Urs Schreiber

(New York University, Abu Dhabi & Czech Academy of Science, Prague)

The Higher Structure of 11-dimensional Supergravity

talk at

Souriau 2019

IHP, Paris 2019

based on joint work with

Hisham Sati and Domenico Fiorenza

arXiv:1903.02834, arXiv:1904.10207

Abstract.

Souriau's work has shown the immense fruitfulness of giving basic concepts in theoretical physics a precise mathematical foundation. This program of mathematical physics has been outstandingly successful throughout the 20th century, ranging from the mathematical formalization of classical gravity (via Riemannian geometry/Cartan geometry) and gauge theory (via Chern-Weil theory/differential cohomology) over perturbative renormalization (Schwartz distribution theory/microlocal analysis) to perturbative string theory (2d conformal field theory on all genera). However, the success story got stuck with the 2nd superstring revolution: The D-branes/M-branes and weak/strong-coupling string dualities that constitute the modern picture of putative M-theory (UV-completed 11-dimensional supergravity) have remained mathematical folklore. But it is precisely these non-perturbative effects that, via holographic QCD (intersecting D-brane models) plausibly solve the Millenium Problem of theoretical physics: non-perturbative QCD.

This talk presents recent progress ([FSS19a], [FSS19b]) on identifying mathematical foundations for M-theory in (co-)homotopy theory (aka "higher structures").

References

[FSS19a] D. Fiorenza, H. Sati, U. Schreiber:
The rational higher structure of M-theory,
Proceedings of the LMS-EPSRC Durham Symposium:
Higher Structures in M-Theory, August 2018,
Fortschritte der Physik, 2019
doi:10.1002/prop.201910017
arXiv:1903.02834

[FSS19b] D. Fiorenza, H. Sati, U. Schreiber:
Twisted Cohomotopy implies M-theory anomaly cancellation
arXiv:1904.10207

Theoretical physics:

Source for interesting mathematical folklore:

Concepts	that plausibly can be given a	precise definition.
Statements	that plausibly can be given a	rigorous proof.

Mathematical physics:

Turn folklore	concepts	into	definitions.
Turn folklore	claims	into	propositions.
Turn folklore	arguments	into	proofs.

Pure mathematics:

Run with this formalization.

$\xrightarrow{\text{Examples}}$

In Souriau's footsteps. Historical examples.

theoretical physics	**mathematical physics**	**pure mathematics**
phase space	symplectic manifold	symplectic geometry
quantization	geometric quantization	index theory
higher gauge field	higher connection	differential cohomology, Chern-Weil theory
gravity	metric tensor, vielbein	Riemannian geometry, Cartan geometry
perturbative renormalization	causal perturbation theory	Schwartz distribution theory, microlocal analysis
perturbative string theory	2d SCFT on all genera	Gromov-Witten-type theory, 2-spectral geometry
non-perturbative string theory (branes, dualities, ...) "M-theory"	??	??

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non-perturbative string theory (branes, dualities, ...) "M-theory"	...	...

In Souriau's footsteps. The next step.

theoretical physics	**mathematical physics**	**pure mathematics**
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quantization	geometric quantization	index theory
higher gauge field	higher connection	differential cohomology, Chern-Weil theory
gravity	metric tensor, vielbein	Riemannian geometry, Cartan geometry
perturbative renormalization	causal perturbation theory	Schwartz distribution theory microlocal analysis
perturbative string theory	2d SCFT on all genera	Gromov-Witten-type theory, 2-spectral geometry
non-perturbative string theory (branes, dualities, ...) "M-theory"	*Hypothesis H*: arXiv:1903.02834, arXiv:1904.10207 C-field charge-quantized in twisted Cohomotopy	Cohomotopy theory Ravenel 86: the modern *music of the spheres*.

The foundational principles.

	physics	mathematics
	gauge principle	homotopy theory
&	Pauli exclusion principle	super-geometry
=		**super homotopy theory**

for detailed exposition see:
ncatlab.org/nlab/show/geometry+of+physics+–+supergeometry
ncatlab.org/schreiber/show/Introduction+to+Higher+Supergeometry

Homotopy theory as a metaphysical microscope.

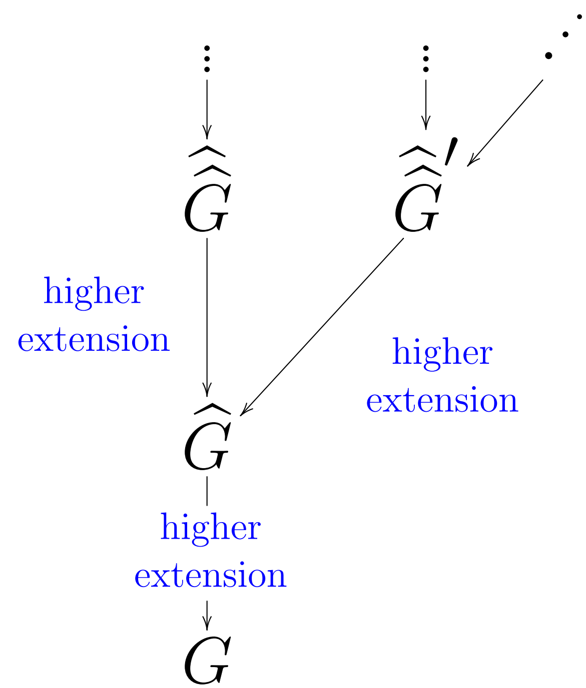
gauge-of-gauge transformations = higher homotopies.

$\Downarrow$

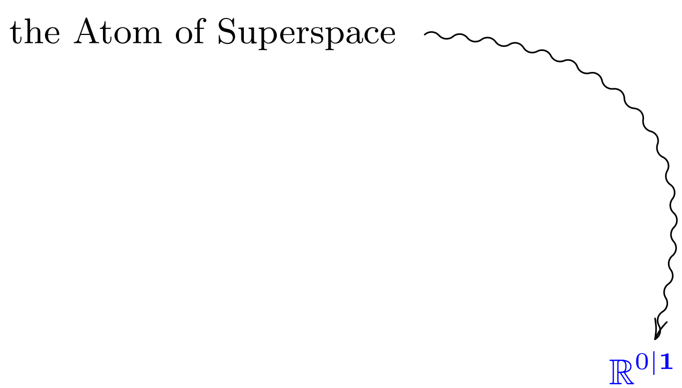
All exact sequences are long.

$\Downarrow$

Out of any group G
emerges a bouquet
of universal invariant
higher central extensions:



Emergence from the superpoint – The brane bouquet.



the Atom of Superspace

$\mathbb{R}^{0|\mathbf{1}}$

regarded with its infinitesimal super group structure

$\mathbb{R}^{0|\mathbf{1}}$

Type I

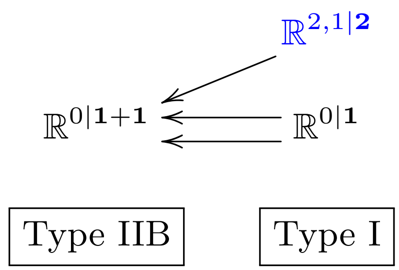
Emergence from the superpoint – The brane bouquet.

$$\mathbb{R}^{0|\mathbf{1}+\mathbf{1}} \leftleftarrows \mathbb{R}^{0|\mathbf{1}}$$

Type IIB   Type I

Emergence from the superpoint – The brane bouquet.

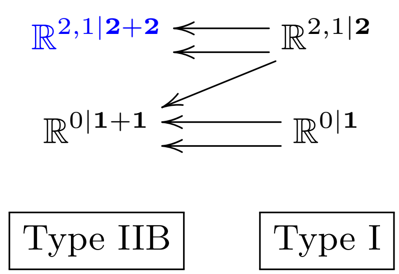
[HS17]



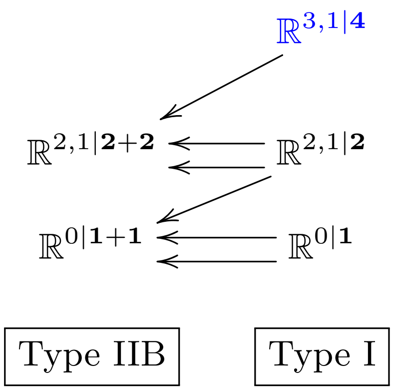
universal central extension: 3d super-Minkowski spacetime

Emergence from the superpoint – The brane bouquet.

[HS17]



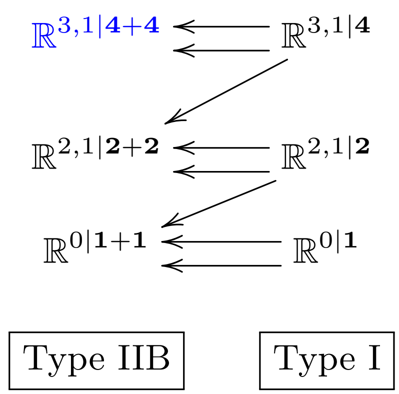
Emergence from the superpoint – The brane bouquet.



[HS17]

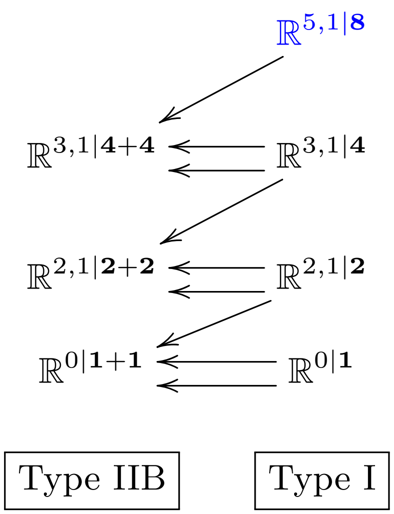
universal invariant central extension: 4d super-Minkowski spacetime

Emergence from the superpoint – The brane bouquet.



[HS17]

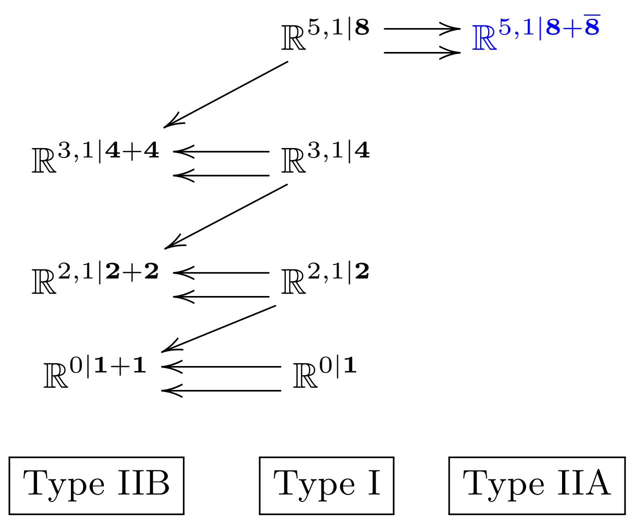
Emergence from the superpoint – The brane bouquet.



[HS17]

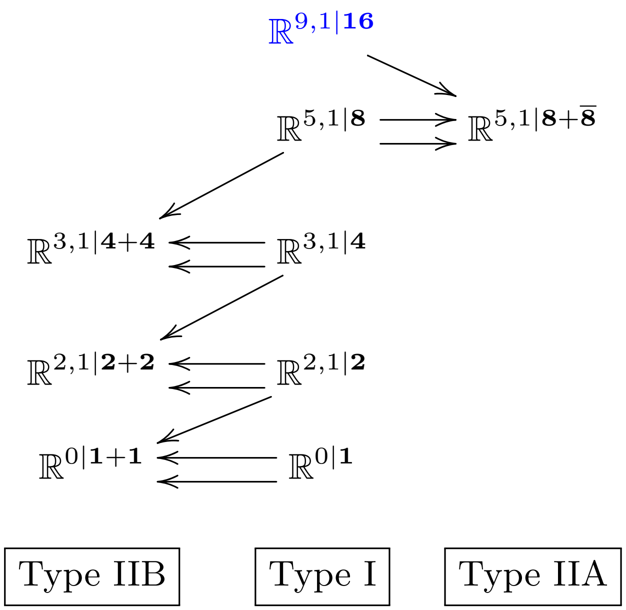
universal invariant central extension: 6d super-Minkowski spacetime

Emergence from the superpoint – The brane bouquet.



[HS17]

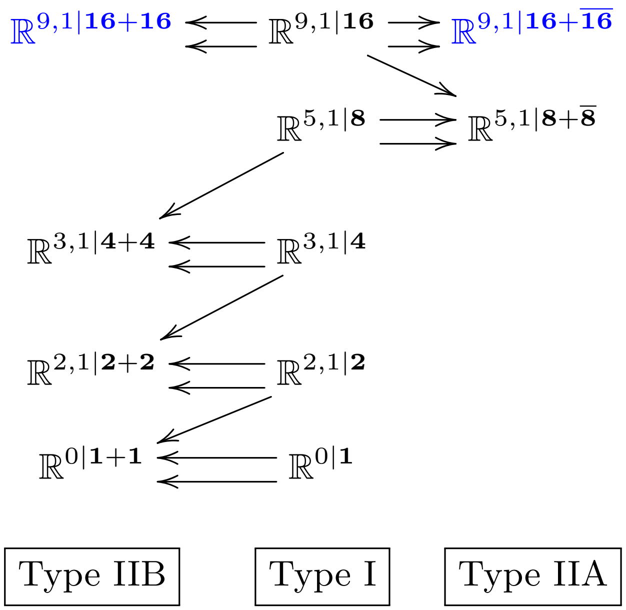
Emergence from the superpoint – The brane bouquet.



[HS17]

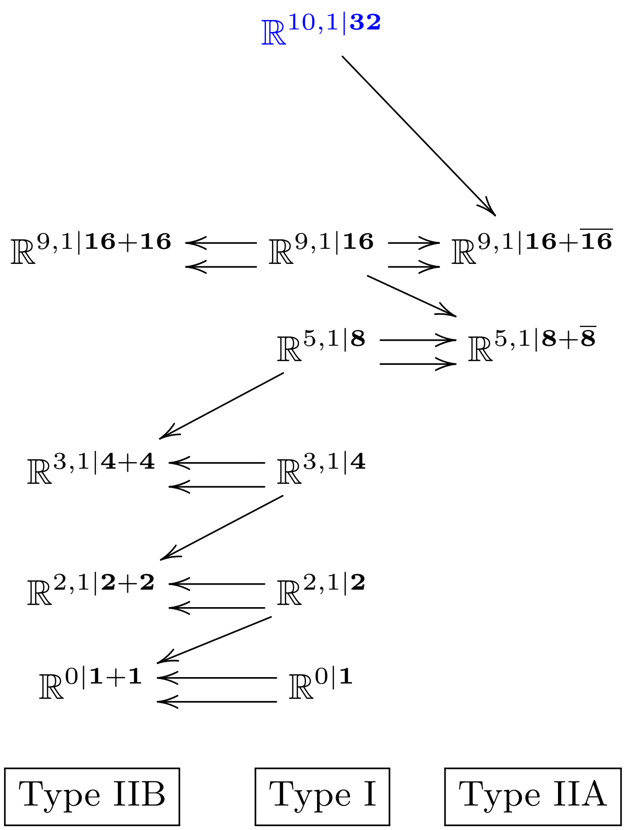
universal invariant central extension: 10d super-Minkowski spacetime

Emergence from the superpoint – The brane bouquet.



[HS17]

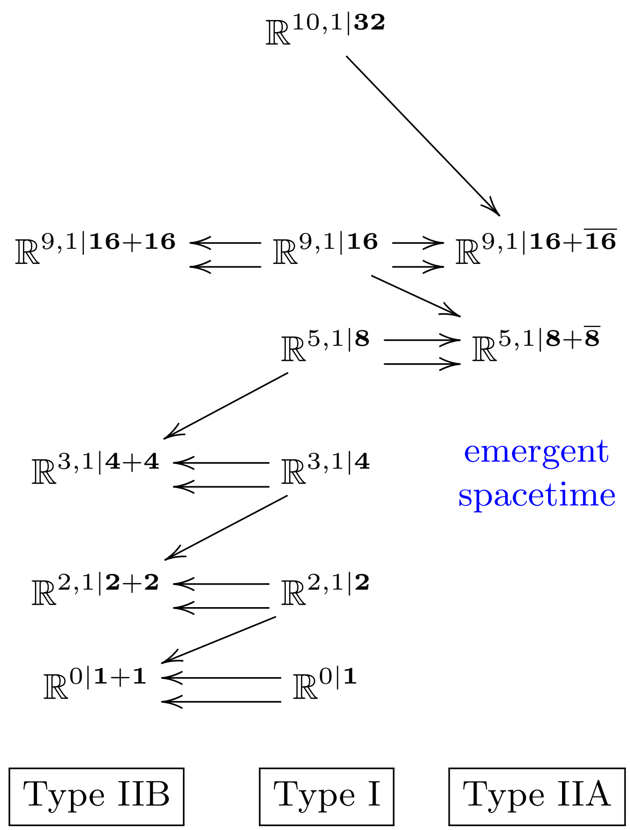
Emergence from the superpoint – The brane bouquet.



[HS17]

universal invariant central extension: 11d super-Minkowski spacetime

Emergence from the superpoint – The brane bouquet.



