Anyonic Defect Branes in TED-K-Theory

Urs Schreiber on joint work with Hisham Sati

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Center for Quantum & Topological Systems

talk via:

Seminario de Categorías @ UNAM, Mexico City, 13 April 2022

RIND Sem. MathPhys & Strings @ U. Mainz, München, Heidelberg, Wien, 09 May 2022

slides and pointers at: `ncatlab.org/schreiber/show/Anyonic+defect+branes+in+TED-K-theory`

I – Equivariant ∞-Bundles

II – TED-K-Theory

III –Anyonic Defect Branes

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I – Equivariant ∞-Bundles

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This part is
a gentle exposition of
the most basic concept
underlying these articles:

Principal ∞-bundles [arXiv:1207.0248/49]
Equivariant Principal ∞-bundles [arXiv:2112.13654]
Proper Orbifold Cohomology [arXiv:2008.01101]

following

Diff. Cohomology in a Cohesive ∞-Topos [arXiv:1310.7930]

Motivation, Overview, Summary and Outlook – in one single slide:

Generalized Cohomology Theories $\longleftrightarrow$ **Cohesive Higher Fiber Bundles**

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Cohomology		**Higher Bundles**
non-abelian	$\leftrightarrow$	general fibers
twisted	$\leftrightarrow$	associated
differential	$\leftrightarrow$	cohesive
G-equivariant	$\leftrightarrow$	sliced over $\mathbf{B}G$

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A major phenomenon/subtlety is that the last two aspects go hand-in-hand:

Proper G-equivariance corresponds to the **cohesive slice** over $\mathbf{B}G$,

while

Borel equivariance corresponds just to the **slice of shapes**.

2-Groupoids

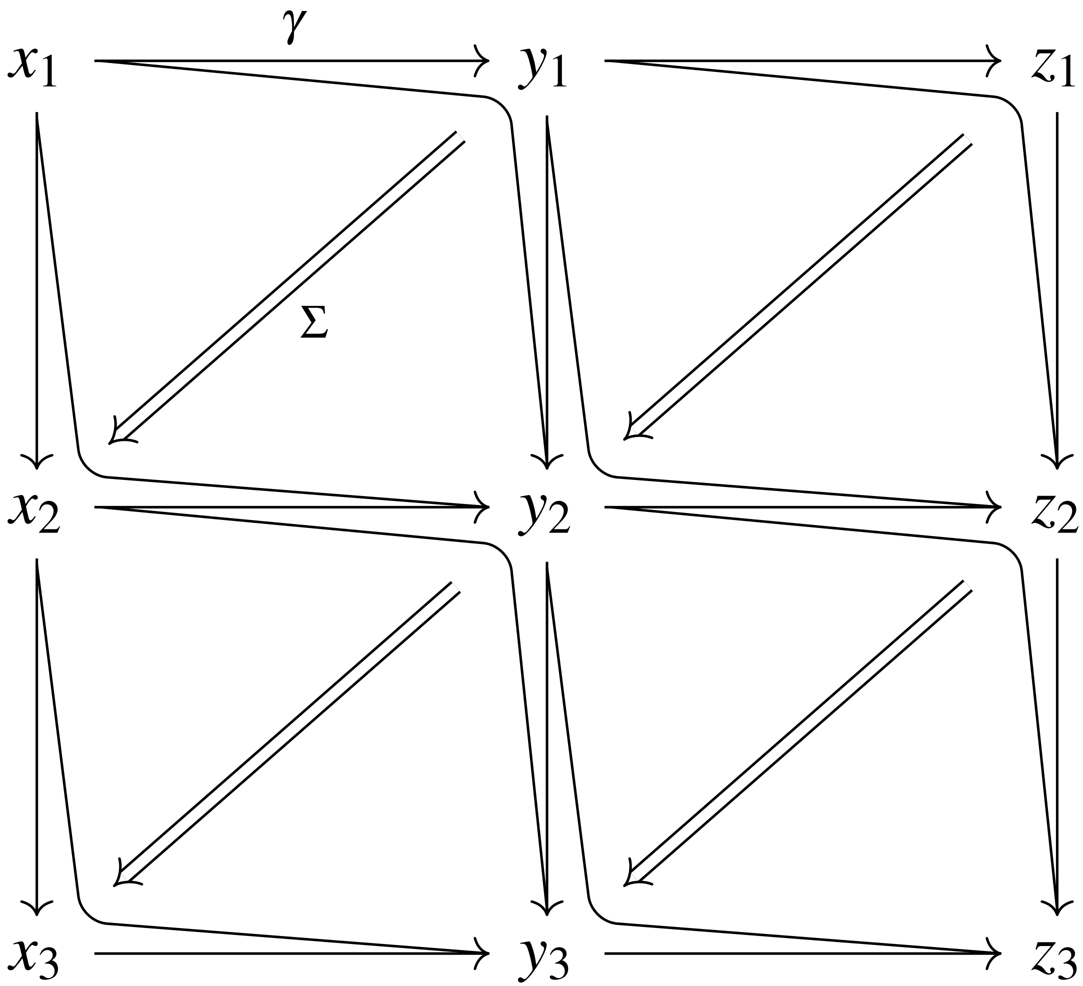
2-Groupoids are the algebra of 2-dimensional pasting,
such that all composition is associative and invertible:

In general we need n-groupoids for $n \in \{1, 2, 3, \cdots, \infty\}$
but for sake of exposition we may focus on $n = 2$.

2-Groupoids

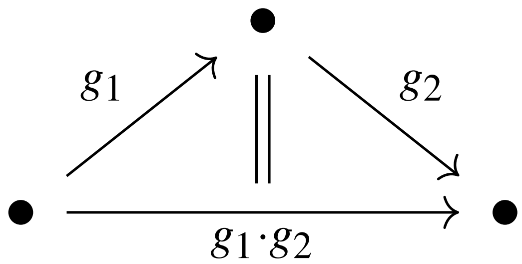
2-Groupoids are the algebra of 2-dimensional pasting,
such that all composition is associative and invertible:

Accurate intuition:
homotopy classes of surfaces Σ relative boundary paths γ
in a topological space:



2-Groupoids – Examples.

