Higher theta functions and higher CS/WZW holography

Proposing an interdisciplinary mathematics research project based at Surrey, UK

Urs Schreiber

Career path

PhD in Essen-Duisburg (2005) Postdocs in Hamburg, Utrecht, Nijmegen Research fellowships at HIM Bonn and Notre Dame, USA Habilitation in Hamburg (2011) Visiting professorship in Prague (present) need long-term perspective right now

Publications

29+ research articles 13 international coauthors edited survey for Amer. Math Society (2011) preprint monograph

Invited conference talks

plenty, e.g.:			
2014	Oxford, Paris (IHP, Diderot),		
	New York (CUNY), Vienna (ESI),		
2015	Göttingen, Prague,		
	Porto (Amer. + Eur. Math Society meeting),		
	Hamburg (Germ. Math Society meeting, 2 talks),		

Invited lecture series

- 2013 Singapore (NUS), Bonn (MPI), Pittsburgh
- 2014 Edinburgh (Heriot–Watt), Paris (Diderot)
- 2015 Prague, Pittsburgh

Impact and Leadership

'08:



communication platform recording research level mathematics focus on topics related to this propoal

- '10: cited widely by scholarly sites: Wikipedia, PlanetMath, MathOverflow, PhysicsOverflow,...
- open publishing experiment: Publications of the nLab '11: needs dedicated staff and funding
- migrated to server at Carnegie-Mellon '15: funded by DoD MURI grant '14

Background I: The rise of homotopy theory

- '70s: Grothendieck (Fields medal '66) envisions: "algebraic geometry and number theory will reveal deep secrets via homotopy theory"
- '95: Baez (UCR), my de-facto PhD advisor, conjectures cobordism hypothesis: "homotopy theory governs quantum field theory"
- '09: Lurie (Harvard) finds proof, realizes Grothendieck's dream Breakthrough Prize '14, MacArthur Award '14
- '13: Awodey (CMU), Voevodsky (IAS, Fields medal '02) find: "homotopy theory serves as new foundations" DoD MURI Grant '14

At this point my proposal comes in:

Novel method introduced in my Habilitation (2011):

cohesive homotopy theory: "inter-geometric" { differential arithmetic

remarkable consequence:

Claim Θ (Research Hypothesis): cohesive homotopy theory + cobordism hypothesis \Rightarrow higher Theta functions

proven (with D. Fiorenza) but yet unpublished...

...research grant for exploiting this gold mine

Background II: Theta functions

- 19c: hallmark of number theory give famous zeta functions
- '80s: appearance in quantum field theory quantum correlators, vacuum amplitudes

At the heart of two major international research programs:

arithmetic geom.: Langlands program Galois representations \mapsto zeta functions

∫inter-geometric analogy

differential geom.: quantization of 3dCS/2dWZW theory gauge fields \mapsto vacuum amplitudes Background III: Higher CS/WZW holography

- '90: Witten receives Fields medal for new mathematics suggested from quantum field theory
- '97: Maldacena finds *holographic principle* becomes the most cited topic in the new millenium
- '00: Witten relates it to higher dimensional CS/WZW:

	holography		
3dCS	\leftrightarrow	2dWZW	"string"
7dCS	\leftrightarrow	6dWZW	"M5-brane"
11dCS	\leftrightarrow	10dWZW	"RR-fields"

The mathematics for the latter is barely existent...

...but Claim Θ provides just that: \longrightarrow



Objectives – Claim $\Theta \Rightarrow$ novel unified handle on 4 nascent topics:

cutting edge, highly interdisciplinary, broad impact

UK hosts all relevant expertise, esp. at Surrey and vicinity would make UK international focus point for these new developments

Academic Beneficiaries

plenty, due to the novel level of interdisciplinarity. E.g.:

UK

J Gutowski, J McOrist, S Pasquetti, A Torrielli, M Wolf (Surrey) O Ben-Bassat, N Hitchin, D Joyce, Y Kremnitzer, G Segal (Oxford) M Atiyah (Edinburgh), K Buzzard (King's), M Green (Cambridge) T Altenkirch, I Fesenko, S Oblezin (Nottingham), A Schenkel, C Sämann, R Szabo (Heriot–Watt), E Cheng, N Gurski, K Mackenzie, N Strickland (Sheffield), N Gambino (Leeds), D Corfield (Kent), J Ladyman (Bristol), ...

international

D Freed (Austin), J Lurie (Harvard), M Ando (Illinois), N Ganter (Melbourne), U Bunke (Regensburg), A Henriques (Utrecht), G Moore (Rutgers), ...