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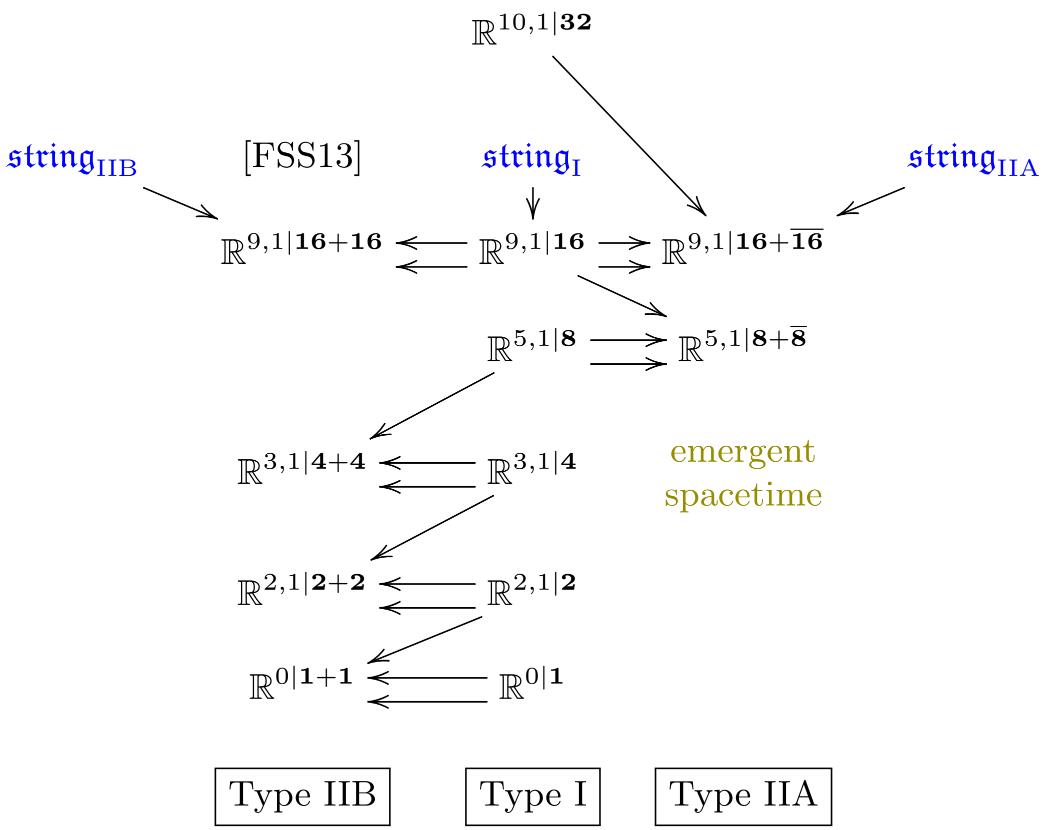
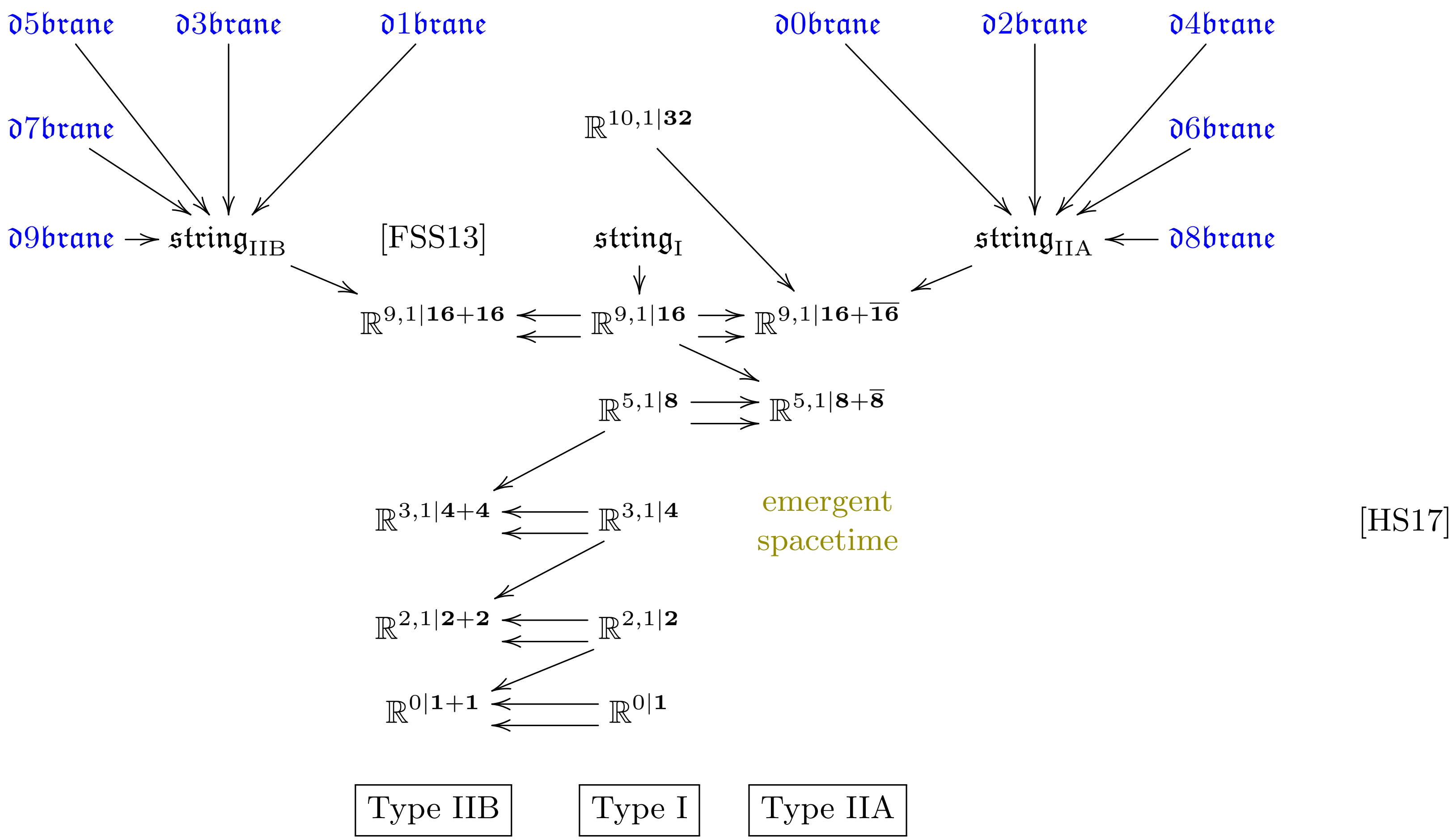


[HS17]

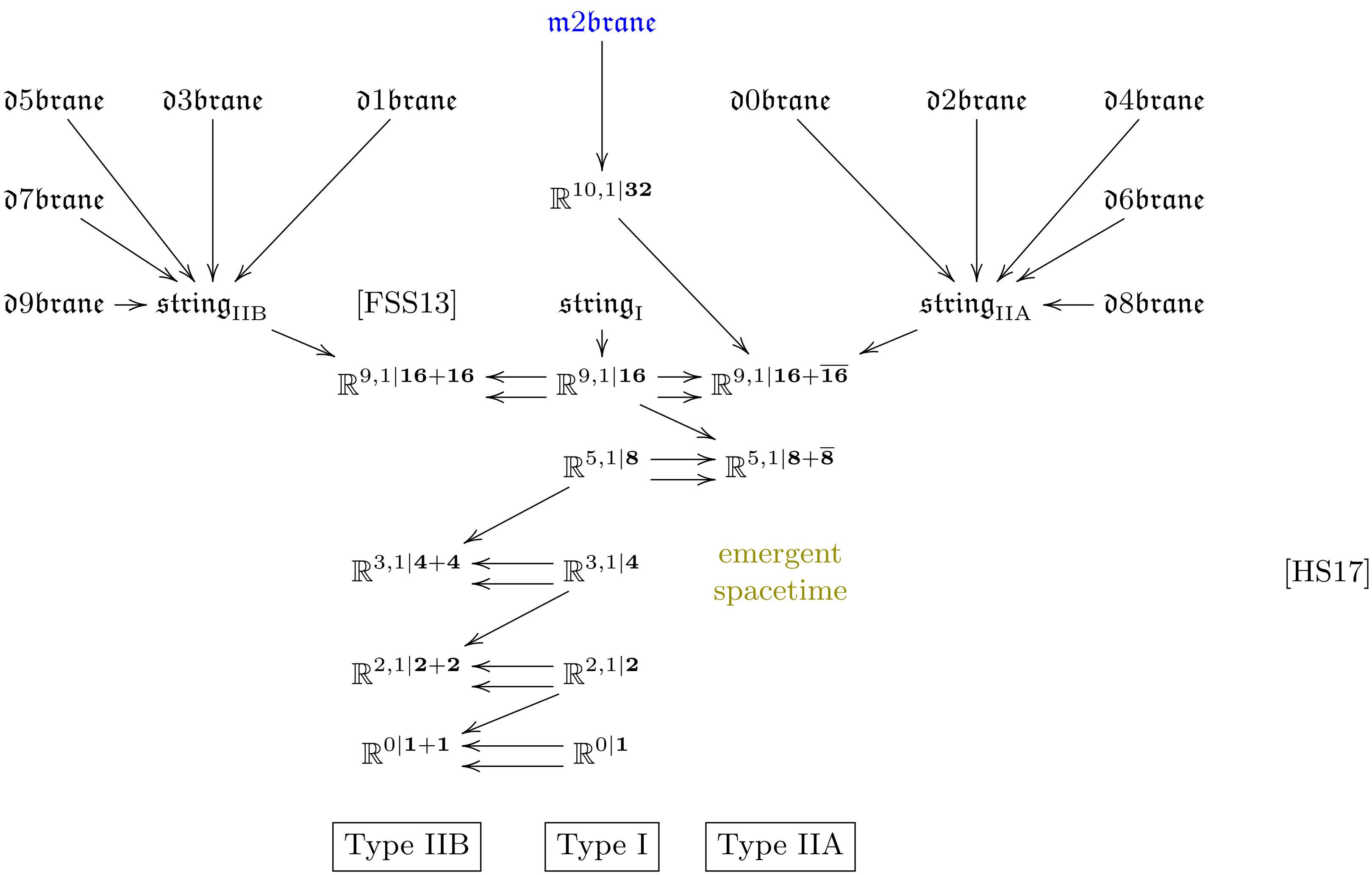
universal *higher* central invariant extension: stringy extended super-spacetimes

Emergence from the superpoint – The brane bouquet.



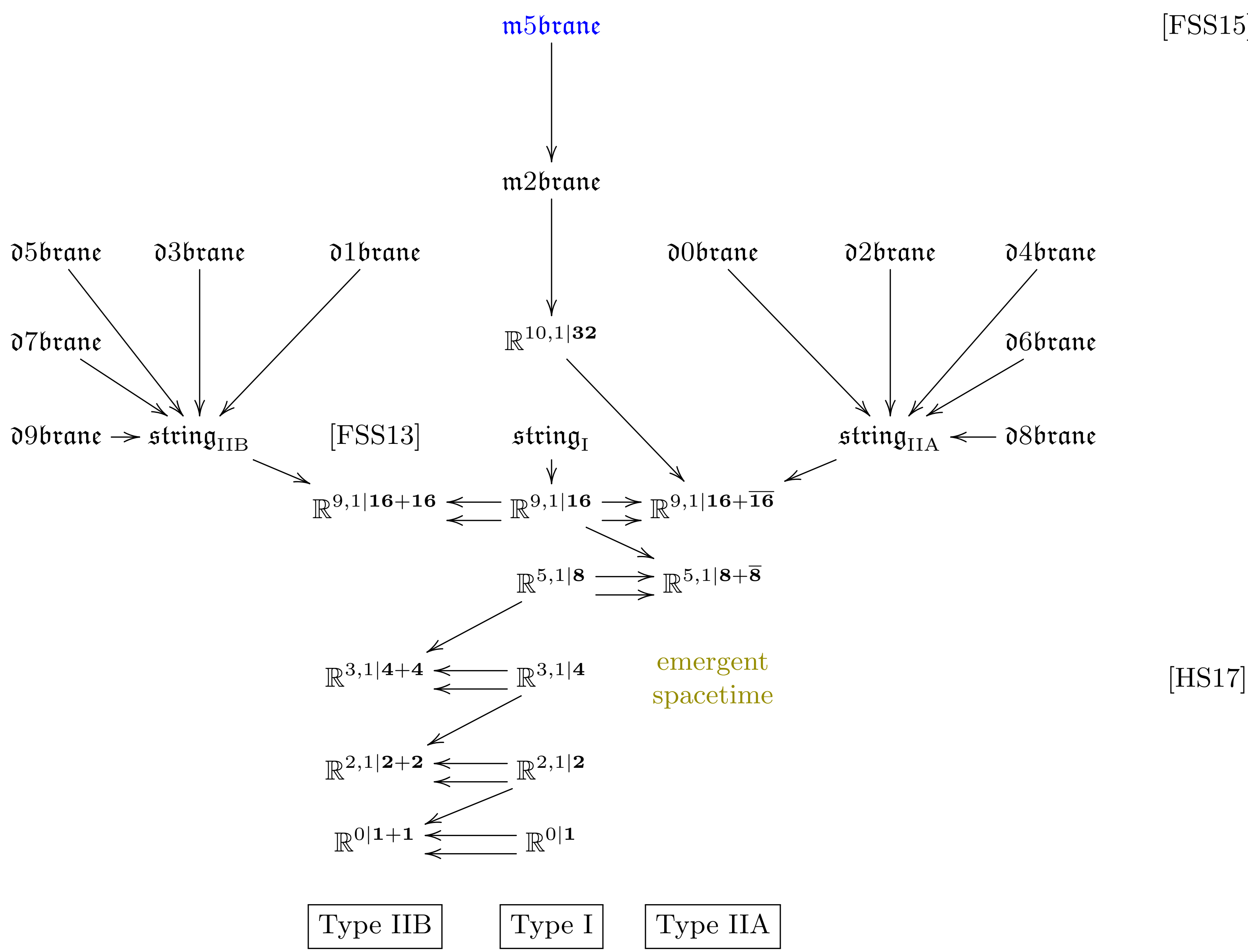
universal *higher* central invariant extension: D-brane extended super-spacetimes

Emergence from the superpoint – The brane bouquet.



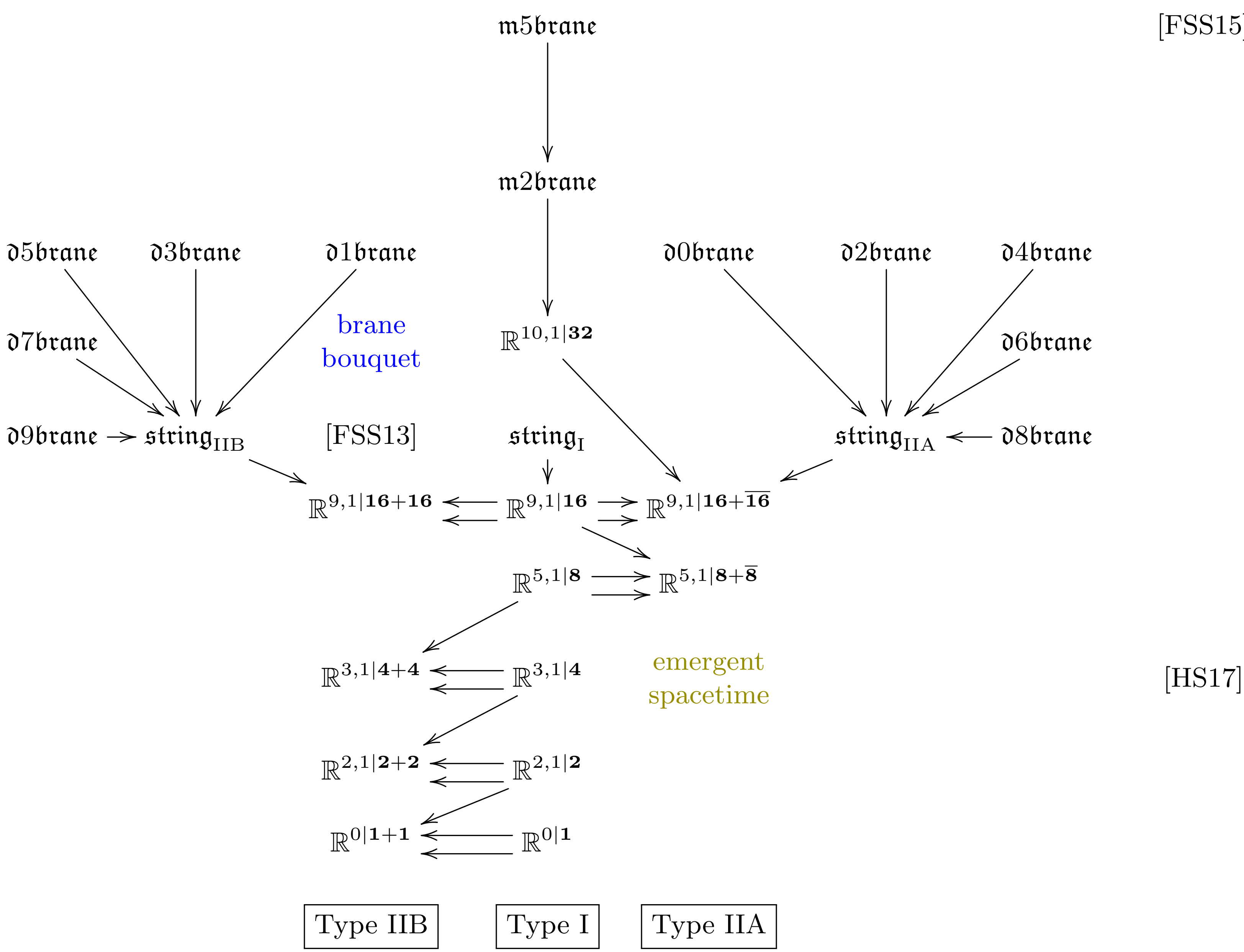
universal *higher* central invariant extension: M2-brane extended super-spacetimes

Emergence from the superpoint – The brane bouquet.