For G a discrete group, there is its *delooping 1-groupoid* – $\boxed{\mathbf{B}G}$:



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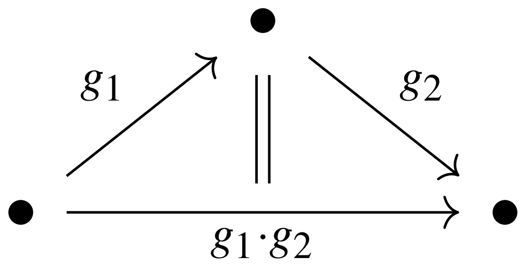

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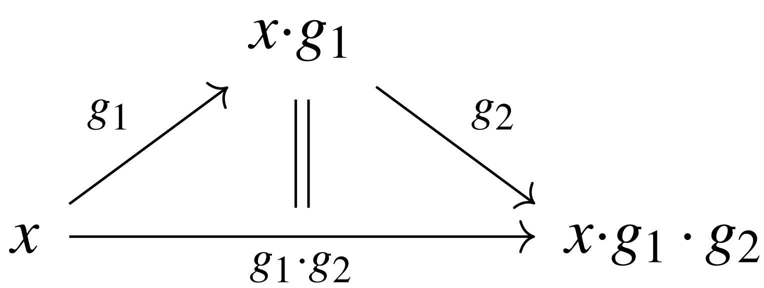
For $X \circlearrowleft G$ a G-action on a set X there is its *action groupoid* or *homotopy quotient* – $\boxed{X /\!\!/ S}$:



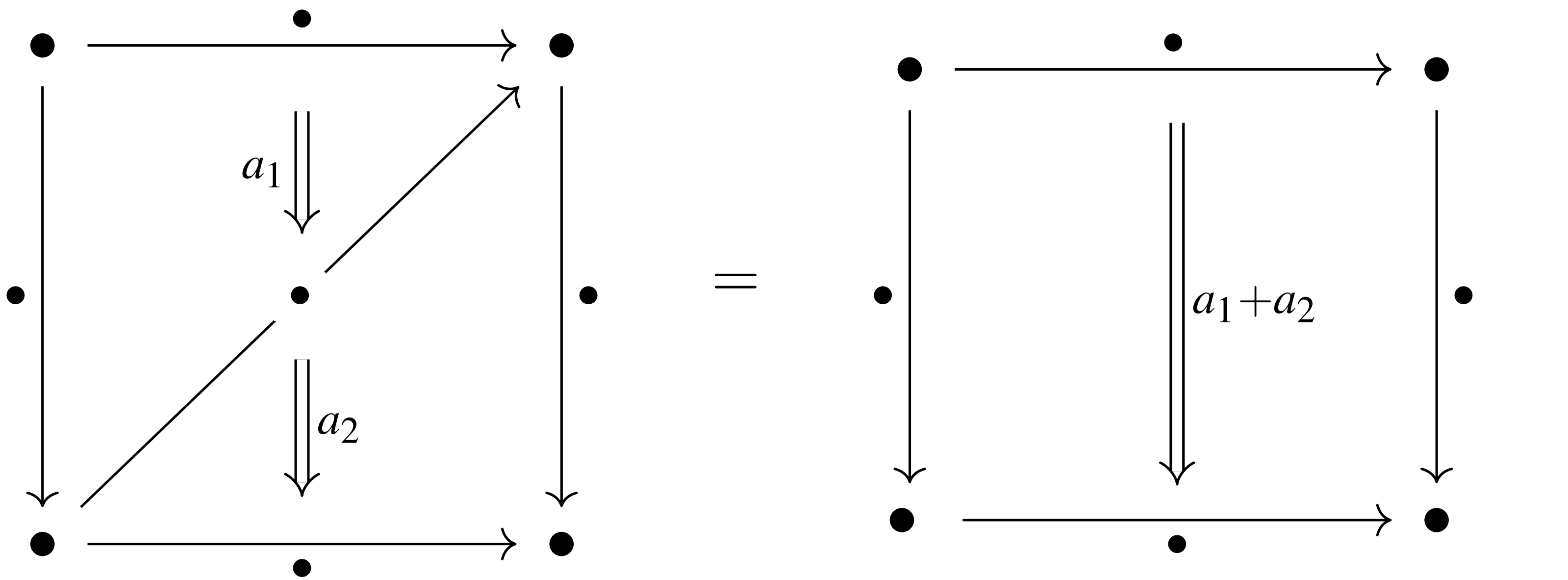
$g_i \in G$
$x \in X$

Hence: $\mathbf{B}G \simeq * /\!\!/ G$.

2-Groupoids – Examples.

For A an *abelian* group there is the *double delooping 2-groupoid*

$$\mathbf{B}^2 A = \mathbf{B}\big(\overbrace{\mathbf{B}A}^{\text{“2-group”}}\big)$$



