o](f, x)) \stackrel{d}{=} f : [\prod_t; x](A, B)}$$