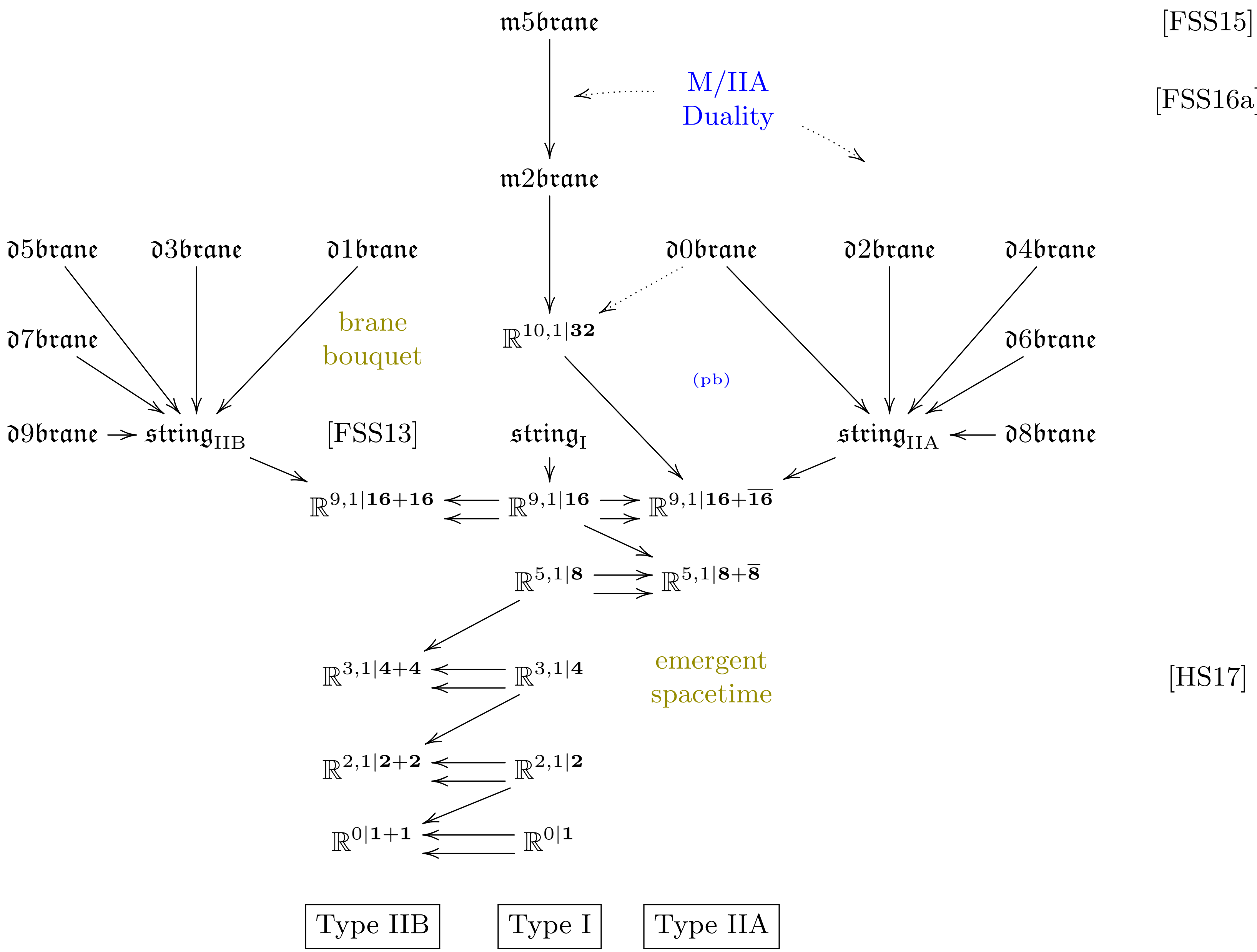


universal *higher* central invariant extension: M5-brane extended super-spacetimes

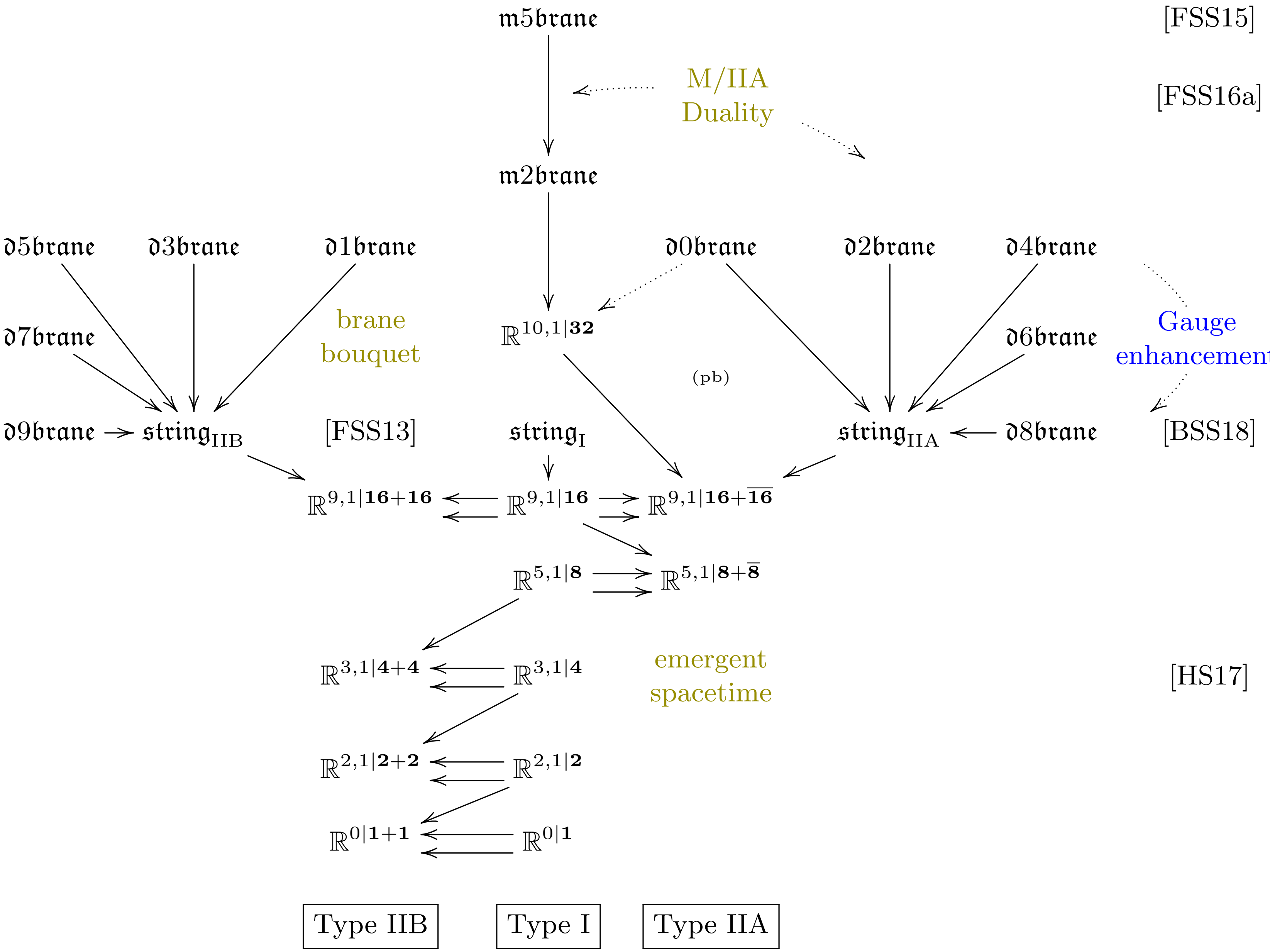
Emergence from the superpoint – The brane bouquet.



Emergence from the superpoint – The brane bouquet.



Emergence from the superpoint – The brane bouquet.



[FSS15]

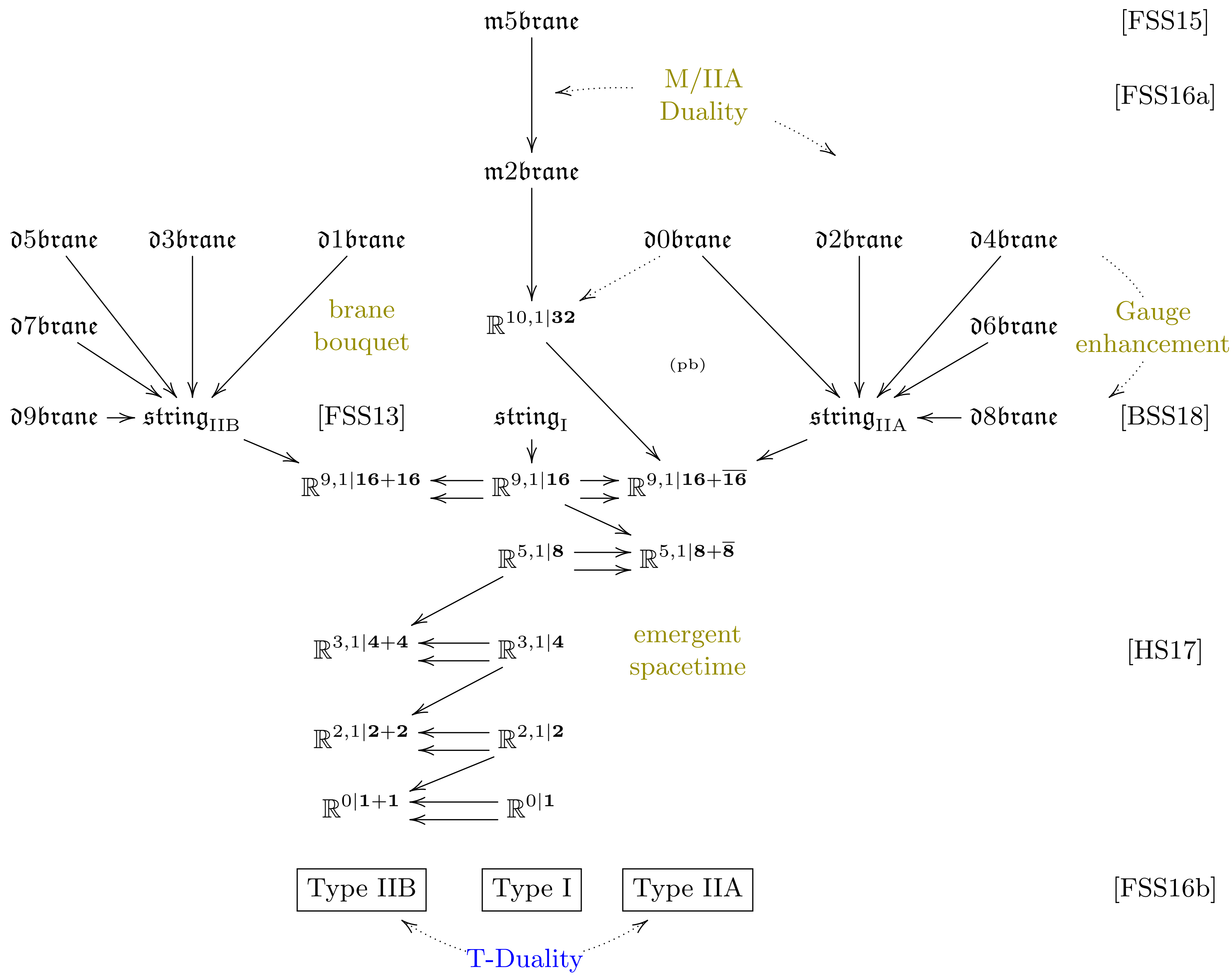
[FSS16a]

[FSS13]

[BSS18]

[HS17]

Emergence from the superpoint – The brane bouquet.



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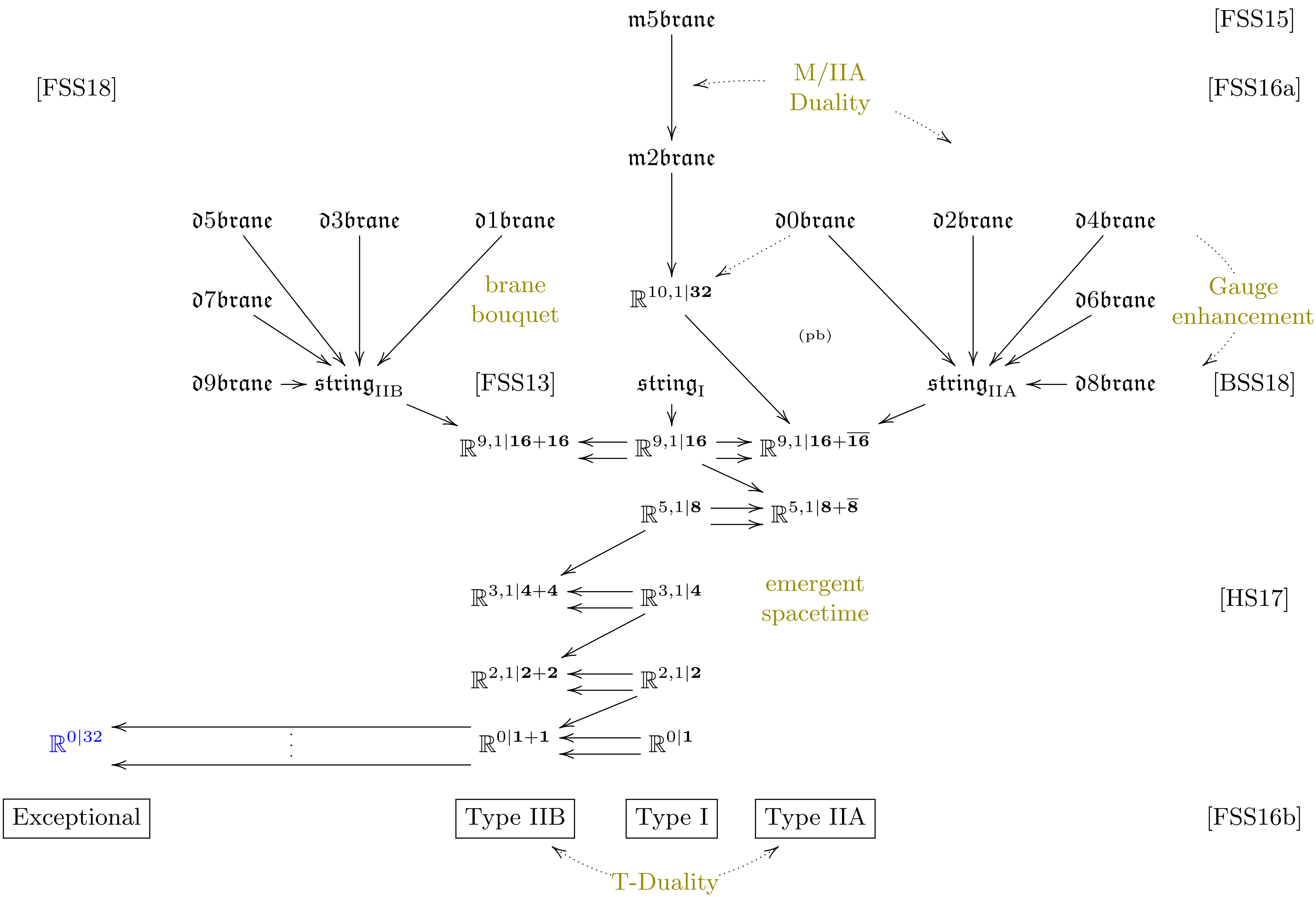

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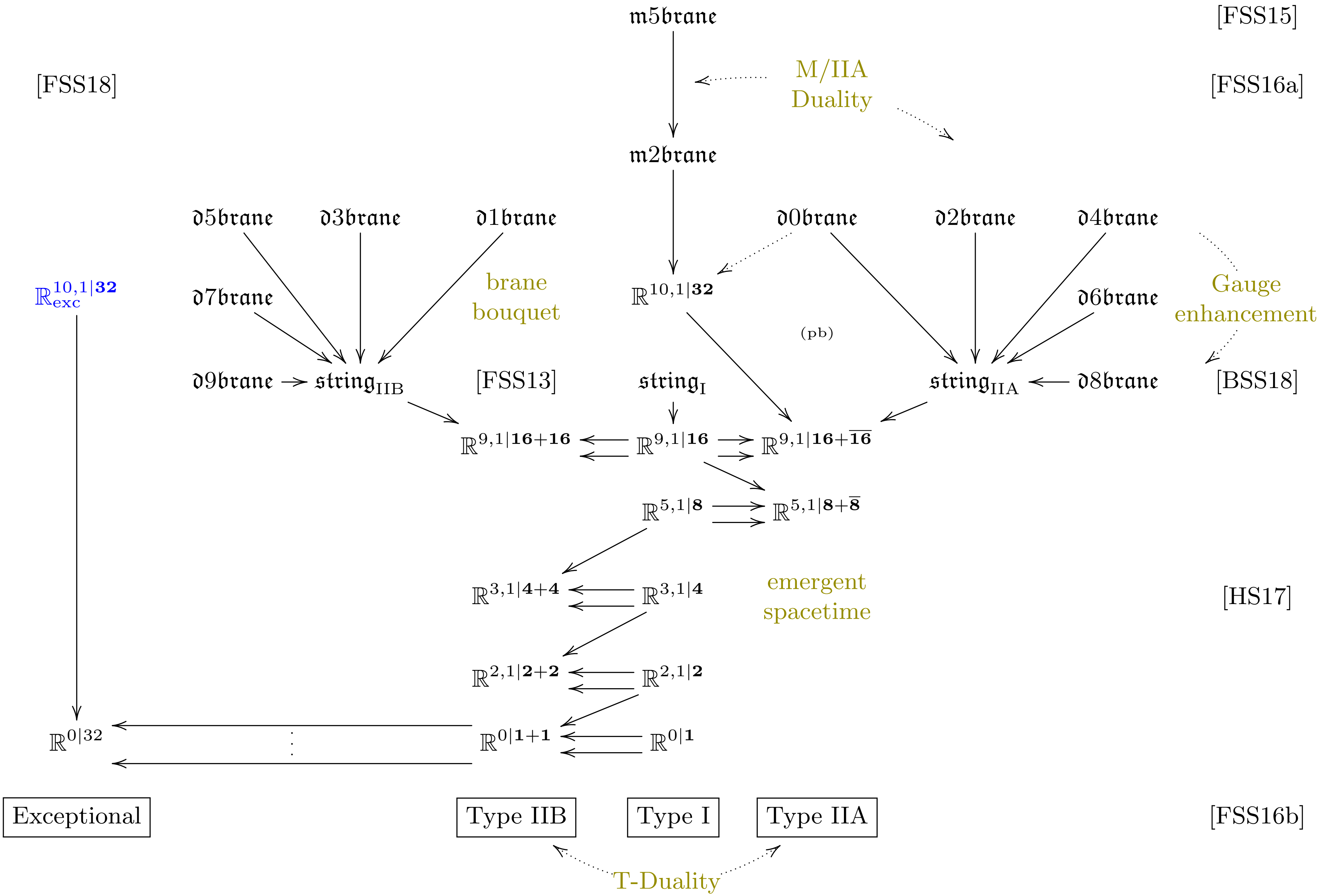