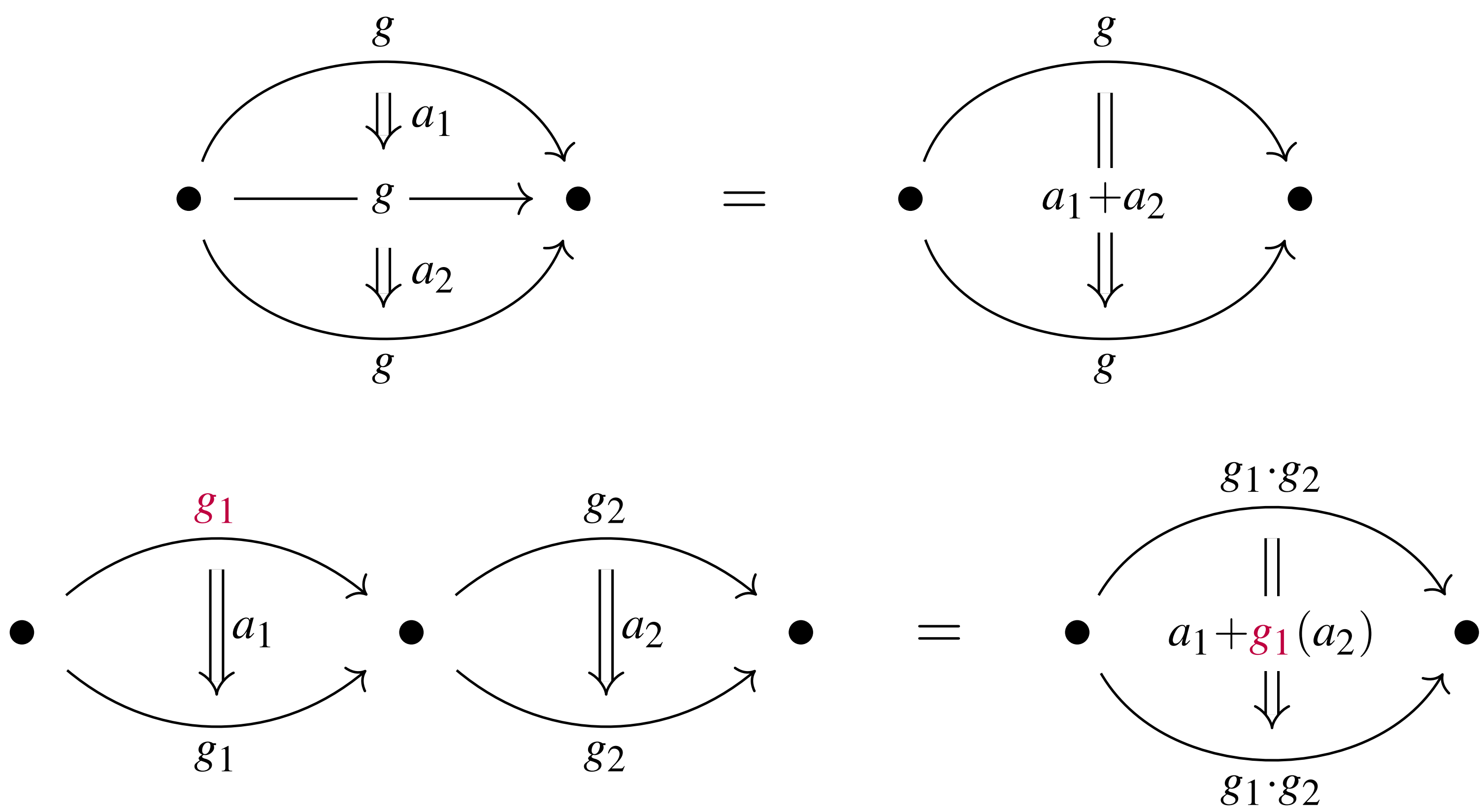
$$a_i \in A$$

2-Groupoids – Examples.

For $A \curvearrowleft G$ a *linear* action, i.e. by group automorphisms,

there is the delooping 2-groupoid $\mathbf{B}\Big(\underbrace{(\mathbf{B}A) \rtimes G}\Big) \;\simeq\; \big(\mathbf{B}^2 A\big) /\!\!/ G$

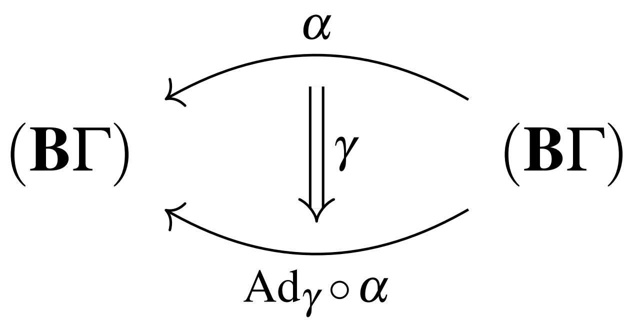
of the *semidirect product 2-group*:



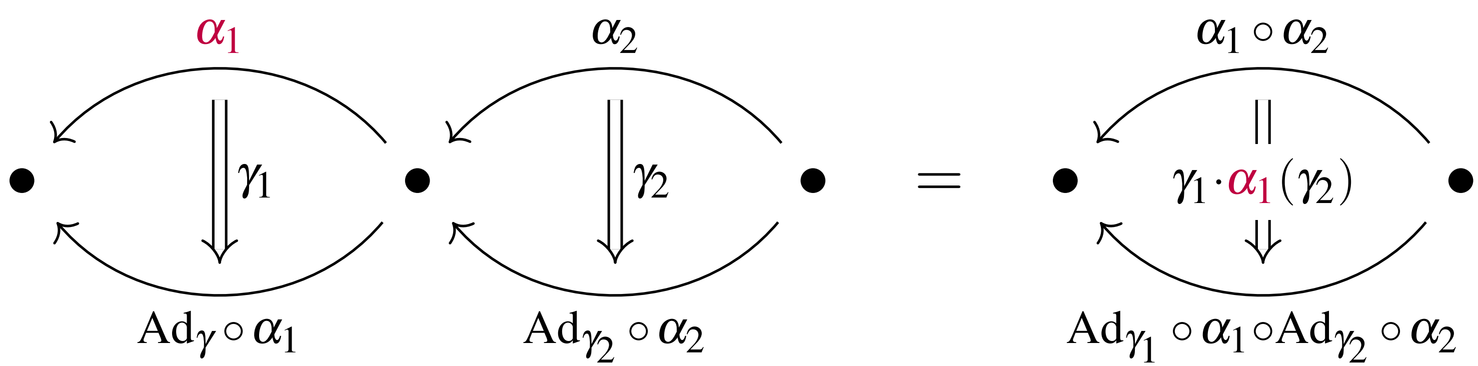
2-Groupoids – Examples.

This is a special case of the delooping of the *automorphism 2-group* of a group Γ:

$$\mathbf{B}\Big(\mathrm{Aut}\big(\mathbf{B}\Gamma\big)\Big) = \mathbf{B}\Big(\overbrace{\mathrm{Aut}(\Gamma)/\!\!/\Gamma}\Big)$$



$$\alpha \in \mathrm{Aut}(\Gamma)$$
$$\gamma \in \Gamma$$
$$\Gamma \xrightarrow{\mathrm{Ad}} \mathrm{Aut}(\Gamma)$$



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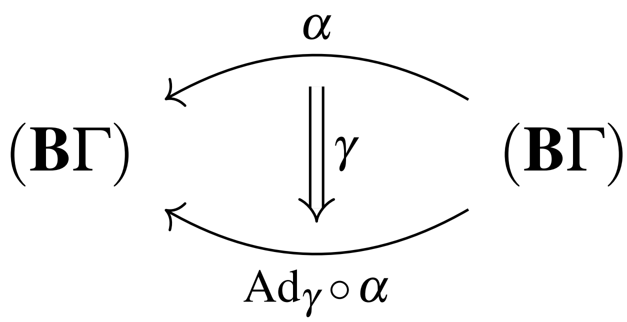

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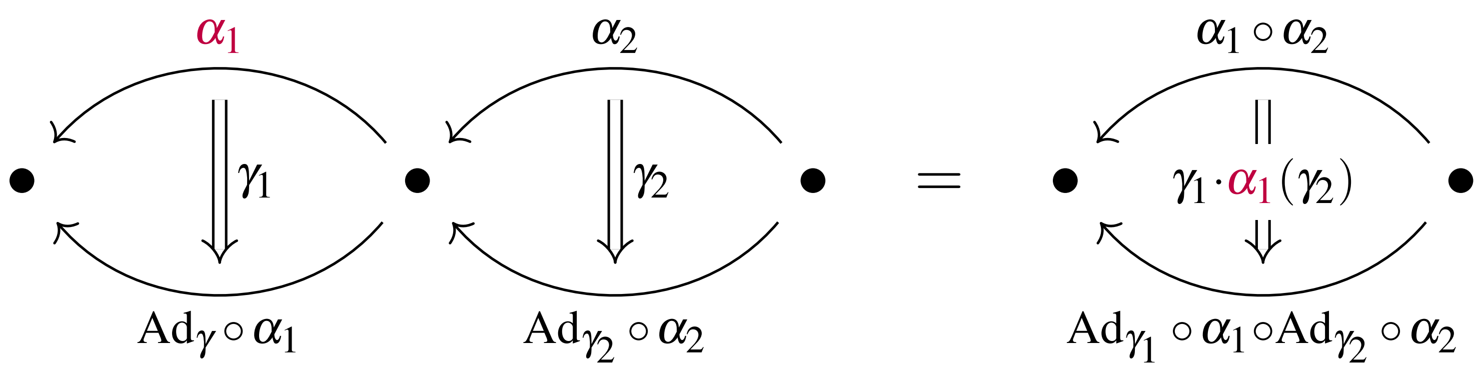

NB: Always need to choose whether actions are right- or left-actions, hence whether group multiplication is opposite or aligned to arrow composition. Before long we want *structure groups* to act *from the left* and *equivariance groups* to act *from the right*.