Workplan - detailed strategy based on my previous work:

Objective 1 Higher modular functors Haugseng's O(n)-action on homotopy fixed points fractional CS on cobord. with Wu-structure generalized cohomology by coherent pushforw orientations in geometric quantizatic generalized cohomology by coherent pushforw Haugseng's O(n)-action on homotopy fixed points fractional CS on cobord. with Wu-structure generalized cohomology by coherent pushforw orientations in geometric quantizatic generalized cohomology by coherent pushforw higher Jacobians for differential cohomology 1-dimensional formal groups formal group formal group formal group laws, height. and Landweber-exactness higher elliptic spectra on moduli stacks sheaves of such spectra on moduli stacks Objective 3 - Higher Langlands correspondence arithmetic cohesion Cocal systems to theta functions in derived arithmetic geometry and study in depth Objective 4 - Higher equiv elliptic cohomology and study in depth	1st year	2nd year	3rd year	4th year	5th year
Objective 1 – Higher modular functors Haugseng's conjecture O(n)-action on higher CS theory homotopy fixed points (anomaly cancellation) fractional CS on cobord. with Wu-structure orientations in generalized cohomology geometric quantization by coherent pushforw Objective 2 – Higher topological modular forms –			work out Obj 1–4 in the eory example of 7d CS	-Examples and Applications	
Objective 2 – Higher topological modular forms higher Jacobians for differential cohomology 1-dimensional formal groups in variation of Artin-Mazur and Landweber-exactness of Mathematic cohesion 1-dimensional formal groups in variation of Artin-Mazur and Landweber-exactness of Local systems to theta functions in derived arithmetic geometry in derived arithmetic geometry sheaves of such spectra on moduli stacks Objective 3 – Higher Langlands correspondence local systems to theta functions in derived arithmetic geometry compare to existing Langlands theory and study in depth Objective 4 – Higher equiv elliptic cohomology Objective 4 – Higher equiv elliptic cohomology	Haugseng's $O($ conjecture hig	n)-action on homotopy fixed pointer CS theory (anomaly cancellat	Objective 1 – Higher modular fund ints fractional CS ion) on cobord. with Wu-structure	ctors orientations in generalized cohomology	geometric quantization by coherent pushforward
Objective 3 - Higher Langlands correspondence further develop model arithmetic cohesion higher CS theories as cocycles in algebraic K-theory biology local systems to theta functions in derived arithmetic geometry objective 4 - Higher equiv elliptic cohomology compare to existing Langlands theory and study in depth	higher Jacobians for differential cohomology	Obje 1-dimensional formal groups for r in variation of Artin-Mazur a	c tive 2 – Higher topological modu ormal group laws, height, ind Landweber-exactness higher elli	Ilar forms ptic spectra	sheaves of such spectra on moduli stacks
Objective 4 – Higher equiv elliptic cohomology	further develop model arithmetic cohesion	Obje higher CS theories as cocycles in algebraic K-theor	ctive 3 – Higher Langlands correspondent of the second systems to the second systems to the second s	ions compare try and study	to existing Langlands theory / in depth
to M _G in Lurie's survey Obj 1-3 to 2-equivariance of the general cohomology theories equivariant cohomology t		Obje relate Loc _G as in Obj 1–3 relat to M _G in Lurie's survey Obj	ective 4 – Higher equiv elliptic coh te CS theories in deduce ge 1–3 to 2-equivariance of the gen	neral <i>n-equivariance</i> eral cohomology theories	construct the corresponding equivariant cohomology theories

plenty of strong sub-goals \Rightarrow much contingency

Ambitious but Viable

have been developing much theoretical infrastructure

in my monograph and lecture notes

- \Rightarrow preliminary results, immediate starting point of project
- \Rightarrow concrete goals for the PhD student

First results since submission of proposal:

Fiorenza, Schreiber, Valentino: Extensions of mapping class groups from characteristic classes arXiv:1503.00888

Fiorenza, Sati, Schreiber: The WZW term of the M5-brane and differential cohomotopy arXiv:1506.07557 First Impact – invited to contribute to book collection:

Baez, Cartier, Cisinski, Joyal , Kapranov, Kontsevich, Manin, Marcolli, Penrose, Schreiber, Simpson et al.:

New Spaces for Mathematics and Physics IHP Paris (end of 2015) funded by European Research Council

Expert referee judgement

"ambition of the proposed methodology is outstanding" "scope of the areas is remarkable" "can truly be called mathematics of the 21st century" "wish there were more EPSRC proposals of this type"

Claim Θ