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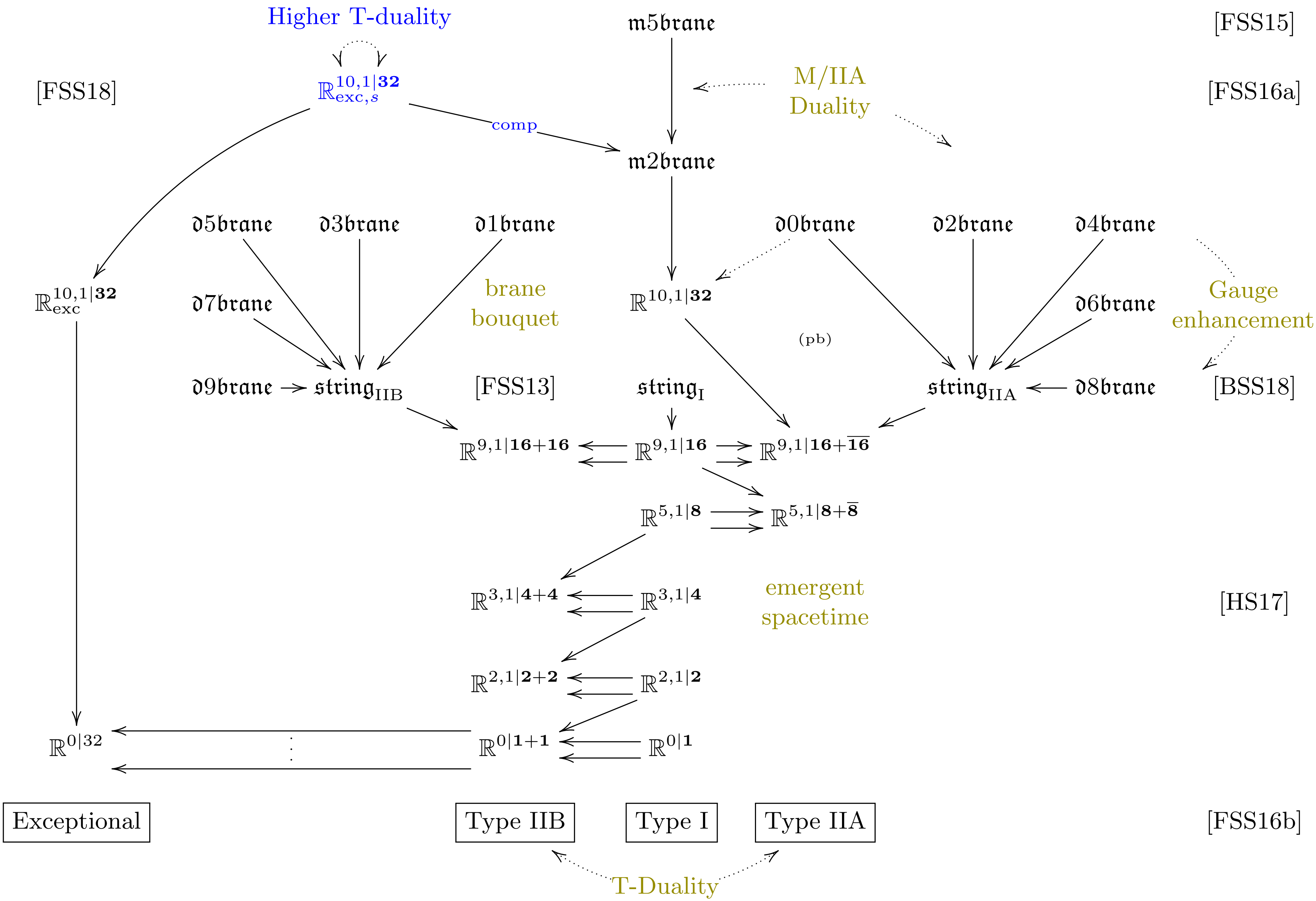

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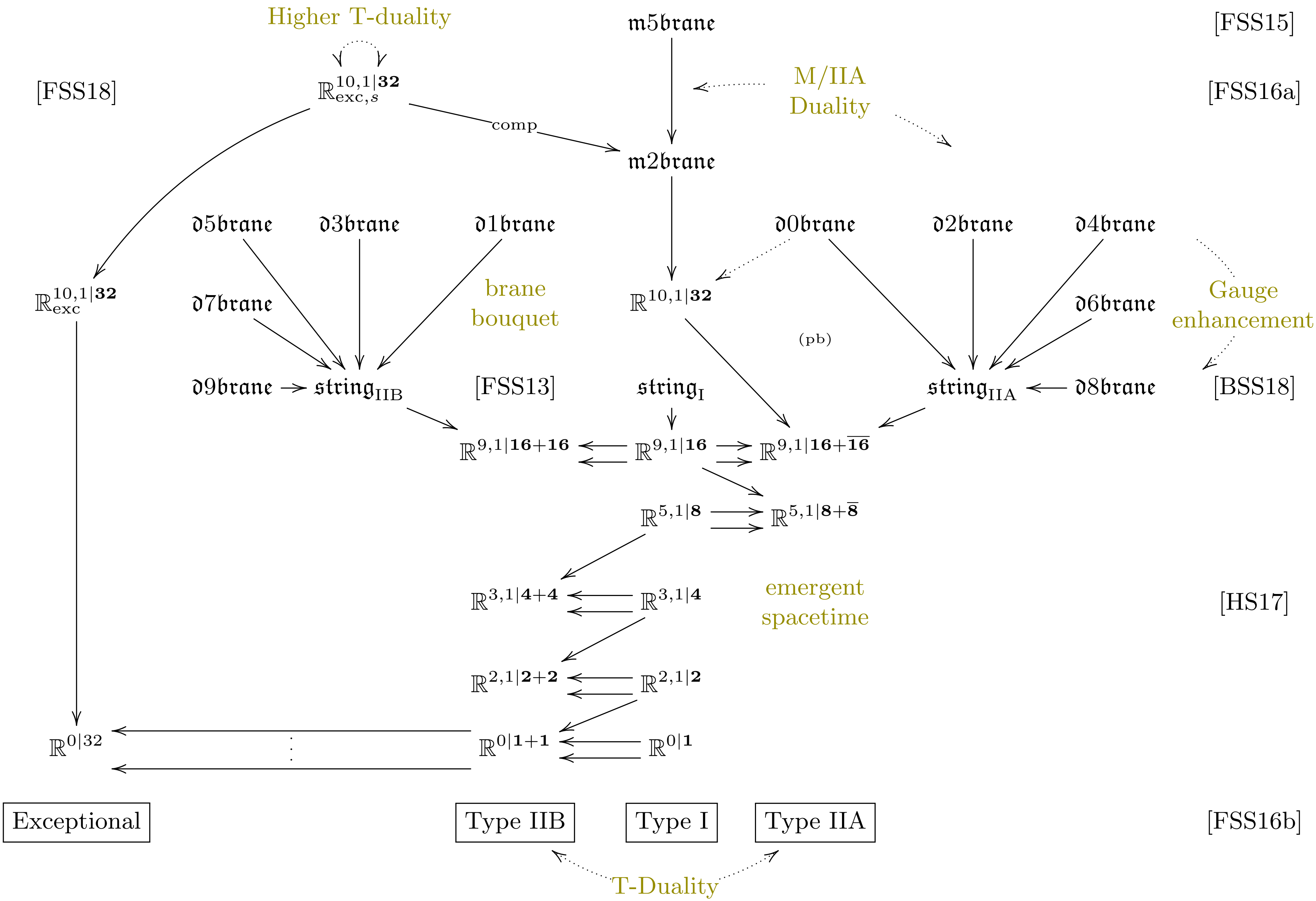

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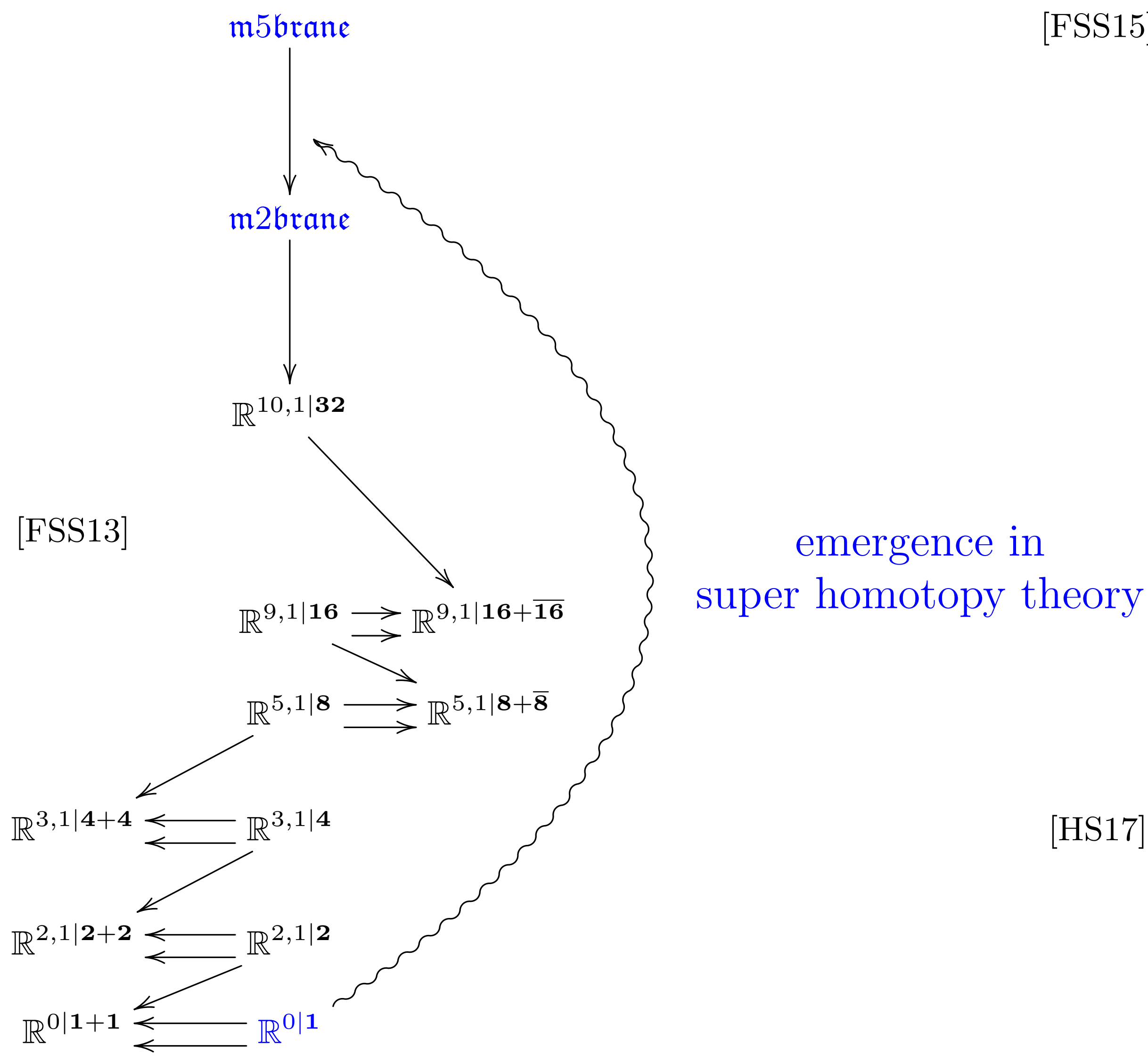


homotopical emergence of fundamental M-branes from the Atom of Superspace

Emergence from the superpoint – The brane bouquet.

[FSS15]

$$
\begin{array}{c}
\mathfrak{m5brane} \\
\downarrow \\
\mathfrak{m2brane} \\
\downarrow \\
\mathbb{R}^{10,1|\mathbf{32}}
\end{array}
$$

zoom in on the fundamental M-brane super-extensions

The fundamental M2/M5-brane cocycle

[FSS15]

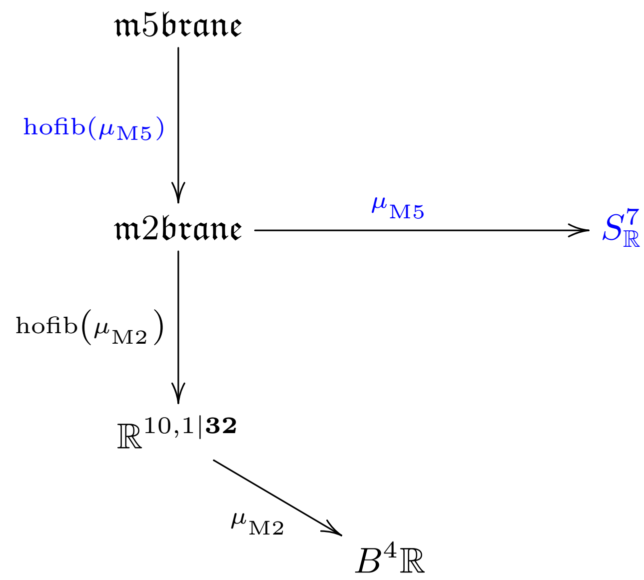
$$
\begin{array}{ccc}
\mathfrak{m5brane} & & \\
\downarrow & & \\
\mathfrak{m2brane} & & \\
\Big\downarrow{\scriptstyle \mathrm{hofib}(\mu_{\mathrm{M2}})} & & \\
\mathbb{R}^{10,1|\mathbf{32}} & \xrightarrow{\ \mu_{\mathrm{M2}}\ } & B^4\mathbb{R}
\end{array}
$$

$$\mu_{\mathrm{M2}} = dL^{\mathrm{WZW}}_{\mathrm{M2}} = \tfrac{i}{2}\left(\overline{\psi}\Gamma_{a_1 a_2}\psi\right) \wedge e^{a_1} \wedge e^{a_2}$$

the WZW-curvature of the Green-Schwarz-type sigma-model super-membrane

The fundamental M2/M5-brane cocycle

[FSS15]

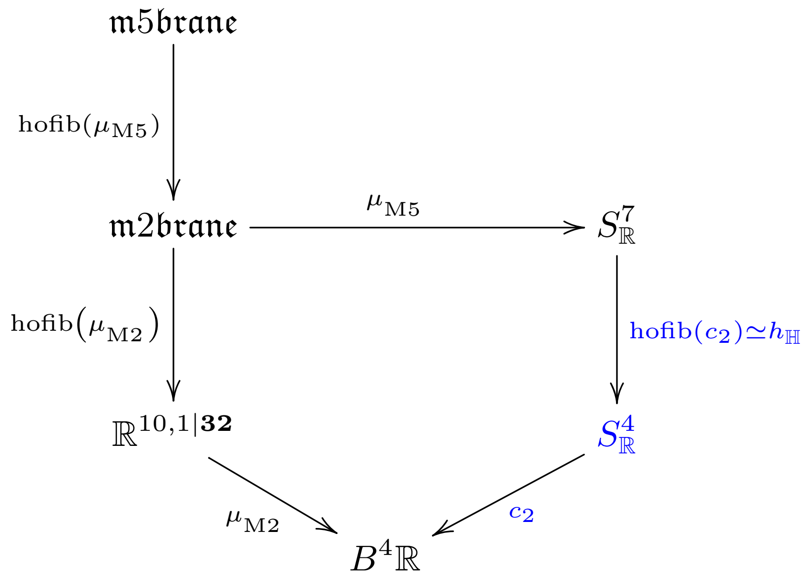


$$\mu_{\mathrm{M5}} = dL^{\mathrm{WZW}}_{\mathrm{M5}} = \tfrac{1}{5!}\left(\overline{\psi}\Gamma_{a_1\cdots a_5}\psi\right) \wedge e^{a_1} \wedge \cdots \wedge e^{a_5} + c_3 \wedge \tfrac{i}{2}\left(\overline{\psi}\Gamma_{a_1 a_2}\psi\right) \wedge e^{a_1} \wedge e^{a_2}$$

the WZW-curvature of the Green-Schwarz-type sigma-model super-fivebrane

The fundamental M2/M5-brane cocycle

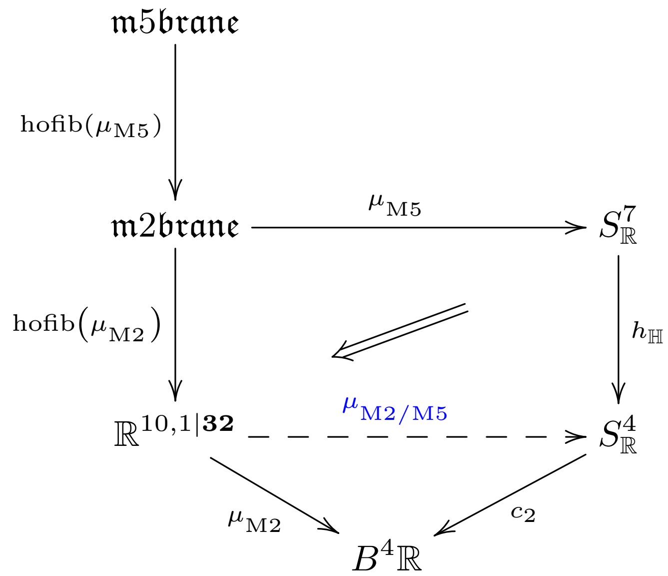
[FSS15]



the quaternionic Hopf fibration $h_{\mathbb{H}}$

The fundamental M2/M5-brane cocycle

[FSS15]



the unified M2/M5-cocycle

The fundamental M2/M5-brane cocycle

[FSS15]

$$\mathbb{R}^{10,1|\mathbf{32}} \xrightarrow{\mu_{\mathrm{M2/M5}}} S^4_{\mathbb{R}}$$

the unified M2/M5-cocycle is in rational Cohomotopy in degree 4

The fundamental M2/M5-brane cocycle

[FSS15]

$$\mathbb{R}^{10,1|\mathbf{32}} \xrightarrow{\;\mu_{\mathrm{M2/M5}}\;} S^4_{\mathbb{R}}$$

$$\tfrac{i}{2}\left(\overline{\psi}\Gamma_{a_1 a_2}\psi\right) \wedge e^{a_1} \wedge e^{a_2} \longleftarrow\!\dashv G_4$$

$$\tfrac{1}{5!}\left(\overline{\psi}\Gamma_{a_1 \cdots a_5}\psi\right) \wedge e^{a_1} \cdots e^{a_5} \longleftarrow\!\dashv G_7$$

Sullivan model: $\mathcal{O}\left(S^4_{\mathbb{R}}\right) \simeq \mathbb{R}[G_4, G_7] / \begin{pmatrix} dG_4 = 0 \\ dG_7 = -\frac{1}{2} G_4 \wedge G_4 \end{pmatrix}$

= 11d supergravity equations of motion of the C-field ([Sati13, Sect. 2.5])

The fundamental M2/M5-brane cocycle

[FSS15]

$$\mathbb{R}^{10,1|\mathbf{32}} \xrightarrow{\mu_{\mathrm{M2/M5}}} S^4_{\mathbb{R}}$$

the unified M2/M5-cocycle

The fundamental M2/M5-brane cocycle

[FSS15]

$$\mathbb{R}^{10,1|\mathbf{32}} \xrightarrow{\;\mu_{\mathrm{M2/M5}}\;} S^4_{\mathbb{R}}$$

$$\Big\downarrow \text{ double dimensional reduction \& gauge enhancement}$$

$$\mathbb{R}^{9,1|\mathbf{16}+\overline{\mathbf{16}}} \xrightarrow[\;\mu_{F1/D_{2p}}\;]{} \mathrm{ku}_{/\!/B^2\mathbb{R}}$$

