1 Flux Quantization

In higher gauge theories [Al24, §2][FSS19-HighM][JSSW19], *flux* of *field lines* is sourced by branes (§1.1), and *flux quantization* makes fluxes/charges form a discrete space, reflecting individual brane sources (§1.2). A choice of flux quantization is a hypothesis about the non-perturbative completion of the given theory (§1.3).

Tradition, originating in the ancient past, is to define any physical theory by a *stationary action principle* embodied by a Lagrangian density (e.g. [HT92]), from which one systematically extracts a perturbative phase space in the guise of a BRST-BV complex. But flux-quantization laws used to be imposed in ad-hoc fashion to "cancel anomalies" (cf. pp. 22).

In contrast, we observe [SS23-FQ] that all admissible flux-quantization laws $\mathcal{A}$ are algebro-topologically determined by the duality-symmetric form of the Bianchi identity or Gauß law (29) satisfied by the flux densities.

Hypothesizing an admissible flux quantization law $\mathcal{A}$, the non-perturbative phase space is the moduli stack of differential $\mathcal{A}$-cohomology on any Cauchy surface.
Among the admissible flux-quantization laws is typically an "evident" one. In traditional examples like electromagnetic or RR-fields this evident choice is the traditional choice, whose hypothetical nature tends to be forgotten.

The "Hypothesis H" is essentially nothing but the corresponding evident choice of flux quantization for the C-field in 11d supergravity.

The reason why this was not so "evident" earlier is that the admissible flux-quantization laws of the C-field are *non-abelian* (unstable) forms of generalized cohomology (owing to the non-linear sourcing of M2-brane flux by M5-brane flux), whose theory was fully established only in [FSS23-Char].

Survey of Part 1		**General Higher gauge field**	**A-field** in $D=4$	**B&RR-field** in $D=10$	**C-field** in $D=11$
§1.1	Flux densities	$\vec{F} \equiv \Big(F^{(i)} \in \Omega^{\deg_i}_{\mathrm{dR}}(X^D)\Big)_{i \in I}$	F_2 magnetic G_2 electric	H_3 NS5 H_7 F1 $F_{2\bullet}$ $D_{8-2\bullet}$	G_4 M5 G_7 M2
	Self-duality	$\star\vec{F} = \vec{\mu}(\vec{F})$	$\star F_2 = G_2$	$\star H_3 = H_7$ $\star F_{2\bullet} = F_{10-2\bullet}$	$\star G_4 = G_7$
	Bianchi identities	$\mathrm{d}\vec{F} = \vec{P}(\vec{F})$	$\mathrm{d}F_2 = 0$ $\mathrm{d}G_2 = 0$	$\mathrm{d}H_3 = 0$ $\mathrm{d}H_7 = 0$ $\mathrm{d}F_{2\bullet} = H_3 \wedge F_{2\bullet-2}$	$\mathrm{d}G_4 = 0$ $\mathrm{d}G_7 = -\tfrac{1}{2}G_4 \wedge G_4$
§1.2	CE-algebra of characteristic L_∞-algebra	$\mathrm{CE}(\mathfrak{a}) \equiv$ $\mathbb{R}[\vec{b}]\big/\big(\mathrm{d}\vec{b} = \vec{P}(\vec{b})\big)$	$\mathrm{d}f_2 = 0$ $\mathrm{d}g_2 = 0$	$\mathrm{d}h_3 = 0$ $\mathrm{d}h_7 = 0$ $\mathrm{d}f_{2\bullet} = h_3 \wedge f_{2\bullet-2}$	$\mathrm{d}g_4 = 0$ $\mathrm{d}g_7 = -\tfrac{1}{2}g_4 \wedge g_4$
	Solution space of fluxes on $X^D = \mathbb{R}^{0,1} \times X^d$	Gauß law = $\mathfrak{a}$-closedness $\mathbf{\Omega}_{\mathrm{dR}}\big(X^d;\,\mathfrak{a}\big)_{\mathrm{flat}} \equiv$ $\mathrm{Hom}\big(\mathrm{CE}(\mathfrak{a}),\, \Omega^\bullet_{\mathrm{dR}}(X^d)\big)$	$\Omega^2_{\mathrm{dR}}(X^d)_{\mathrm{clsd}}$ $\times\, \Omega^2_{\mathrm{dR}}(X^d)_{\mathrm{clsd}}$ can. momenta	3-twisted de Rham cocycles	"4-twisted" de Rham cocycles
	Characteristic L_∞-algebra	$\mathfrak{a}$	$b\mathfrak{u}(1) \oplus b\mathfrak{u}(1)$	$\big[b_2, v_{2\bullet-1}\big] = v_{2\bullet+1}$	$\big[v_3,\, v_3\big] = v_6$ M-theory gauge algebra
	as rational Whitehead L_∞-algebra	$\mathfrak{a} \simeq \mathfrak{l}\mathcal{A}$	$\mathfrak{l}\big(B^2\mathbb{Z} \times B^2\mathbb{Z}\big)$	$\mathfrak{l}\Big(\big(\mathrm{KU}_0 /\!\!/ B^2\mathbb{Z}\big) \times B^7\mathbb{Z}\Big)$	$\mathfrak{l}\big(S^4\big)$
§1.3	Evident choice of classifying space	$\mathcal{A}$	$B^2\mathbb{Z} \times B^2\mathbb{Z}$ Dirac's hypothesis	$\big(\mathrm{KU}_0 /\!\!/ B^2\mathbb{Z}\big) \times B^7\mathbb{Z}$ Hypothesis K	S^4 Hypothesis H
	Corresponding cohomology theory	generalized cohomology	ordinary cohomology	twisted K-theory	unstable CoHomotopy
	Flux-quantized phase space	$\mathbf{\Omega}_{\mathrm{dR}}\big(X^d;\mathfrak{a}\big)_{\mathrm{flat}} \underset{L^{\mathbb{R}}\mathcal{A}(X^d)}{\times} \mathcal{A}(X^d)$	differential cohomology	differential twisted K-theory	differential CoHomotopy