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Notice:

(1) $(\mathbf{B}A) \rtimes G$ is a non-abelian 2-group iff G is a non-abelian group;

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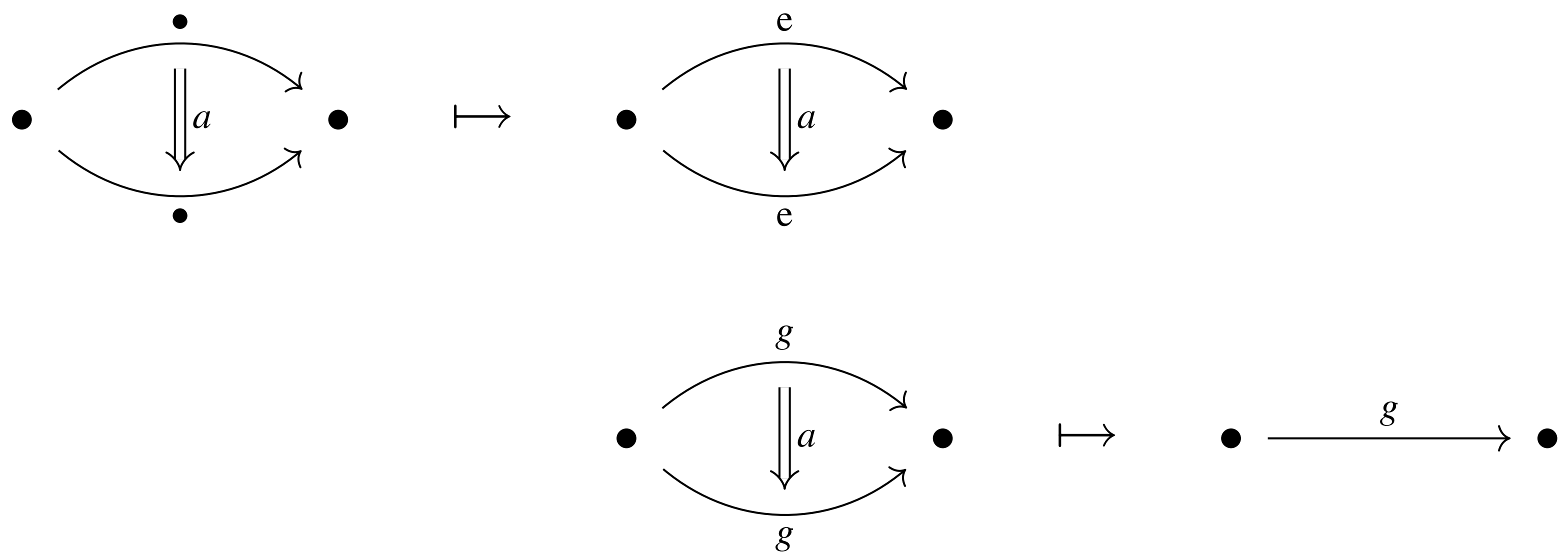
(2) its delooping sits in this fiber sequence:

$$
\begin{array}{ccccc}
 & & \mathbf{B}\Big((\mathbf{B}A) \rtimes G\Big) & & \\
 & & \| & & \\
\mathbf{B}^2 A & \xrightarrow{\ \mathrm{fib}(p)\ } & (\mathbf{B}^2 A) /\!\!/ G & \underset{\in \mathrm{KanFib}}{\xrightarrow{\ p\ }} & \mathbf{B}G
\end{array}
$$



2-Groupoids – 2-Functors.

A *2-functor* is a map between 2-groupoids respecting identities and composition.

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E.g.: if $\mathbb{Z} \circlearrowleft \mathbb{Z}_2$ by sign inversion, and $G \xrightarrow{\sigma} \mathbb{Z}_2$ a homomorphism then
2nd group cohomology of G with coefficients in $G \underset{\sigma}{\circlearrowright} \mathbb{Z}$ is 2-functors:

$$
\begin{array}{ccc}
\mathbf{B}G & \xrightarrow{\text{2-functor = 2-cocycle}} & (\mathbf{B}^2 A) /\!\!/ \mathbb{Z}_2 \\
& {}_{\mathbf{B}\sigma}\searrow \quad \mathbf{B}\mathbb{Z}_2 \quad \swarrow &
\end{array}
$$

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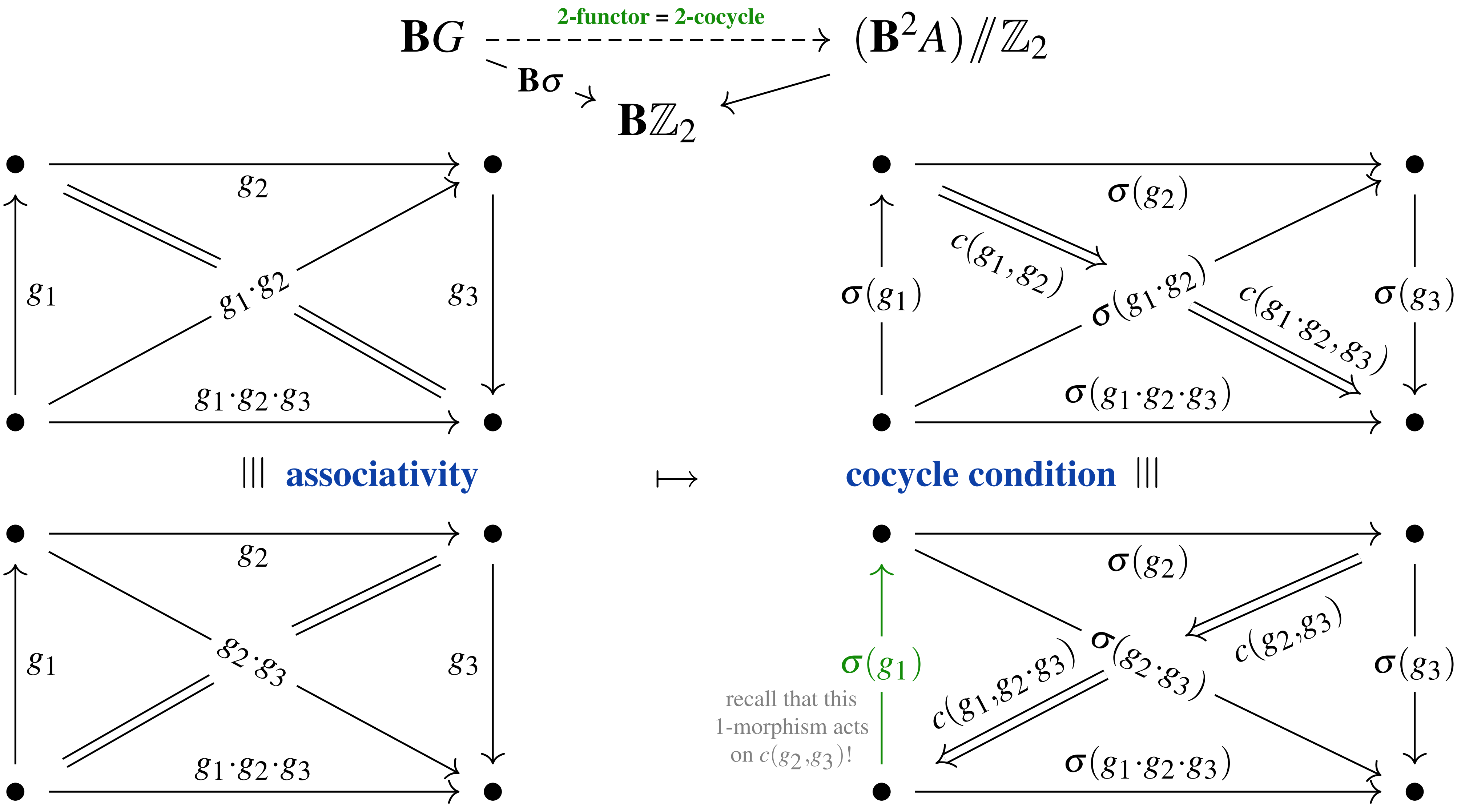