D-brane charge in twisted K-theory, rationally

[BSS18]

The rational conclusion [FSS19a]

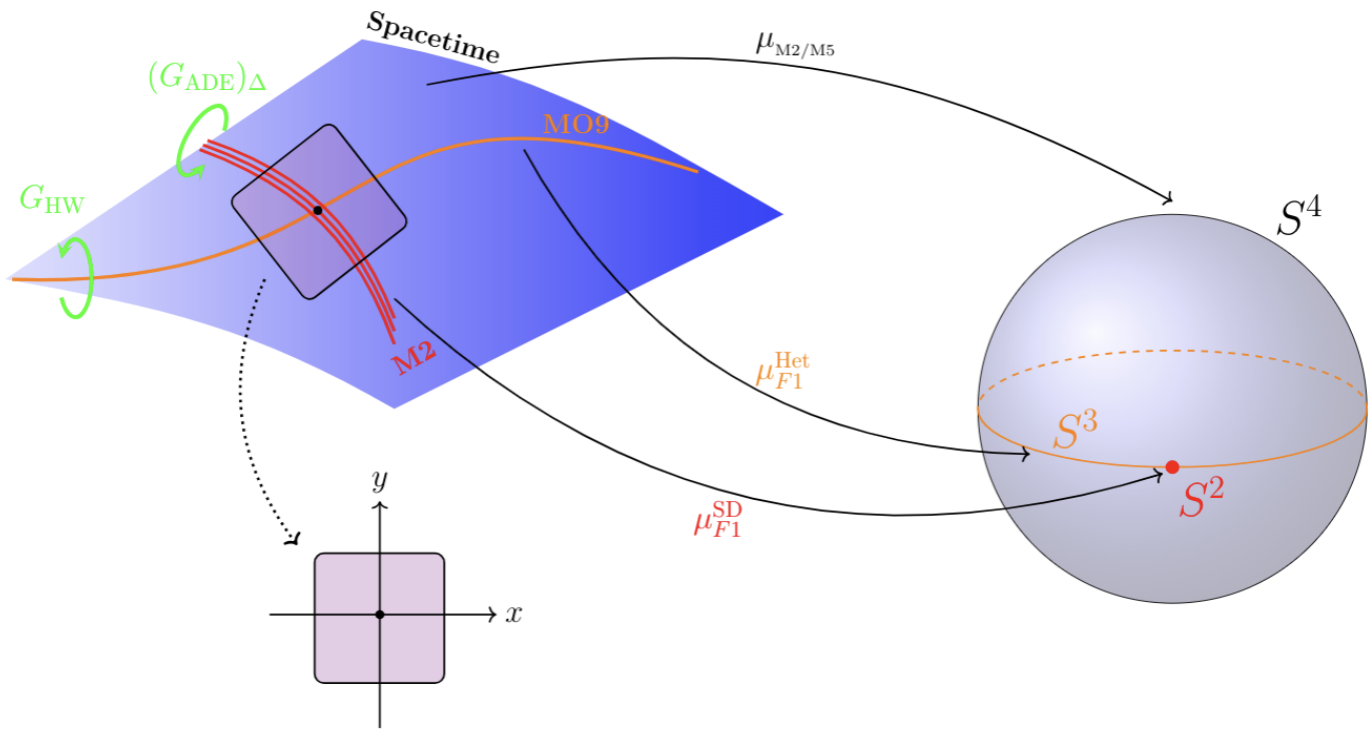
$$\text{In}\left\{\begin{array}{c}\text{infinitesimal}\\ \text{rational}\end{array}\right\}\text{approximation}$$

brane charge quantization follows from first principles

and reveals this situation:

brane species	**cohomology theory of charge quantization**
D-branes	twisted K-theory
M-branes	twisted Cohomotopy

1. *Hypothesis H:* C-field is a cocycle in twisted differential Cohomotopy of 11d super-orbifold spacetimes
2. lifting super-tangent-space-wise the fundamental M2/M5-brane cocycle.



3. *Check:* Compare the resulting rigorous observables to the M-theory folklore.

topological sector [FSS19b] →

super
spacetime X

Consider a spacetime manifold...

super
spacetime X

frame bundle TX

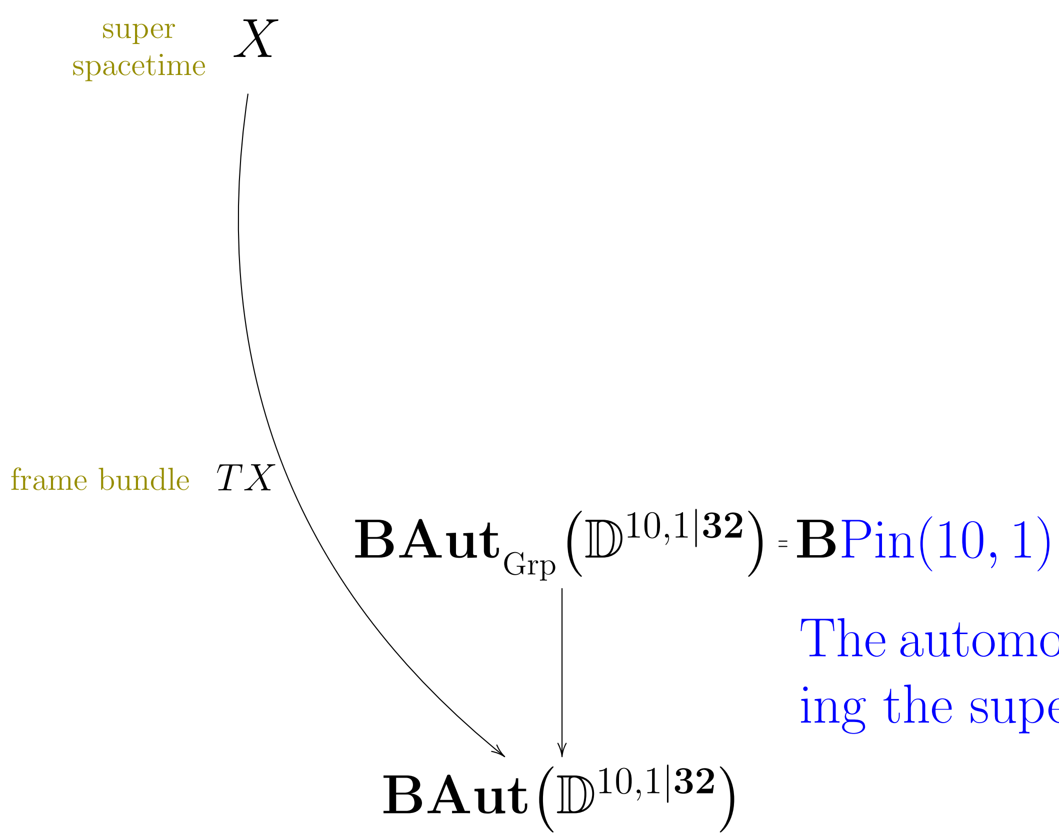
Consider a spacetime manifold
and its frame bundle.

super spacetime X

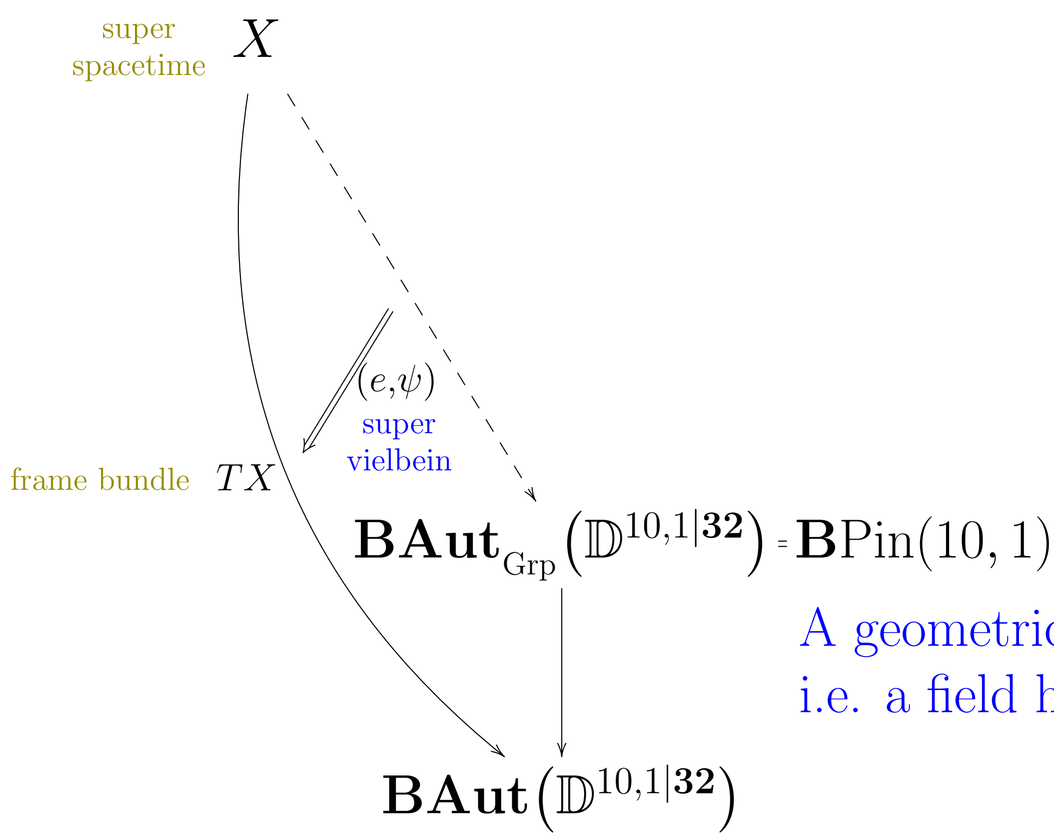
frame bundle TX

$\mathbf{BAut}\big(\mathbb{D}^{10,1|\mathbf{32}}\big)$

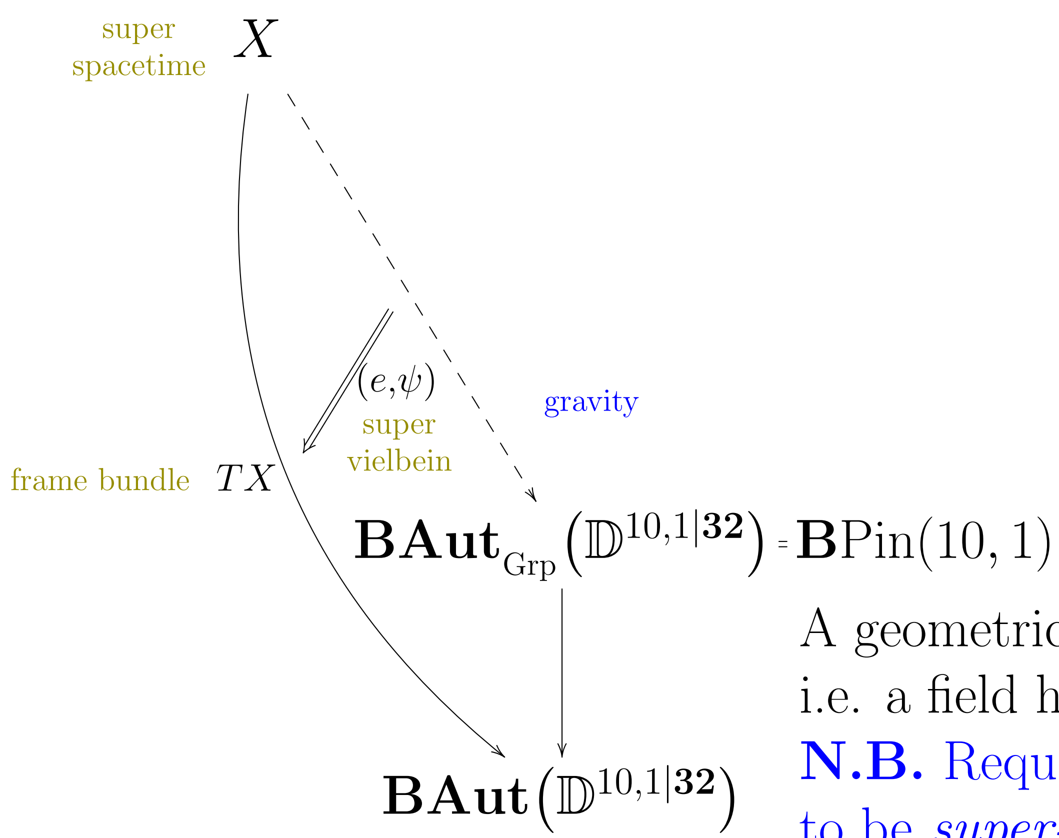
Consider a spacetime manifold
and its frame bundle.
The frames are infinitesimal neighbourhoods $\mathbb{D}^{10,1|\mathbf{32}}$ in super-Minkowski spacetime $\mathbb{R}^{10,1|\mathbf{32}}$.



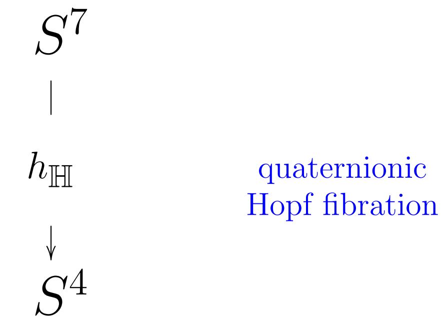
The automorphism group of $\mathbb{D}^{10,1|\mathbf{32}}$ preserving the super-group structure is $\mathrm{Pin}(10,1)$

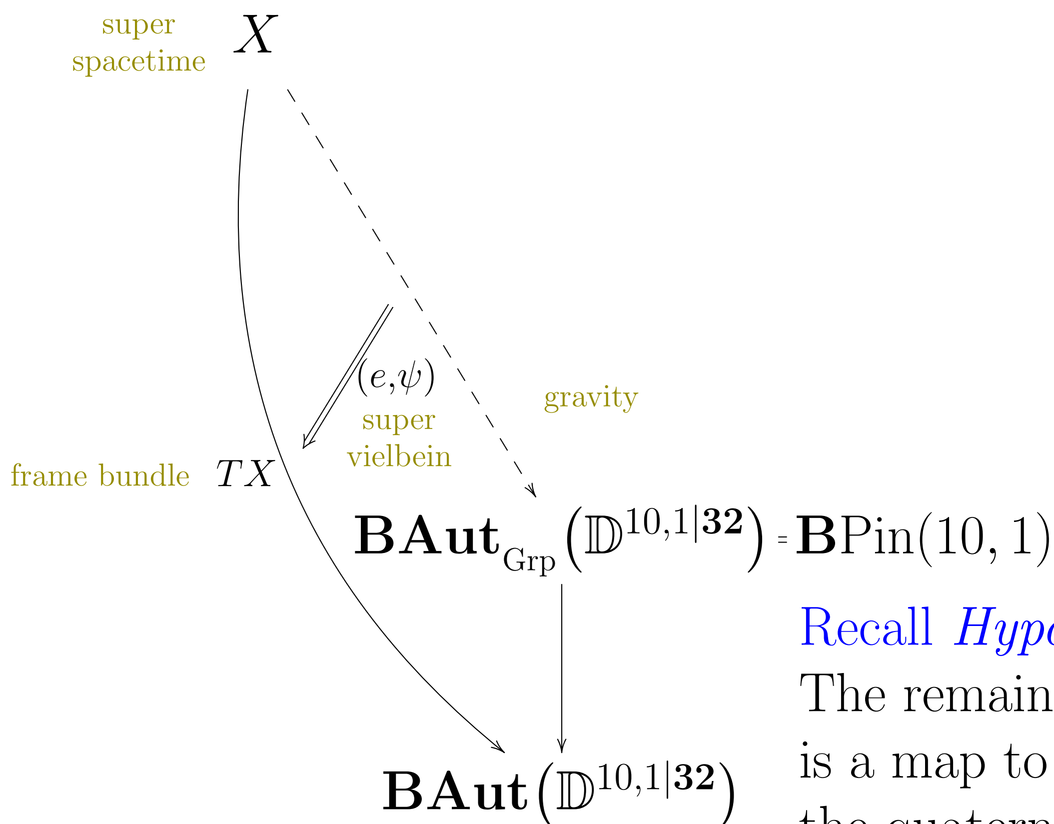


A geometric lift is a super-vielbein field
i.e. a field history of 11d supergravity.

