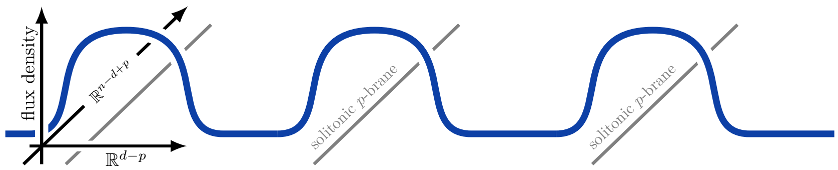
A cohomotopical ADHM construction. Pullback of the volume form on S^n along the Cohomotopy charge map (118) assigns to solitonic codim $< n$ branes (p. 75) their flux density, cf. eq. (2).



[SS24-Cnf]: This map Φ represents the cohomotopical character, and thus induces a shape-equivalence $\widehat{\Phi}$ to differential Cohomotopy, showing that the **configuration space is a gauge-fixed phase space of multi-core solitons** representing every solitonic Cohomotopy charge sector.

$$
\begin{array}{ccccc}
 & \int \mathrm{Conf}\big(\mathbb{R}^{d-p}_{\sqcup\{\infty\}},\, \mathbb{R}^{n-d+p}_{\cup\{\infty\}}\big) & \underset{\text{May-Segal thm.}}{=} & \int \mathrm{Map}^{*/}\big(\mathbb{R}^{d-p}_{\cup\{\infty\}} \wedge \mathbb{R}^{n-d+p}_{\sqcup\{\infty\}},\, S^n\big) & \\
{}^{\eta^{\int}}\nearrow & & \nearrow \quad \searrow{}^{\int\Phi} & & \\
\mathrm{Conf}\big(\mathbb{R}^{d-p}_{\sqcup\{\infty\}},\, \mathbb{R}^{n-d+p}_{\cup\{\infty\}}\big) & \overset{\widehat{\Phi}}{\dashrightarrow} & \widehat{\pi}^n\big(\mathbb{R}^{d-p}_{\cup\{\infty\}} \wedge \mathbb{R}^{n-d+p}_{\sqcup\{\infty\}}\big) & & \int \Omega_{\mathrm{dR}}\big(\mathbb{R}^{d-p}_{\cup\{\infty\}} \wedge \mathbb{R}^{n-d+p}_{\sqcup\{\infty\}};\, \mathfrak{l}S^n\big)_{\mathrm{clsd}} \\
{}_{\Phi}\searrow & & \searrow \quad \nearrow{}^{\eta^{\int}} & & \\
 & \boldsymbol{\Omega}_{\mathrm{dR}}\Big(\mathbb{R}^{d-p}_{\cup\{\infty\}} \wedge \mathbb{R}^{n-d+p}_{\sqcup\{\infty\}};\, \mathfrak{l}S^n\Big)_{\mathrm{clsd}} & & &
\end{array}
$$

Intersecting solitonic brane charges in Cohomotopy. Noticing that the n-flux density arising this way has vanishing cup-square (simply by degree reasons in low codimension) hence behaves linearly, the gauge-fixed phase space of *intersecting* flat branes of low codimension must be the fiber product of these configuration spaces [SS22-Cnf, Ex. 2.3].

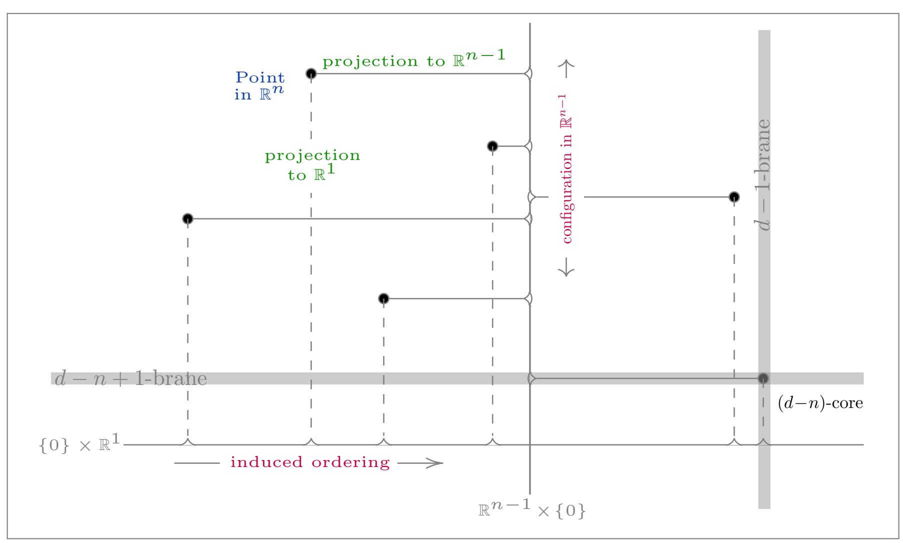
Gauge enhancement on domain wall intersections. In the special case that one of the intersecting brane species is of codimension=1 something remarkable happens [SS22-Cnf, Prop. 2.4. 2.11]: The fiber product of the "labelled" configuration spaces (p. 75) is homotopy-equivalent to a configuration space of *ordered* points in the remaining $n-1$ transverse dimensions that may no longer escape to ∞:

$$
\underbrace{\overbrace{\mathrm{Conf}\big(\mathbb{R}^1_{\sqcup\{\infty\}},\, \mathbb{R}^{n-1}_{\cup\{\infty\}}\big)}^{\substack{n\text{-Cohomotopy moduli of} \\ \text{solitonic codim} = 1\text{-branes}}} \underset{\mathrm{Conf}\left(\mathbb{R}^n_{\cup\{\infty\}}\right)}{\times} \overbrace{\mathrm{Conf}\big(\mathbb{R}^{n-1}_{\sqcup\{\infty\}},\, \mathbb{R}^1_{\cup\{\infty\}}\big)}^{\substack{n\text{-Cohomotopy moduli of} \\ \text{solitonic codim} = (n-1)\text{ branes}}}}_{\text{their intersection}}
$$

The phase space of intersections of cod = 1 with cod = $(n-1)$-branes flux-quantized in n-Cohomotopy is configurations of ordered points in $\mathbb{R}^{n-1}$

homotopy equivalence

$$
\underbrace{\coprod_{N \in \mathbb{N}} \mathrm{Conf}_{\{1,\cdots,N\}}\big(\mathbb{R}^{n-1}\big)}_{\substack{\text{configuration space} \\ \text{of } N \text{ ordered points}}} = \left\{ \quad \right\} \tag{119}
$$



Now, the homotopy type of such configuration spaces where points are no longer allowed to escape to ∞ is quite rich (see eg. [Kn18]) considerably richer than that of the "labeled" configuration spaces on p. 75. With Hypothesis H this provides a substantiation of the expection of rich physics appearing on intersecting branes. We next check this by computing the lightcone quantum observables of these configurations.