2-Groupoids with smooth structure.

A *smooth 2-groupoid* $\mathscr{X}$ is given by a rule

which to each chart $\mathbb{R}^n$, $n \in \mathbb{N}$, assigns the plain 2-groupoid $\mathrm{Probe}(\mathbb{R}^n, \mathscr{X})$

of ways of smoothly mapping $\mathbb{R}^n$ into the would-be $\mathscr{X}$

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$$
\begin{array}{rcccl}
\text{a chart} & \mathbb{R}^{n_1} & \mapsto & \mathrm{Probe}(\mathbb{R}^{n_1}, \mathscr{X}) & \text{set of probes by this chart} \\
 & {\scriptstyle\text{smooth}}\big\downarrow f & & f^* \big\uparrow {\scriptstyle\text{pullback}} & \\
\text{another chart} & \mathbb{R}^{n_2} & \mapsto & \mathrm{Probe}(\mathbb{R}^{n_2}, \mathscr{X}) &
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such that this respects composition and identities of smooth functions f.

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So $\mathrm{Probe}(*, \mathscr{X}) = \mathrm{Probe}(\mathbb{R}^0, \mathscr{X})$ is the underlying 2-groupoid

and the system of $\mathrm{Probe}(\mathbb{R}^{\bullet > 0}, \mathscr{X})$ is *smooth structure* on it.

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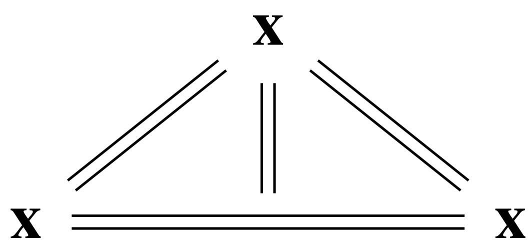
Grothendieck (1965): “functorial geometry”

common jargon: “pre-2-stacks on the site of Cartesian spaces”

2-Groupoids with smooth structure – Examples.

If X is a topological space, then as a smooth 2-groupoid it's this assignment:

$$\mathrm{X} \quad : \quad \mathbb{R}^n \quad \mapsto \quad \mathrm{Probe}(\mathbb{R}^n, \mathrm{X}) := C^0\left(\mathbb{R}^n, \mathrm{X}\right)$$



$$\mathbf{x} \in C^0\left(\mathbb{R}^n, \mathrm{X}\right)$$

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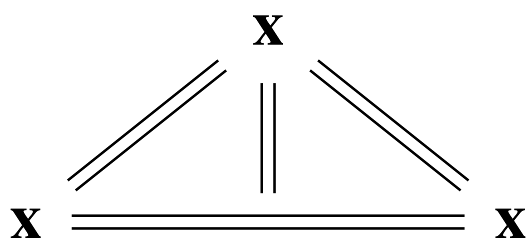

$$\mathbf{x} \in C^\infty(\mathbb{R}^n, \mathrm{X})$$

(also known as X in its incarnation as a *diffeological space*).

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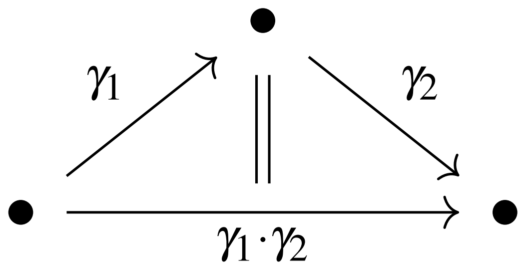
If Γ a *Lie* group, then the sets of smooth functions $C^\infty(\mathbb{R}^n,\Gamma)$ are plain groups, and the *smooth delooping groupoid* $\mathbf{B}\Gamma$ is:

$$\mathbf{B}\Gamma \quad : \quad \mathbb{R}^n \quad \mapsto \quad \mathrm{Probe}(\mathbb{R}^n, \mathbf{B}\Gamma) := \mathbf{B}\big(C^\infty(\mathbb{R}^n, \Gamma)\big)$$



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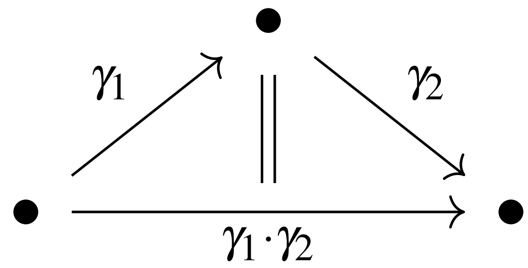

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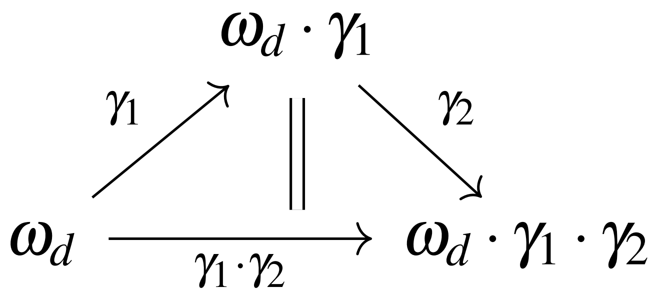
If V is a Γ-representation, then the *smooth moduli space of V-valued differential forms* is

$$\Omega^d_{\mathrm{dR}}(-;V)/\!\!/\Gamma \quad : \quad \mathbb{R}^n \quad \mapsto \quad \Omega^d_{\mathrm{dR}}(\mathbb{R}^n;V)/\!\!/\Gamma$$



$$\gamma_i \in C^\infty(\mathbb{R}^n, \Gamma)$$

$$\omega_d \in \Omega^d_{\mathrm{dR}}(\mathbb{R}^n;V)$$

2-Groupoids with smooth structure – As smooth homotopy types.