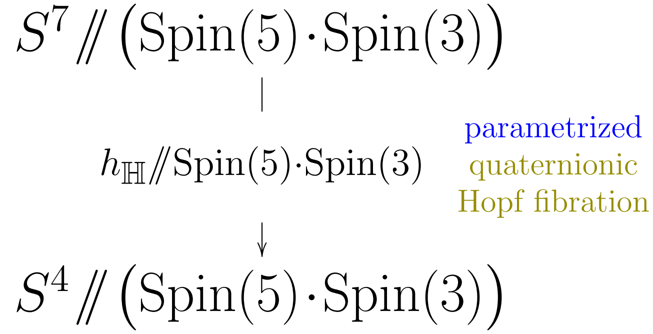


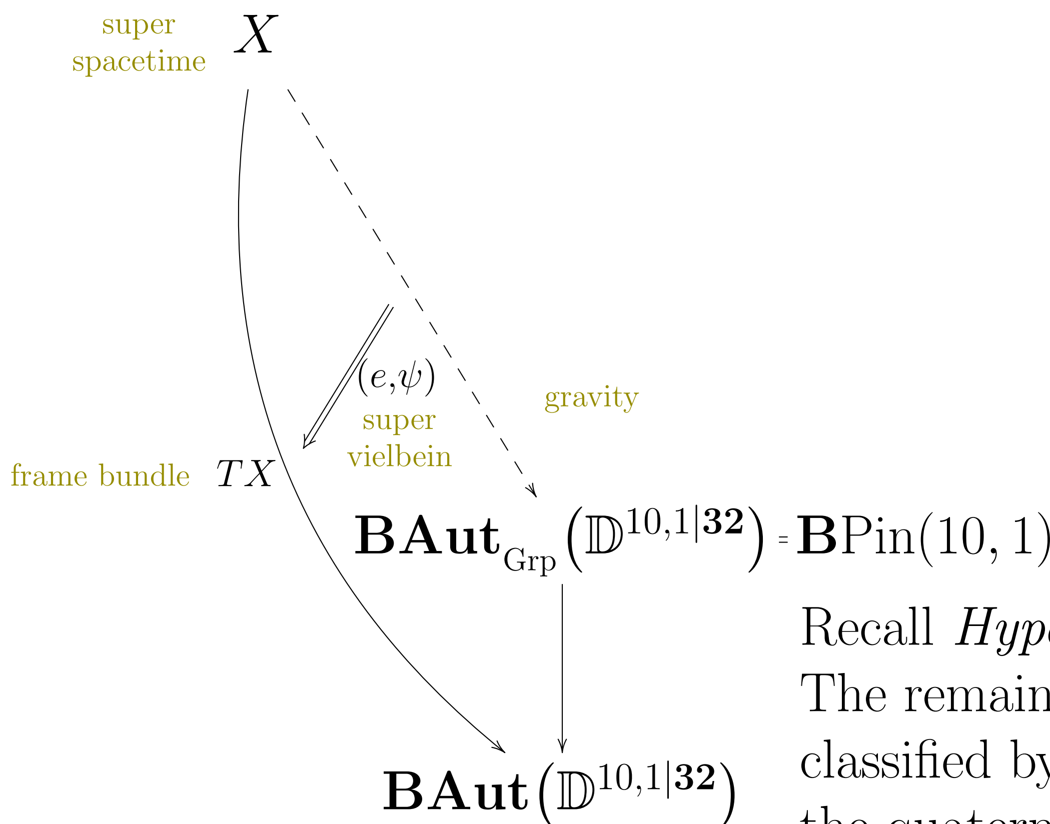
A geometric lift is a super-vielbein field.
i.e. a field history of 11d supergravity.
N.B. Requiring this G-structure
to be *super*-torsion free
is equivalent to 11d Einstein equations!
[CaLe93, How97], see [CGMT05, 2.4]





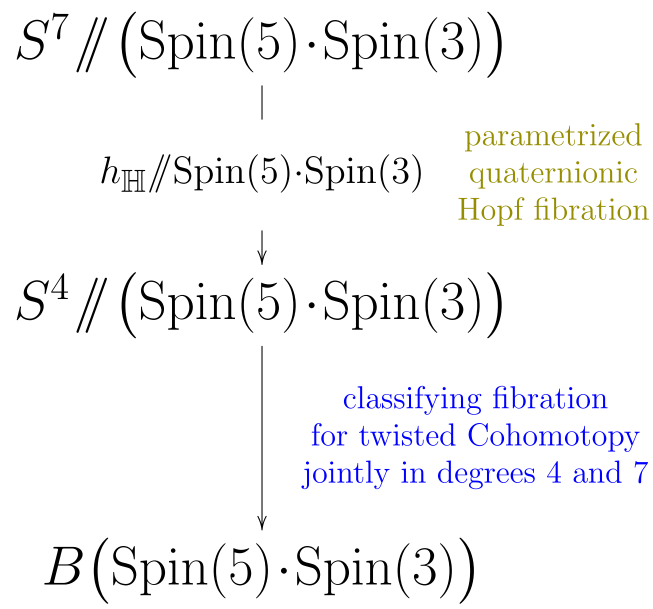
Recall *Hypothesis H* [FSS19b]:
The remaining higher gauge field (C-field)
is a map to the (co-)domain of
the quaternionic Hopf fibration.

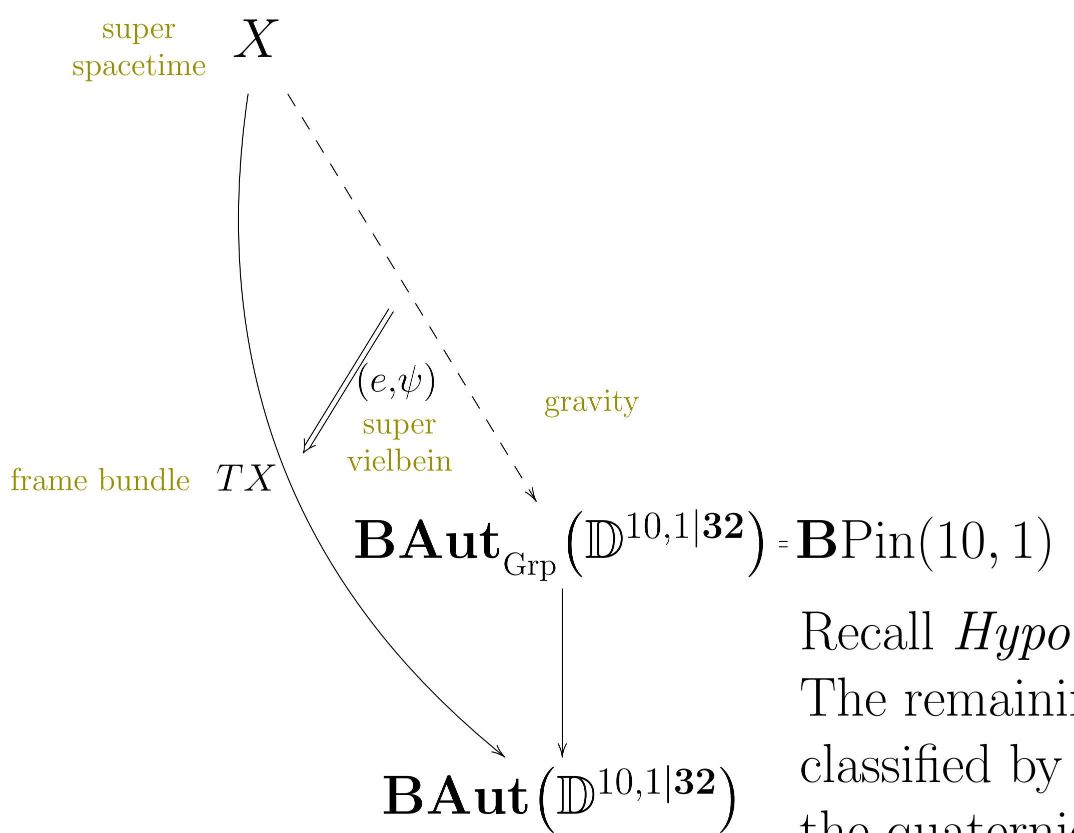




Recall *Hypothesis H* [FSS19b]:
The remaining higher gauge field (C-field) classified by map to the (co-)domain of the quaternionic Hopf fibration.

Prop. $\Rightarrow$ Global twist is in
$\mathrm{Spin}(5)\cdot\mathrm{Spin}(3) \simeq \mathrm{Sp}(2)\cdot\mathrm{Sp}(1)$





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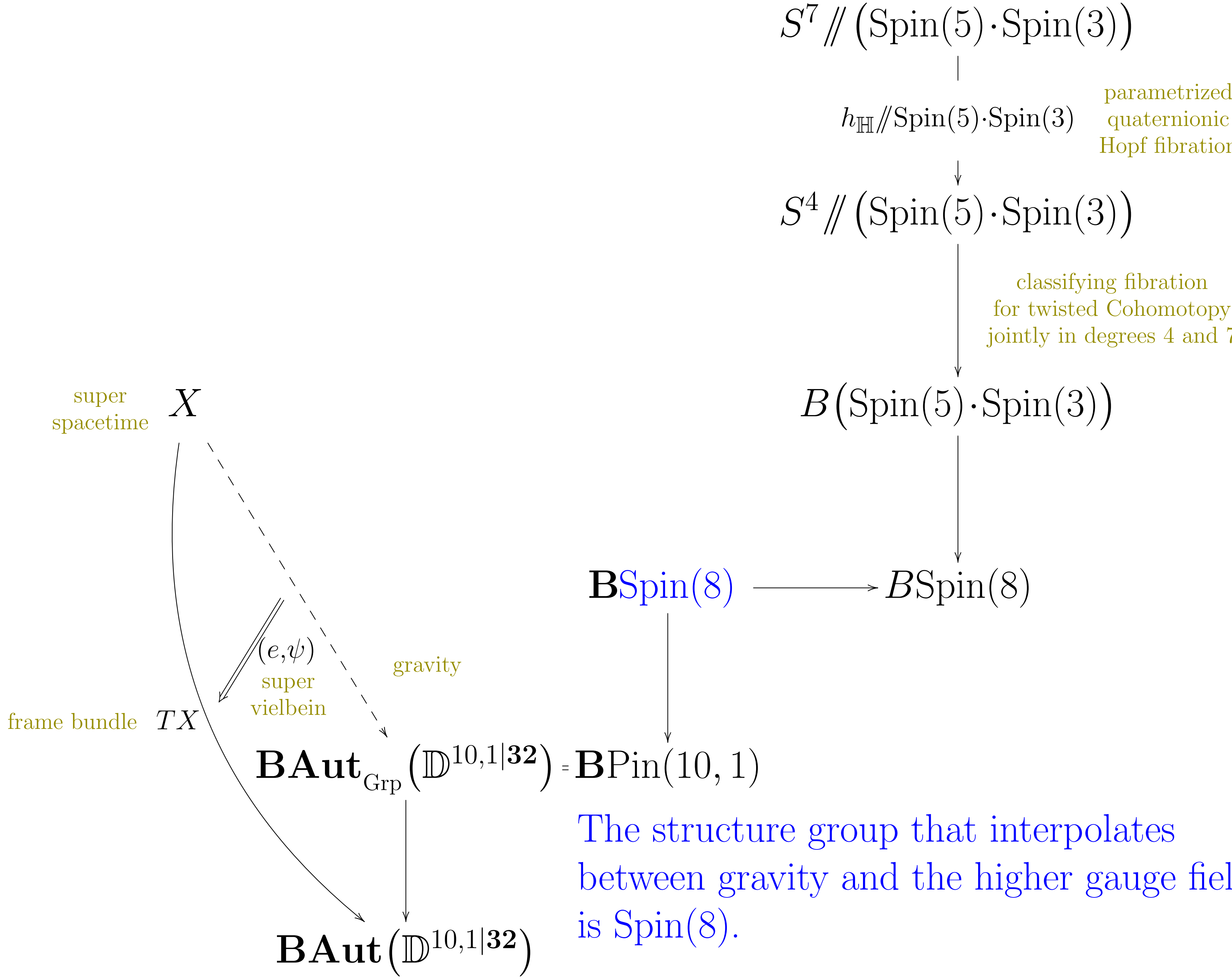
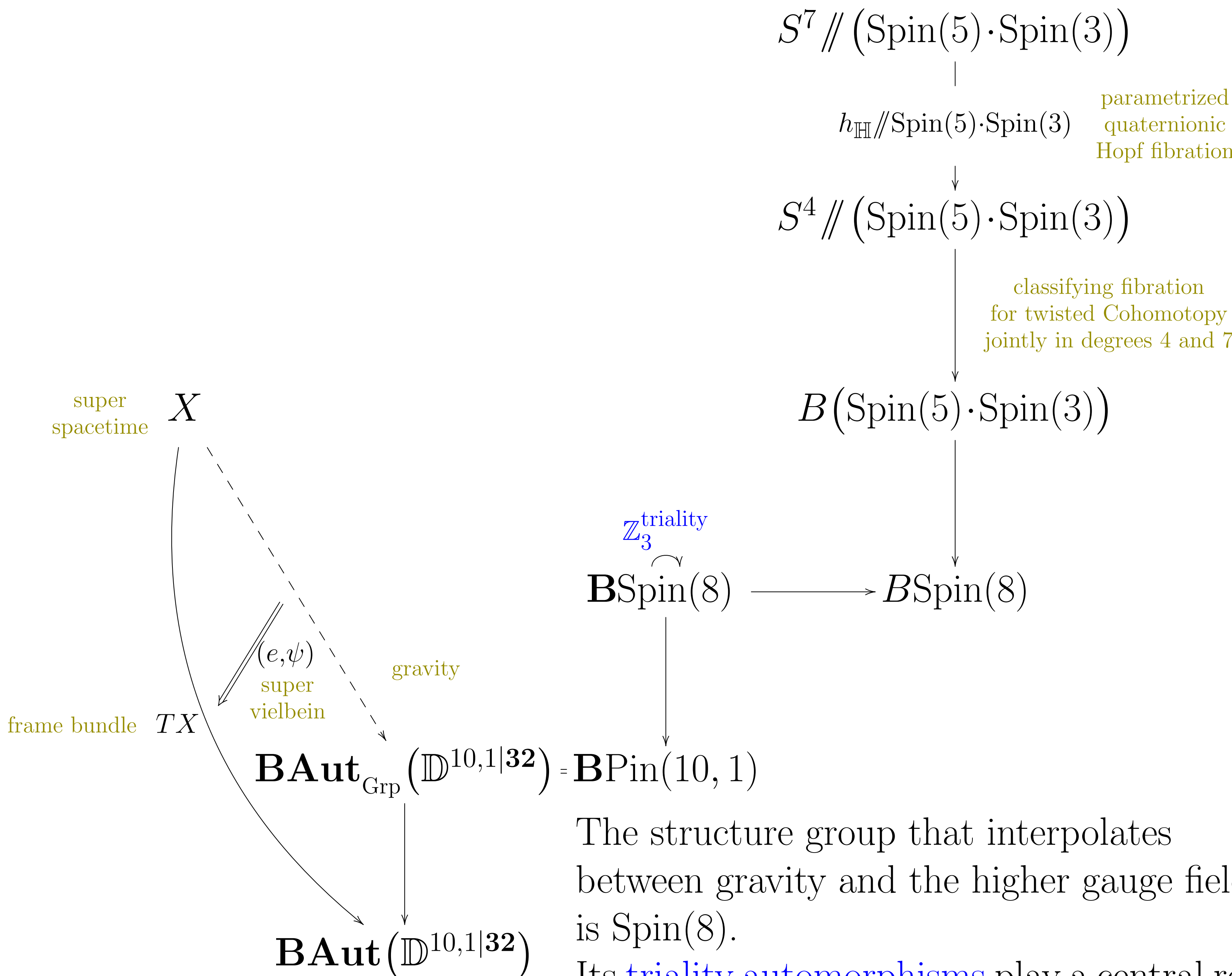
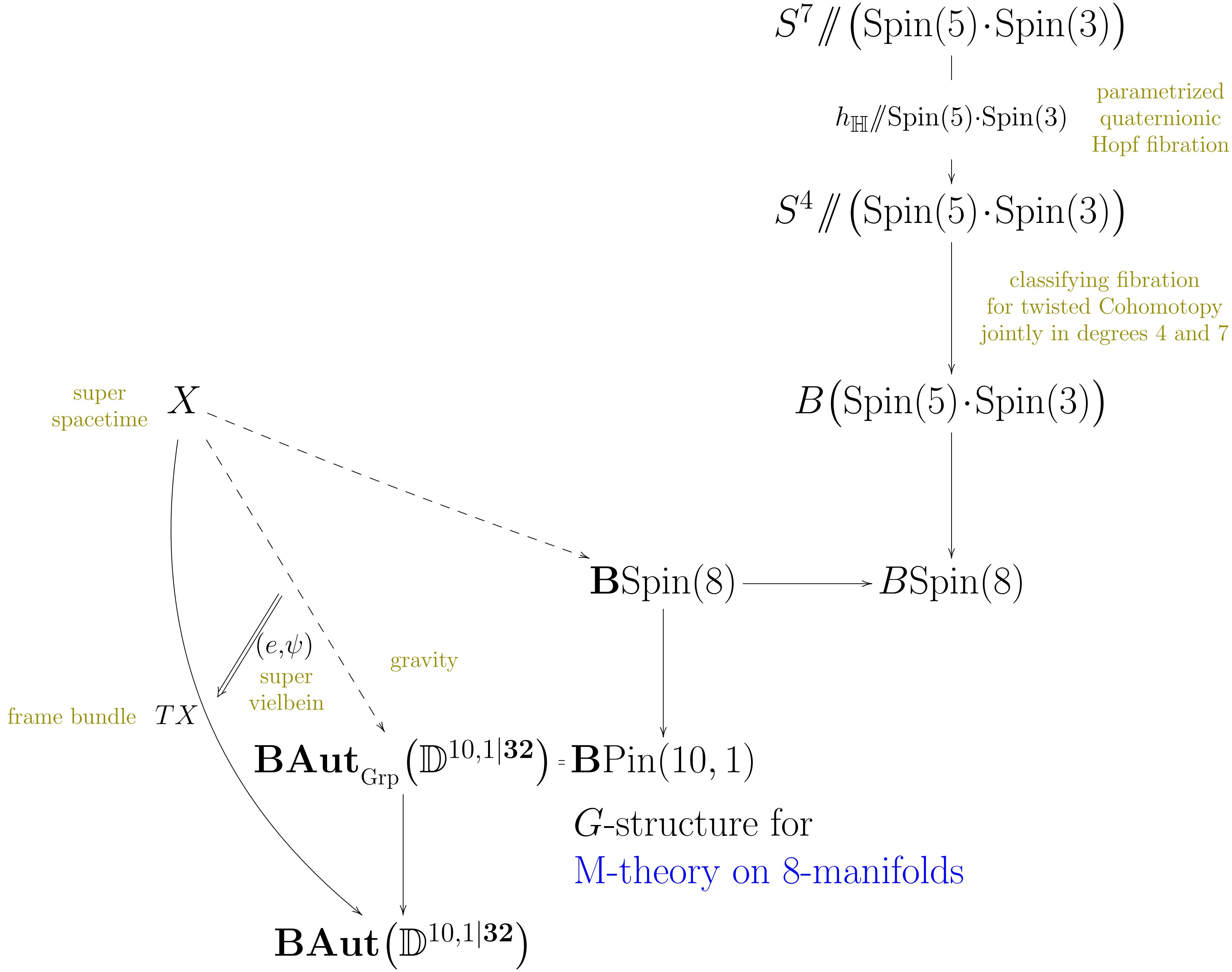


