Entanglement of Sections: The pushout of entangled and parameterized quantum information

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Abstract

Recently Freedman & Hastings asked [FH23] for a mathematical theory that would unify quantum entanglement/tensor-structure with parameterized/bundle-structure via their amalgamation (a hypothetical pushout) along bare quantum (information) theory — a question motivated by the role that vector bundles of spaces of quantum states play in the K-theoretic classification of topological phases of matter.

As a proposed answer to this question, we first make precise a form of the relevant pushout diagram in monoidal category theory. Then we prove that the pushout produces what is known as the *external* tensor product on vector bundles/K-classes, or rather on flat such bundles (flat K-theory), i.e., those equipped with monodromy encoding topological Berry phases. The external tensor product was recently highlighted in the context of topological phases of matter in [Me20] and through our work in quantum programming theory [SS23b] but has not otherwise found due attention in quantum theory yet.

The bulk of our result is a further homotopy-theoretic enhancement of the situation to the "derived category" (∞-category) of flat ∞-vector bundles ("∞-local systems") equipped with the "derived functor" of the external tensor product. Concretely, we present an integral model category of simplicial functors into simplicial $\mathbb{K}$-chain complexes which conveniently presents the ∞-category of parameterized $H\mathbb{K}$-module spectra over varying base spaces and is equipped with homotopically well-behaved external tensor product structure.

In concluding, we indicate how this model category provides candidate categorical semantics for the "Motivic Yoga"-fragment of the recently constructed Linear Homotopy Type Theory (`LHoTT`) [Ri22], which in [SS23b] we describe as a quantum programming language with classical control, dynamic lifting and topological effects. In particular, this serves to express the homotopical construction of topological anyonic braid quantum gates recently described in [MSS23].

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