Smooth 2-groupoids are *models* for *smooth 2-stacks* aka **smooth homotopy 2-types**

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A smooth 2-functor $\mathscr{X} \xrightarrow{f} \mathscr{Y}$ is called:

PrjFib	*projective fibration*	iff for each $\mathbb{R}^n$, every $k+1$-morphism in $\mathrm{Probe}(\mathbb{R}^n, \mathscr{Y})$ that starts at k-morphisms which come from $\mathrm{Probe}(\mathbb{R}^n, \mathscr{X})$ lifts compatibly to a $k+1$-morphism in $\mathrm{Probe}(\mathbb{R}^n, \mathscr{X})$
LWEq	*local weak equivalence*	iff for every $\mathbb{R}^n$ there exists an open ball $0 \in \mathbb{D}^n_\varepsilon \overset{i}{\hookrightarrow} \mathbb{R}^n$ such that $\mathrm{Probe}(\mathbb{R}^n, f)_{\vert i}$ is a weak homotopy equivalence namely an iso on the evident homotopy groups
PrjCof	*projective cofibration*	if (Dugger's sufficient condition): for all k, the spaces of k-morphisms are disjoint unions of charts $\mathbb{R}^n$ (for any n-s)

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Fact/Def.: *Maps of 2-stacks* are *modeled* by

$$\varnothing \underset{\in \mathrm{PrjCof}}{\xrightarrow{\text{cof. domain}}} \widehat{\mathscr{X}} \underset{\text{smooth 2-functor}}{\xrightarrow{\phi}} \mathscr{Y} \underset{\in \mathrm{PrjFib}}{\xrightarrow{\text{fib. co-domain}}} *$$

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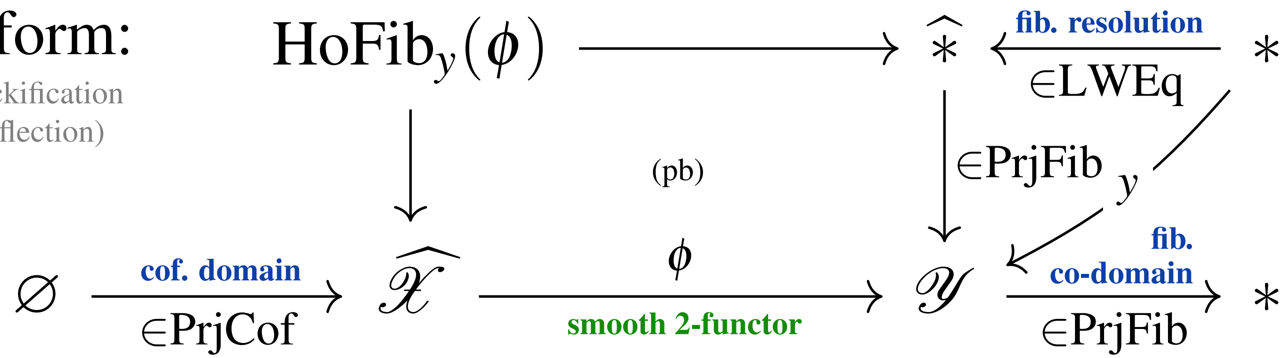
Fact/Def.: *Maps of 2-stacks* and their *homotopy fibers* are *modeled* by pullbacks of this form:

(because 2-stackification is an ∞-lex reflection)



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A smooth 2-functor $\mathscr{X} \xrightarrow{f} \mathscr{Y}$ is called:

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LWEq	*local* *weak*	alence s
PrjCof	*projec* *cofibration*	disjoint unions of charts $\mathbb{R}^n$ (for any n-s)

Definition/Example:
For $\widehat{\mathscr{X}}$ projectively cofibrant
and $\mathscr{Y}$ projectively fibrant
the smooth *mapping 2-stack* is modeled by

$$\mathrm{Map}\big(\widehat{\mathscr{X}}, \mathscr{Y}\big) \;:\; \mathbb{R}^n \mapsto \mathrm{Hom}\big(\widehat{\mathscr{X}} \times \mathbb{R}^n, \mathscr{Y}\big)$$

2-groupoid of smooth 2-functors

Fact/Def.: *Maps of 2-stacks* and their *homotopy fibers* are *modeled* by pullbacks of this form:

(because 2-stackification is an ∞-lex reflection)

$$\begin{tikzcd}
 & \mathrm{HoFib}_y(\phi) \arrow[rr] \arrow[d] \arrow[drr, phantom, "\text{(pb)}"] & & \widehat{*} \arrow[d, "\in \mathrm{PrjFib}"] & * \arrow[l, "\text{fib. resolution}"', "\in \mathrm{LWEq}"] \arrow[dl, "y"] \\
\varnothing \arrow[r, "\text{cof. domain}", "\in \mathrm{PrjCof}"'] & \widehat{\mathscr{X}} \arrow[rr, "\phi", "\text{smooth 2-functor}"'] & & \mathscr{Y} \arrow[r, "\text{fib. co-domain}", "\in \mathrm{PrjFib}"'] & *
\end{tikzcd}$$

2-Groupoids with smooth structure – Homotopy fiber sequences.

$$\mathrm{U}(1) \underset{\text{locally trivial circle-extension}}{\hookrightarrow} \Gamma \twoheadrightarrow \Gamma/\mathrm{U}(1) \quad \text{smooth group}$$

$$\Gamma/\mathrm{U}(1) \overset{\in \mathrm{LWEq}}{\longleftarrow} \Gamma /\!\!/ \mathrm{U}(1)$$

$$\Gamma /\!\!/ \mathrm{U}(1) \underset{\in \mathrm{PrjFib}}{\longrightarrow} \mathbf{B}\mathrm{U}(1) \longrightarrow \mathbf{B}\Gamma \longrightarrow \mathbf{B}(\Gamma/\mathrm{U}(1))$$

$$\mathbf{B}(\Gamma/\mathrm{U}(1)) \overset{\in \mathrm{LWEq}}{\longleftarrow} \mathbf{B}\Gamma /\!\!/ \mathbf{B}\mathrm{U}(1)$$

$$\mathbf{B}\Gamma /\!\!/ \mathbf{B}\mathrm{U}(1) \underset{\text{2-cocycle classifying the extension}}{\longrightarrow} \mathbf{B}^2\mathrm{U}(1)$$

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smooth group

locally trivial circle-extension

$$\Gamma/\mathrm{U}(1) \xleftarrow{\in \mathrm{LWEq}} \Gamma /\!\!/ \mathrm{U}(1)$$