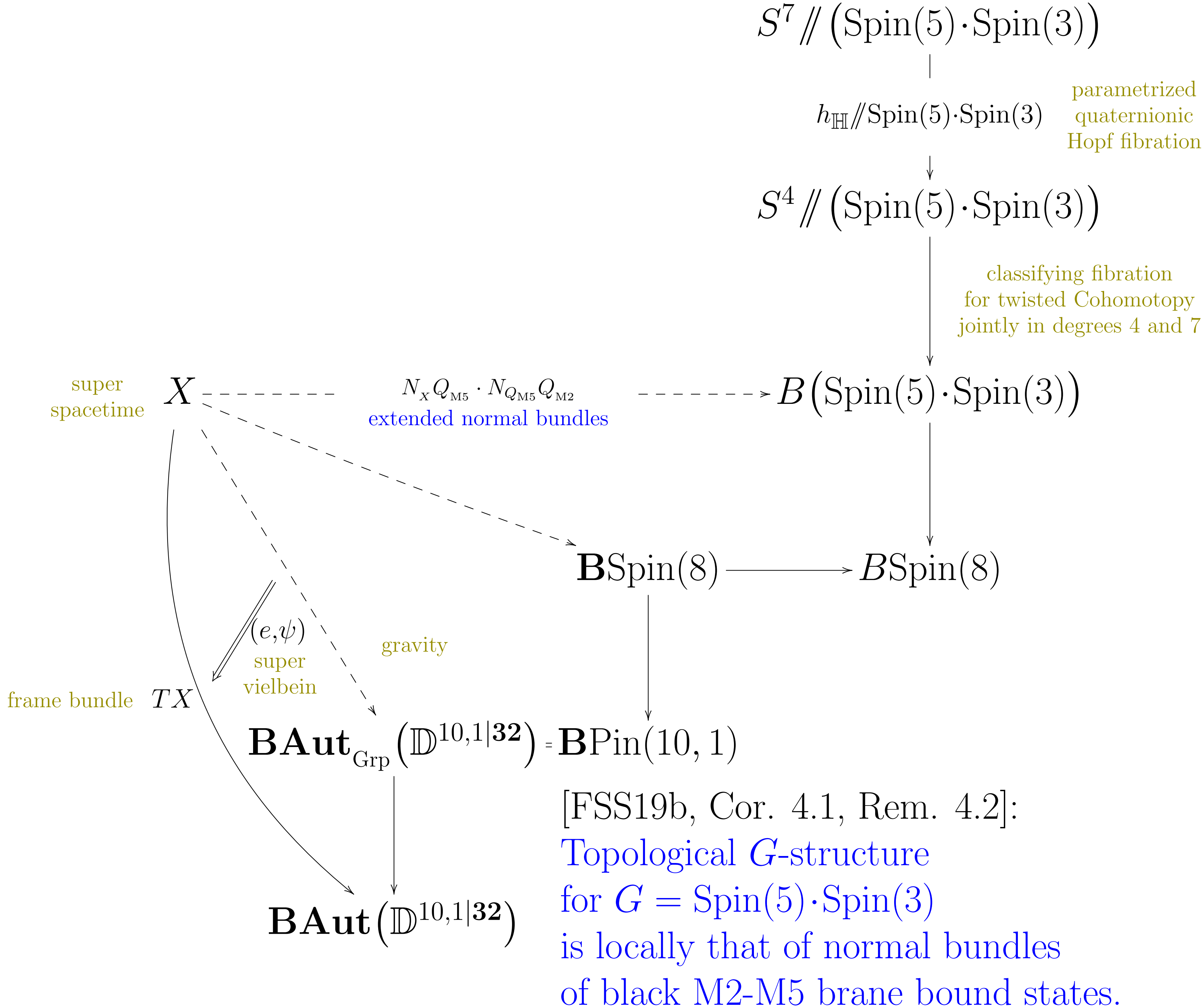
The structure group that interpolates between gravity and the higher gauge field is Spin(8).



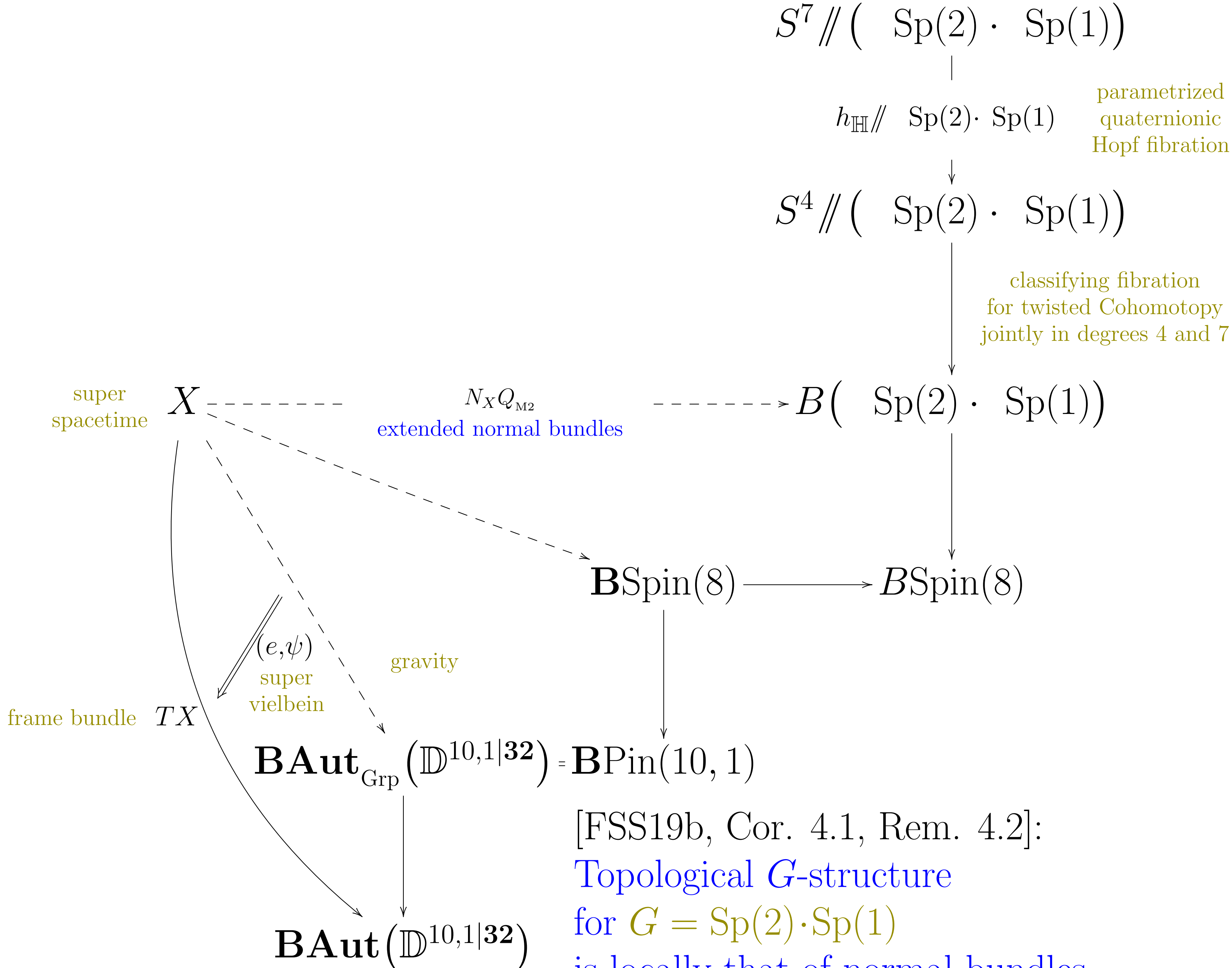
The structure group that interpolates between gravity and the higher gauge field is Spin(8).
Its triality automorphisms play a central role (glossed over here, for exposition).



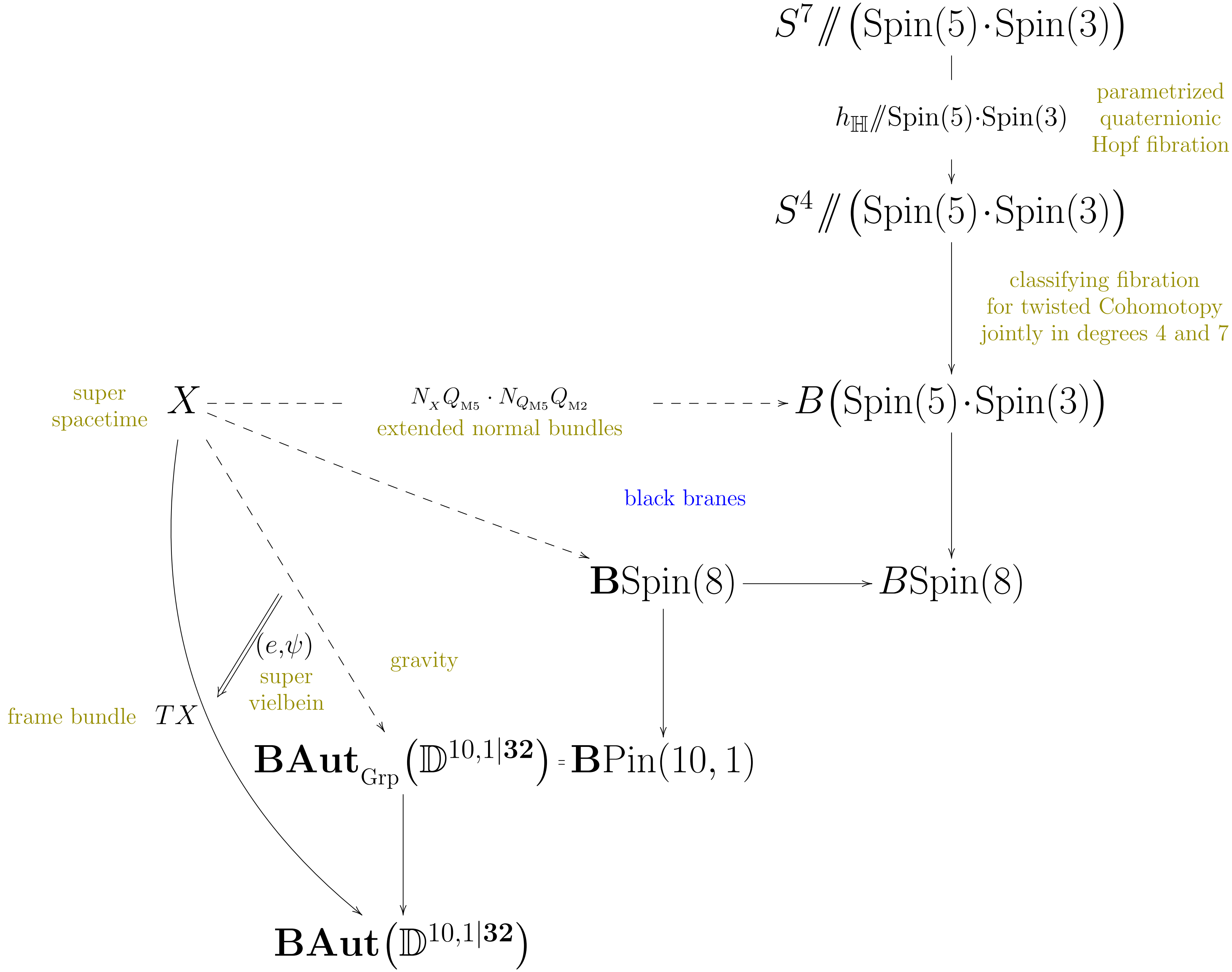
$S^7 /\!\!/ \big(\mathrm{Spin}(5)\cdot\mathrm{Spin}(3)\big)$
$h_{\mathbb{H}} /\!\!/ \mathrm{Spin}(5)\cdot\mathrm{Spin}(3)$
parametrized quaternionic Hopf fibration
$S^4 /\!\!/ \big(\mathrm{Spin}(5)\cdot\mathrm{Spin}(3)\big)$
classifying fibration for twisted Cohomotopy jointly in degrees 4 and 7
$B\big(\mathrm{Spin}(5)\cdot\mathrm{Spin}(3)\big)$
super spacetime
X
$\mathbf{B}\mathrm{Spin}(8)$
$B\mathrm{Spin}(8)$
(e,ψ)
super vielbein
gravity
frame bundle
TX
$\mathbf{BAut}_{\mathrm{Grp}}\big(\mathbb{D}^{10,1|\mathbf{32}}\big) = \mathbf{B}\mathrm{Pin}(10,1)$
G-structure for
M-theory on 8-manifolds
$\mathbf{BAut}\big(\mathbb{D}^{10,1|\mathbf{32}}\big)$



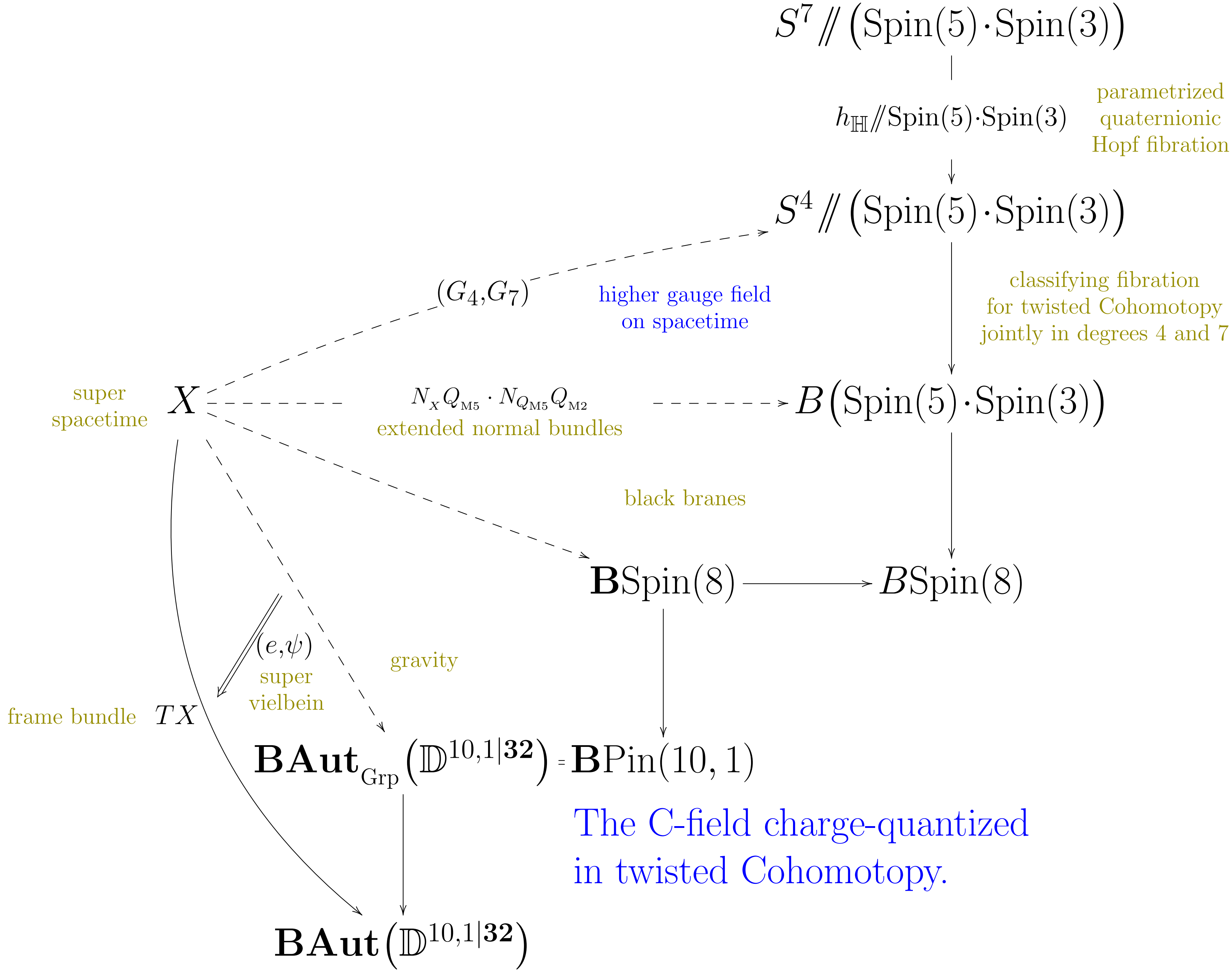
S^7 // (Spin(5)·Spin(3))
h_ℍ // Spin(5)·Spin(3)
parametrized
quaternionic
Hopf fibration
S^4 // (Spin(5)·Spin(3))
classifying fibration
for twisted Cohomotopy
jointly in degrees 4 and 7
super
spacetime
X
N_X Q_M5 · N_{Q_M5} Q_M2
extended normal bundles
B(Spin(5)·Spin(3))
BSpin(8)
BSpin(8)
(e,ψ)
super
vielbein
gravity
frame bundle
TX
BAut_Grp(𝔻^{10,1|32}) = BPin(10, 1)
BAut(𝔻^{10,1|32})
[FSS19b, Cor. 4.1, Rem. 4.2]:
Topological G-structure
for G = Spin(5)·Spin(3)
is locally that of normal bundles
of black M2-M5 brane bound states.



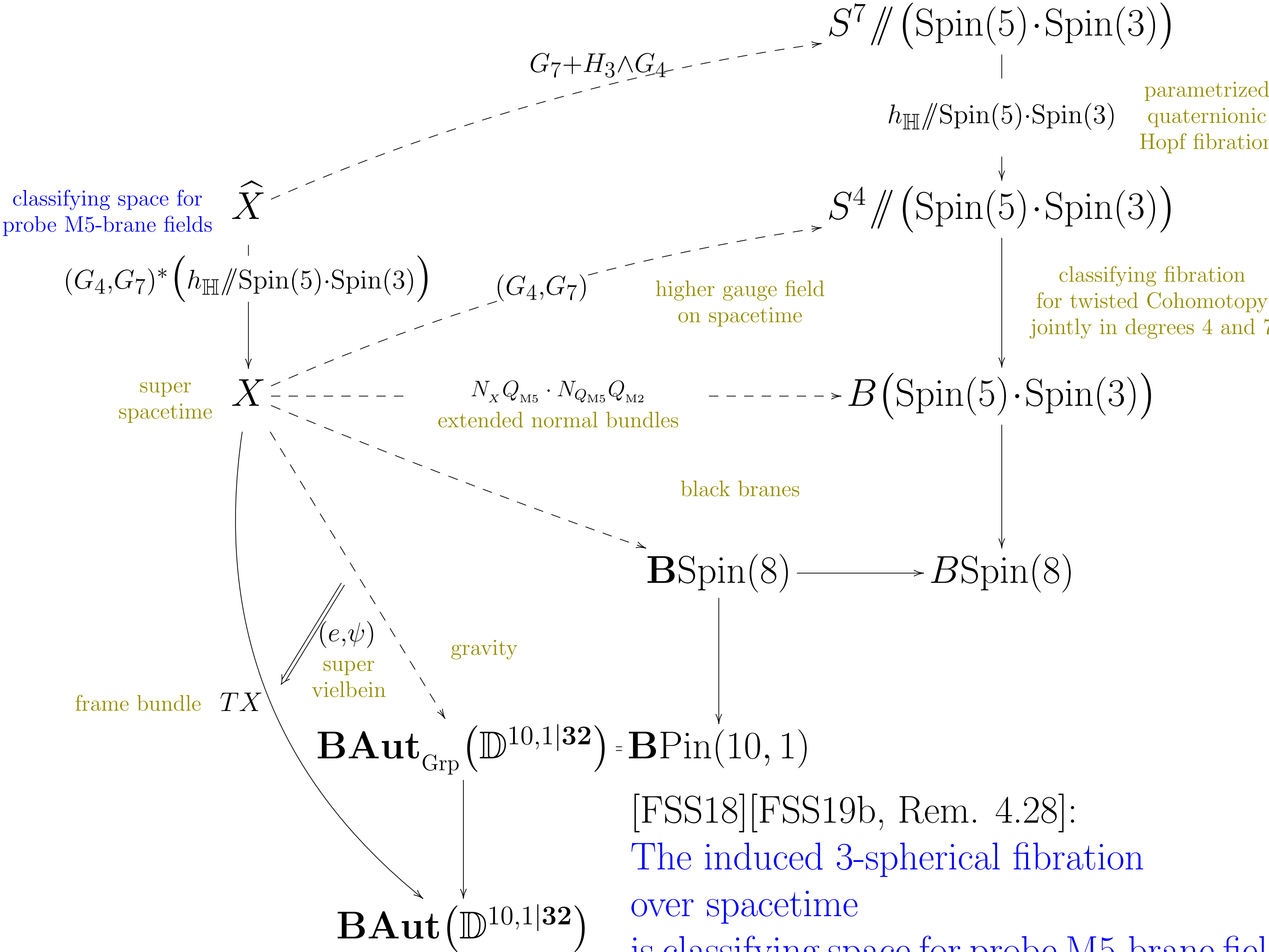
[FSS19b, Cor. 4.1, Rem. 4.2]:
Topological G-structure
for $G = \mathrm{Sp}(2)\cdot\mathrm{Sp}(1)$
is locally that of normal bundles
of BPS black M2 branes.



