The **formalization of the difference between singular and solitonioc branes** is via choices of *domains* on which the flux densities are actually defined (following [SS23-HpH2, §2.1]).

Type of brane	$\Leftrightarrow$	**Domain of flux density** (37)
Singular brane	complement of brane Σ inside spacetime X, removing the singular locus from spacetime	$X \setminus \Sigma$
Solitonic brane	Alexandroff-compactification of transverse space $\Sigma^\perp$, adjoining a "transverse point at infinity" to spacetime	$\Sigma^\perp_{\cup\{\infty\}}$

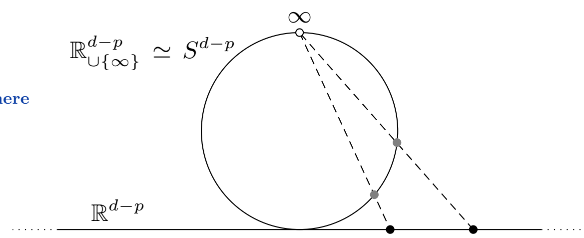
This is most transparent for the special case of "flat" branes in flat Minkowski spacetime:

– **singular branes** have spacetime singularities which are *removed from spacetime*: the field flux sourced by the singularity is that through spheres in the normal bundle around these loci and *would diverge* at the singular brane locus (cf. (13) below):

$$\overset{\text{bulk}}{\mathbb{R}^{d+1}} \setminus \overset{\text{singular brane}}{\mathbb{R}^{p+1}} \underset{\text{homeomorphism}}{\simeq} \overset{\text{punctured transverse space}}{\big(\mathbb{R}^{d-p} \setminus \{0\}\big) \times \mathbb{R}^{p+1}} \underset{\text{homotopy equivalence}}{\simeq} \overset{\text{encircling sphere}}{S^{d-p-1}} \tag{3}$$

– **solitonic branes** are witnessed by non-singular "local bumps" in the flux densities: Their flux *vanishes at infinity*, which means that it is measured on the 1-point compactification of their transverse space, which is again a sphere:

$$\overset{\text{solitonic brane}}{\mathbb{R}^{p+1}_{\cup\{\infty\}}} \wedge \underbrace{\overset{\text{transv. space}}{\mathbb{R}^{d-p}_{\cup\{\infty\}}}}_{\text{with point at infinity}} \simeq \mathbb{R}^{d-p}_{\cup\{\infty\}} \underset{\text{homeo}}{\simeq} \overset{\text{transverse sphere}}{S^{d-p}} \tag{4}$$



Towards flux quantization. The laws of flux discussed so far are laws of "classical physics": By themselves, they do not explain, for instance, why the flux carried by Abrikosov vortices (p. 7) is *quantized* to appear in integer multiples of a unit flux, or why, as argued long ago by Dirac, magnetic monopoles would be quantized to appear in integer multiples of unit charged monopoles. Apparently the electromagnetic flux density $F_2 = \Omega^2_{\mathrm{dR}}(X)$ is just one aspect of the true nature of the electromagnetic field.

In modern mathematical language, the argument underlying *Dirac charge quantization* says that an electromagnetic field configuration on a spacetime X *also* involves a "charge map" $c : X \to B\mathrm{U}(1)$ to the *classifying space* of the circle group. This may be understood as the infinite complex projective space $B\mathrm{U}(1) \underset{\text{whe}}{\simeq} \mathbb{C}P^\infty$, but crucially it is a *classifying space* for ordinary integral cohomology in degree 2, meaning that homotopy classes of such maps are in natural bijection with $H^2(X;\mathbb{Z})$.

Formalizing generalized flux quantization is the topic of §1.2.