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2-cocyle classifying the extension

$$\begin{matrix} c \\ \| \\ c \end{matrix} \;\mapsto\; \begin{matrix} \gamma \\ \| \\ \gamma \end{matrix} \;\mapsto\; \begin{matrix} [\gamma] \\ \| \\ [\gamma] \end{matrix}$$

$$\begin{matrix} \gamma \\ \downarrow{\scriptstyle c} \\ c\cdot\gamma \end{matrix} \;\mapsto\; \begin{matrix} \bullet \\ \downarrow{\scriptstyle c} \\ \bullet \end{matrix} \;\mapsto\; \begin{matrix} \bullet \\ \downarrow{\scriptstyle \gamma} \\ \bullet \end{matrix} \;\mapsto\; \begin{matrix} \bullet \\ \downarrow{\scriptstyle [\gamma]} \\ \bullet \end{matrix}$$

$$\begin{matrix} \bullet \\ \gamma \left(\overset{c}{\Longrightarrow} \right) c\cdot\gamma \\ \bullet \end{matrix} \;\mapsto\; \begin{matrix} \bullet \\ \left(\overset{c}{\Longrightarrow} \right) \\ \bullet \end{matrix}$$

For example, write $\mathrm{U}(n),\ n \in \mathbb{N} \sqcup \{\omega\}$
for the unitary group on a countably-dimensional complex Hilbert space
and regard this as a smooth group by its "continuous diffeology":

$$\mathrm{U}(n) \ : \ \mathbb{R}^k \ \mapsto \ \mathrm{Probe}(\mathbb{R}^k, \mathrm{U}(n)) \ := \ C^0(\mathbb{R}^k, \mathrm{U}(n))$$

2-Groupoids with smooth structure – Dixmier-Douady class.

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Then we have the following long fiber sequence of smooth 2-groupoids:

$$\begin{array}{ccccccc}
\mathrm{U}(1) \hookrightarrow \mathrm{U}(n) \longrightarrow\!\!\!\!\rightarrow \mathrm{PU}(n) & & & & & & \\
\uparrow{\scriptstyle \in \mathrm{LWEq}} & & & & & & \\
\mathrm{U}(n) /\!\!/ \mathrm{U}(1) & \to & \mathbf{B}\mathrm{U}(1) & \to & \mathbf{B}\mathrm{U}(n) & \longrightarrow & \mathbf{B}\mathrm{PU}(n) \\
 & & & & & & \uparrow{\scriptstyle \in \mathrm{LWEq}} \\
 & & & & & & \mathbf{B}\mathrm{U}(n) /\!\!/ \mathbf{B}\mathrm{U}(1) \to \mathbf{B}^2\mathrm{U}(1)
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& & & & & & \mathbf{B}\mathrm{U}(n)/\!\!/\mathbf{B}\mathrm{U}(1) & \to & \mathbf{B}^2\mathrm{U}(1)
\end{array}$$

This is compatible with complex conjugation, so we have a map of 2-stacks like this:

universal Dixmier-Douady class

$$\mathbf{B}\mathrm{PU}(n)/\!\!/\mathbb{Z}_2 \xleftarrow[\in \mathrm{LWEq}]{} \;\xrightarrow{\mathrm{DD}} \mathbf{B}^2\mathrm{U}(1)/\!\!/\mathbb{Z}_2$$

2-Groupoids with smooth structure – Čech groupoids.

For X a smooth manifold with $\{U_i \hookrightarrow X\}_{i \in I}$ a good open cover, in that

$$\big(\mathbf{x}, (i_1, \cdots, i_n)\big) \in C^\infty\big(\mathbb{R}^m, U_{i_1} \cap \cdots \cap U_{i_n}\big) \quad \Rightarrow \quad U_{i_1} \cap \cdots \cap U_{i_n} \underset{\text{diff}}{\simeq} \mathbb{R}^{\dim(X)},$$

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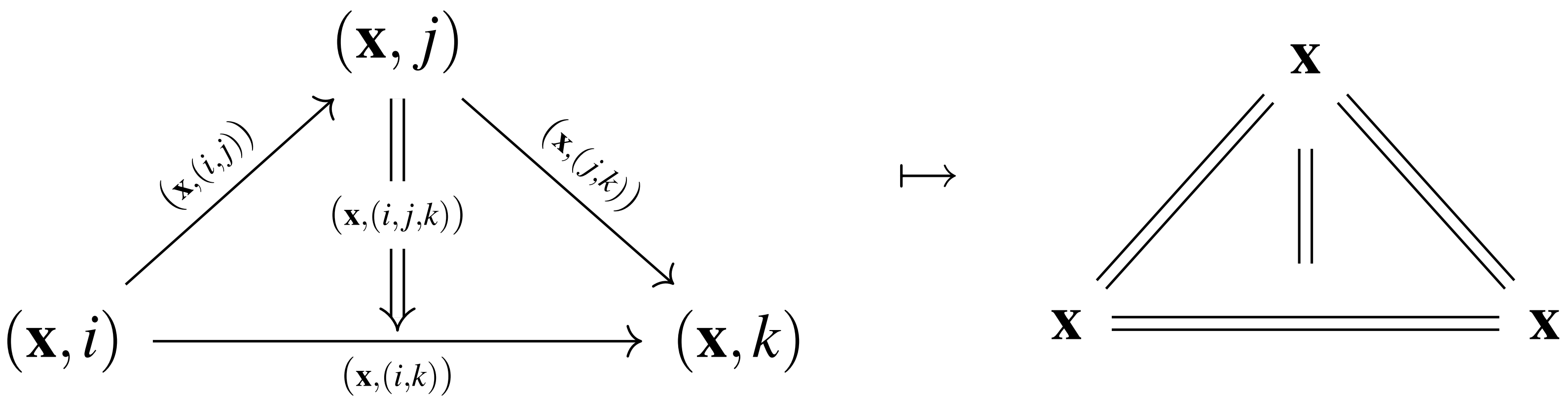
we have the smooth *Čech 2-groupoid*:

$$\varnothing \xrightarrow[\in \mathrm{PrjCof}]{} \underset{}{\overset{\text{Čech 2-groupoid of good open cover}}{\widehat{X}}} \xrightarrow[\in \mathrm{LWEq}]{\text{cofibrant resolution}} X \;\; \text{smooth manifold regarded as smooth 2-groupoid}$$