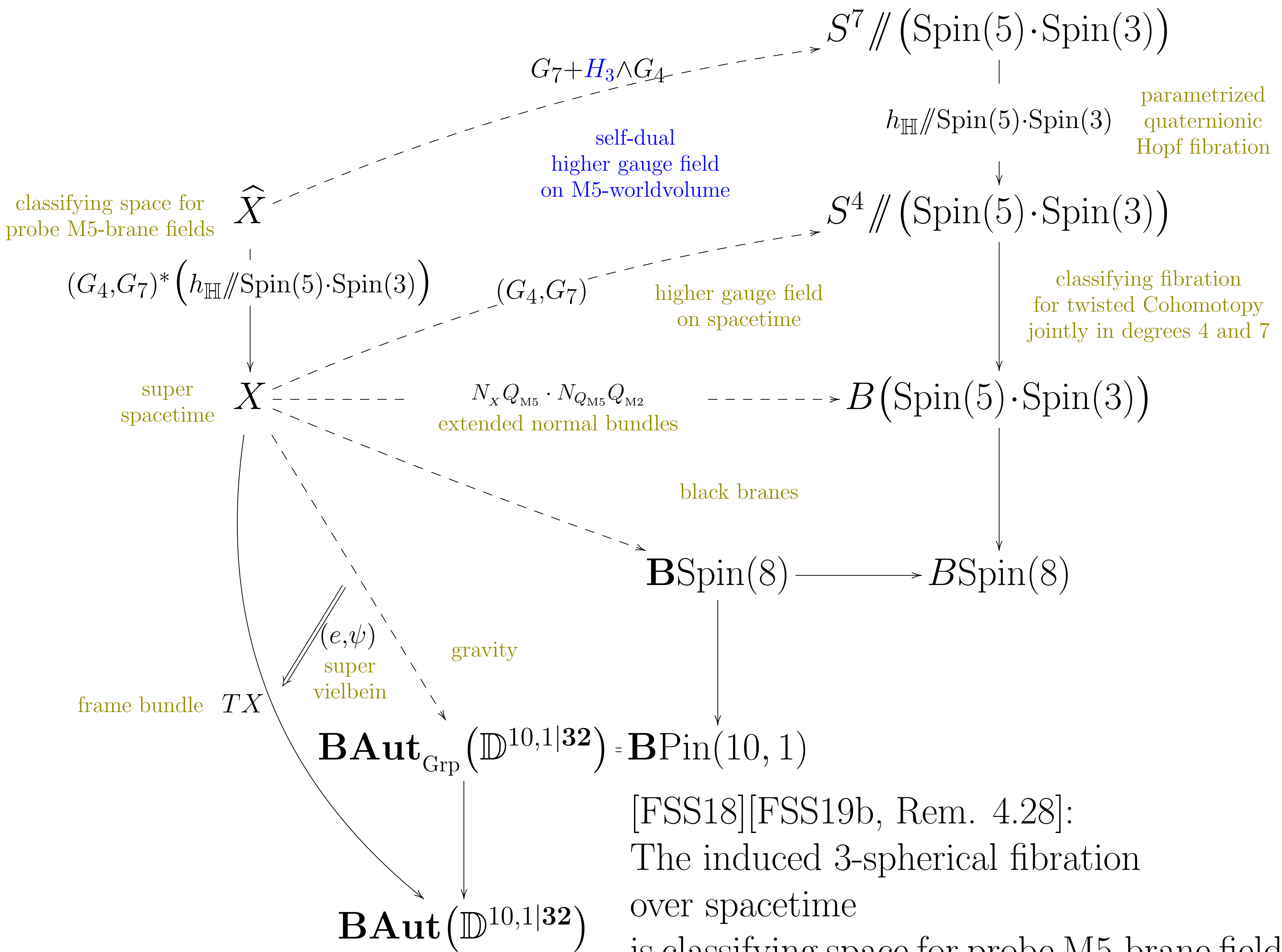
$S^7 /\!\!/ \big(\mathrm{Spin}(5)\cdot\mathrm{Spin}(3)\big)$
$h_{\mathbb{H}} /\!\!/ \mathrm{Spin}(5)\cdot\mathrm{Spin}(3)$
parametrized
quaternionic
Hopf fibration
$S^4 /\!\!/ \big(\mathrm{Spin}(5)\cdot\mathrm{Spin}(3)\big)$
classifying fibration
for twisted Cohomotopy
jointly in degrees 4 and 7
super
spacetime
X
$N_X Q_{\mathrm{M5}} \cdot N_{Q_{\mathrm{M5}}} Q_{\mathrm{M2}}$
extended normal bundles
$B\big(\mathrm{Spin}(5)\cdot\mathrm{Spin}(3)\big)$
black branes
$\mathbf{B}\mathrm{Spin}(8)$
$B\mathrm{Spin}(8)$
(e,ψ)
super
vielbein
gravity
frame bundle
TX
$\mathbf{BAut}_{\mathrm{Grp}}\big(\mathbb{D}^{10,1\vert\mathbf{32}}\big) = \mathbf{B}\mathrm{Pin}(10,1)$
$\mathbf{BAut}\big(\mathbb{D}^{10,1\vert\mathbf{32}}\big)$



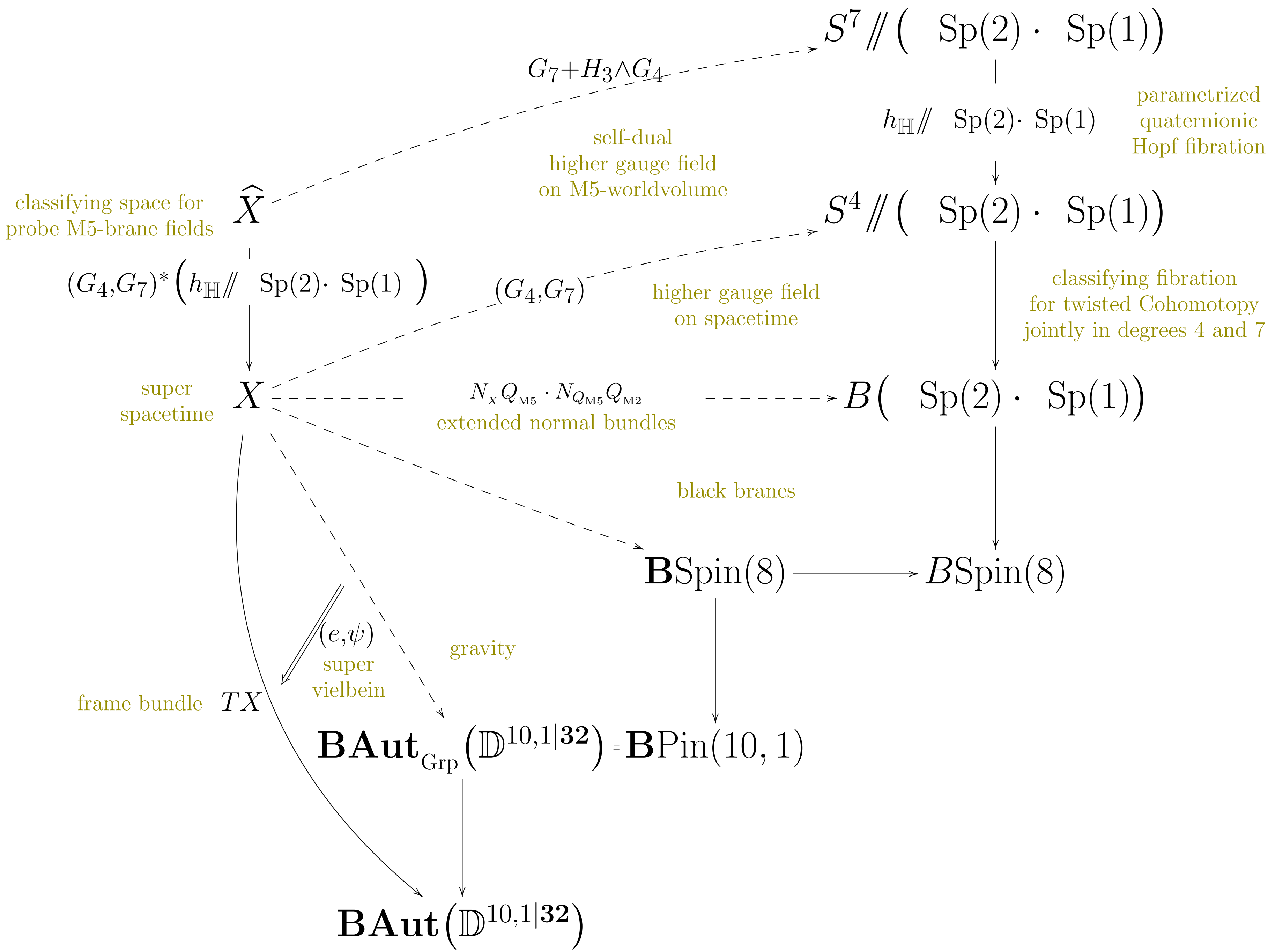
The C-field charge-quantized in twisted Cohomotopy.



[FSS18][FSS19b, Rem. 4.28]:
The induced 3-spherical fibration
over spacetime
is classifying space for probe M5-brane fields.



[FSS18][FSS19b, Rem. 4.28]:
The induced 3-spherical fibration
over spacetime
is classifying space for probe M5-brane fields,
including the self-dual higher gauge field
on the M5-worldvolume.



$S^7 /\!\!/ \big(\mathrm{Sp}(2)\cdot \mathrm{Sp}(1)\big)$
$G_7 + H_3 \wedge G_4$
$h_{\mathbb{H}} /\!\!/ \mathrm{Sp}(2)\cdot \mathrm{Sp}(1)$
parametrized
quaternionic
Hopf fibration
self-dual
higher gauge field
on M5-worldvolume
classifying space for
probe M5-brane fields
$\widehat{X}$
$S^4 /\!\!/ \big(\mathrm{Sp}(2)\cdot \mathrm{Sp}(1)\big)$
$(G_4, G_7)^\ast \big(h_{\mathbb{H}} /\!\!/ \mathrm{Sp}(2)\cdot \mathrm{Sp}(1)\big)$
(G_4, G_7)
higher gauge field
on spacetime
classifying fibration
for twisted Cohomotopy
jointly in degrees 4 and 7
super
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X
$N_X Q_{\mathrm{M5}} \cdot N_{Q_{\mathrm{M5}}} Q_{\mathrm{M2}}$
extended normal bundles
$B\big(\mathrm{Sp}(2)\cdot \mathrm{Sp}(1)\big)$
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$\mathbf{BAut}_{\mathrm{Grp}}\big(\mathbb{D}^{10,1|\mathbf{32}}\big) = \mathbf{B}\mathrm{Pin}(10,1)$
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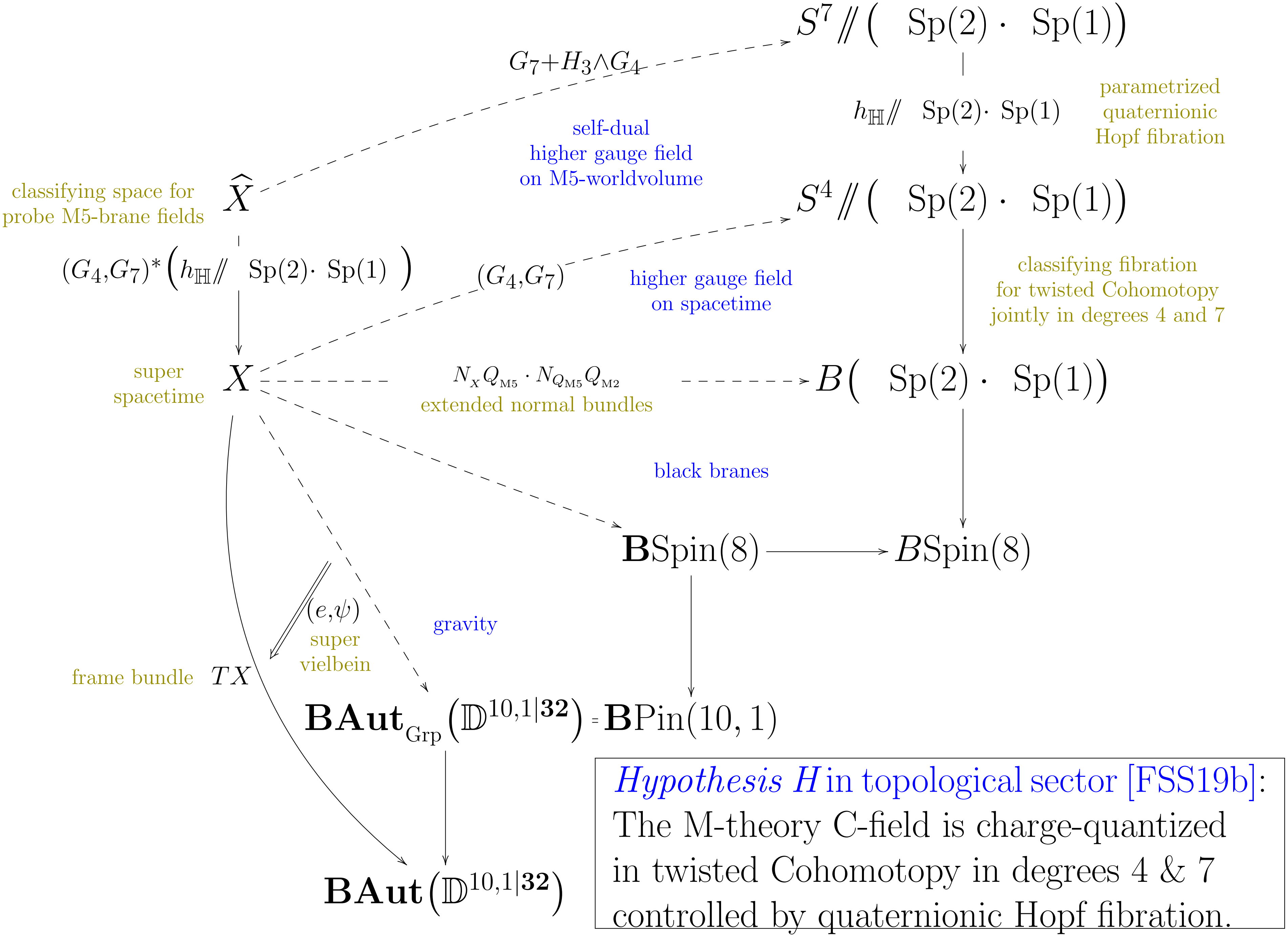

Hypothesis H in topological sector [FSS19b]:
The M-theory C-field is charge-quantized in twisted Cohomotopy in degrees 4 & 7 controlled by quaternionic Hopf fibration.

Theorem [FSS19b] [SS19]:

Hypothesis H rigorously implies
a whole list of subtle consistency conditions
(anomaly cancellation, tadpole cancellation, shifted flux quantization,...)
which have been argued for in the M-theory folklore.

Anomaly cancellation condition		**folklore**	**Hypothesis H**
Half-integral flux quantization	$\underbrace{\left[\, G_4 + \tfrac{1}{4}p_1 \,\right]}_{=:\widetilde{G}_4 \text{ integral flux}} \in H^4(X,\mathbb{Z})$	§2.2	§4.2
Background charge	$\underbrace{q(\widetilde{G}_4)}_{\text{quadratic form}} = \widetilde{G}_4\,(\widetilde{G}_4 - \underbrace{\tfrac{1}{2}p_1}_{=(\widetilde{G}_4)_0}\,)$	§2.4	§4.4
DMW-anomaly cancellation	$W_7(TX) = 0$	§2.1	§4.1
Integral equation of motion	$\underbrace{\mathrm{Sq}^3}_{=\beta \mathrm{Sq}^2}(\widetilde{G}_4) = 0$	§2.3	§4.3
M5-brane anomaly cancellation	$\underbrace{I^{\mathrm{M5}}_{\mathrm{ferm}}}_{\text{chiral fermion}} + \underbrace{I^{\mathrm{M5}}_{\mathrm{sd}}}_{\text{self-dual 3-flux}} + \underbrace{I^{\mathrm{bulk}}_{\mathrm{infl}}}_{\text{bulk inflow}} = 0$	§2.5	§4.5
M2-brane tadpole cancellation	$\underbrace{N_{\mathrm{M2}}}_{\text{number of M2-branes}} + q(\widetilde{G}_4) = I_8$	§2.6	§4.6

This suggests that *Hypothesis H* is a correct assumption
about the mathematical foundations of microscopic M-theory.

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