

2.2.1 Quantum observables on flux-quantized fields.

Recall:

Quantum observables and quantum states.

Given a *star-algebra of quantum observables* consider their

expectation values for a given *quantum state* (which is defined thereby)

respecting involution

and being “positive” on normal observables.

From this a (Hilbert-)space of states is induced (the *GNS construction*) with “ground” state supporting an operator-state correspondence for the induced inner product, reproducing the expectation values in this ground state.

$(\text{Obs}, \cdot, (-)^*)$
$\langle - \rangle : \text{Obs} \xrightarrow{\text{linear}} \mathbb{C}$
$A \in \text{Obs} \vdash \langle A^* \rangle = \overline{\langle A \rangle}$ $A \in \text{Obs} \vdash \langle A^* A \rangle \in \mathbb{R}_{\geq 0} \subset \mathbb{C}$
$\mathcal{H} := \text{Obs} / \{A \in \text{Obs} \mid \langle A^* \cdot A \rangle = 0\}$ $ \psi_0\rangle := [1]$ $A \psi_0\rangle = [A]$ $\langle A\psi_0, B\psi_0\rangle = \langle \psi_0 A^* B \psi_0 \rangle := \langle A^* B \rangle$
$\langle \psi_0 - \psi_0 \rangle = \langle - \rangle$

Non-perturbative quantization of a Poisson-manifold phase space

is a bundle of C^* -algebras

which continuously

deforms the classical observables

satisfying Dirac’s quantization condition.

$\{-, -\} : C^\infty(P) \otimes C^\infty(P) \rightarrow C^\infty(P)$
$\left\{ \text{Obs} \xrightarrow{(-)_\hbar} \text{Obs}_\hbar \in C^* \text{Alg} \right\}_{\hbar \in \mathbb{R}}, \quad f \in C^0(\mathbb{R}), A \in \text{Obs} \vdash$ $fA \in \text{Obs}, (fA)_\hbar = f(\hbar)A_\hbar$
$\forall_{A \in \widehat{\text{Obs}}} \left(\hbar \mapsto A_\hbar \right) \in C^0(\mathbb{R}), \quad A = \sup_\hbar A_\hbar $
$Q : C_{\text{cpt}}^\infty(P) \rightarrow \text{Obs}, \quad Q(-)_0 : C_{\text{cpt}}^\infty(P) \xrightarrow{\text{dense}} A_0$ $\lim_{\hbar \rightarrow 0} \left [Q(f)_\hbar, Q(g)_\hbar] - i\hbar Q(\{f, g\})_\hbar \right = 0.$

Yang-Mills flux observables.

in \mathfrak{g} -Yang-Mills theory on $\mathbb{R}^{0,1} \times X^3$, with an oriented closed surface $\Sigma^2 \hookrightarrow X^3$ measure the weighted integrals of the electric & magnetic flux densities over Σ

$$\begin{aligned} \Phi_E^\omega &= \int_\Sigma \langle \omega, E \rangle & \text{for } \omega \in \Omega_{\text{dR}}^0(X^3; \mathfrak{g}) \\ \Phi_B^\omega &= \int_\Sigma \langle \omega, B \rangle \end{aligned}$$

Proposition [SS23-Qnt]: Non-perturbative quantum observables on quantized YM fluxes.

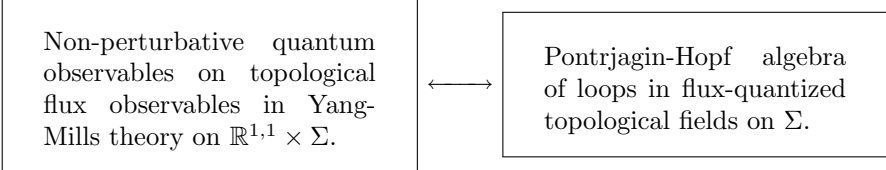
A non-perturbative quantization of the Φ_E and Φ_A

is the Fréchet-group *convolution algebra* of semidirect product group-valued functions on Σ

Hence the *topological flux observables* form a convolution group algebra of cohomology of Σ :

For Maxwell theory on $\mathbb{R}^{1,1} \times \Sigma$ this coincides with the Pontrjagin-Hopf algebra of loops in the moduli of flux-quantized topological field sectors on Σ

This remarkable re-formulation of quantization



we next take as the blueprint for the quantization of topological fluxes in M-theory.