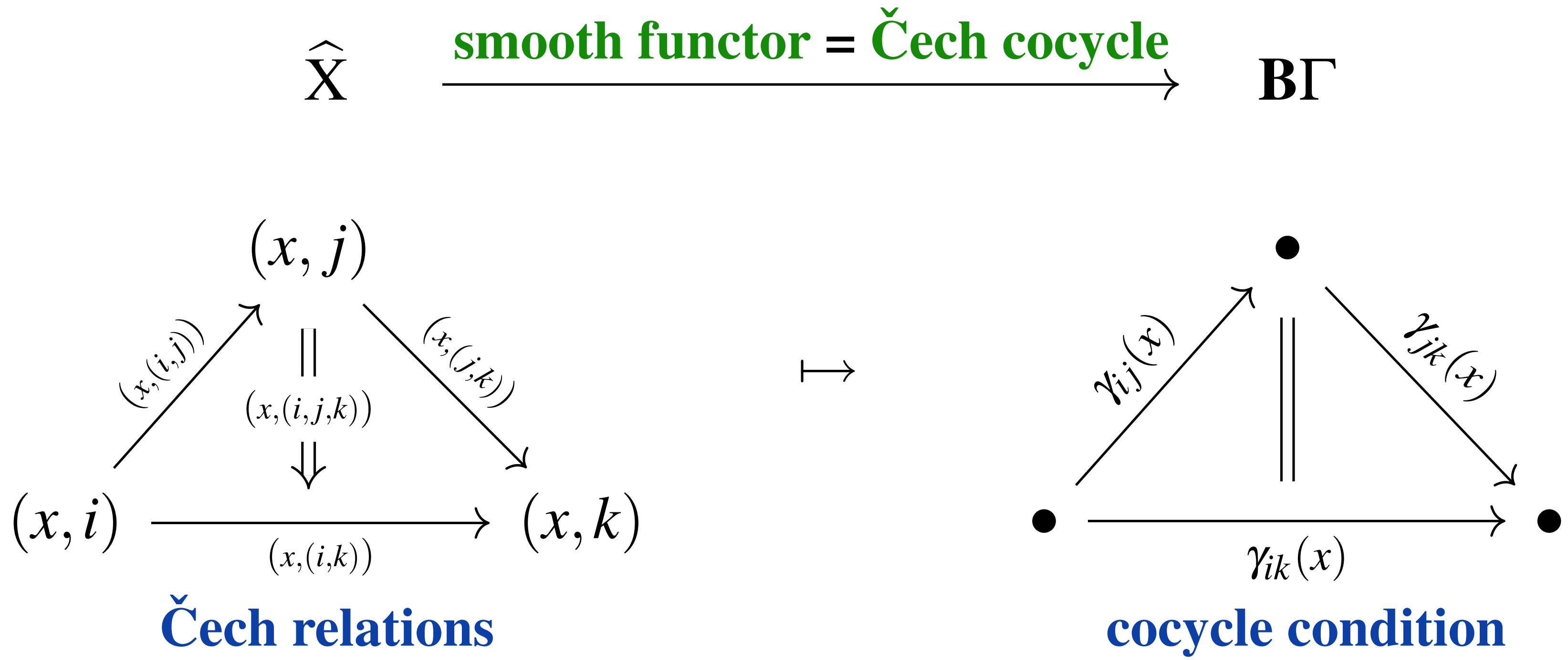
which is a *projectively cofibrant resolution* of X.

2-Groupoids with smooth structure – Čech cocycles.

Smooth 2-functors from such a Čech resolution $\widehat{X} \to X$

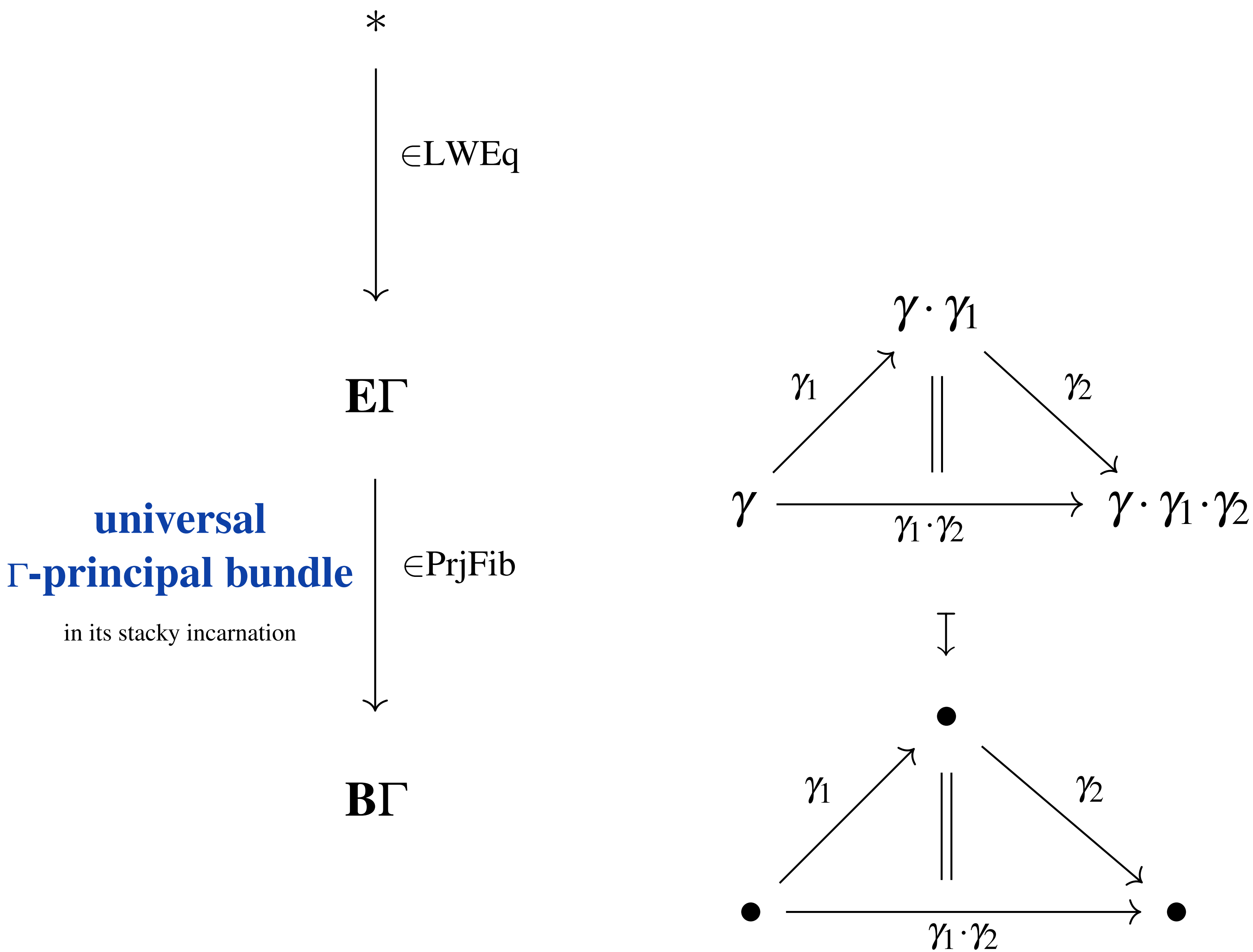
to the delooping $\mathbf{B}\Gamma$ of a Lie group

are *cocycles* in the *Čech cohomology* of X with coefficients in Γ:



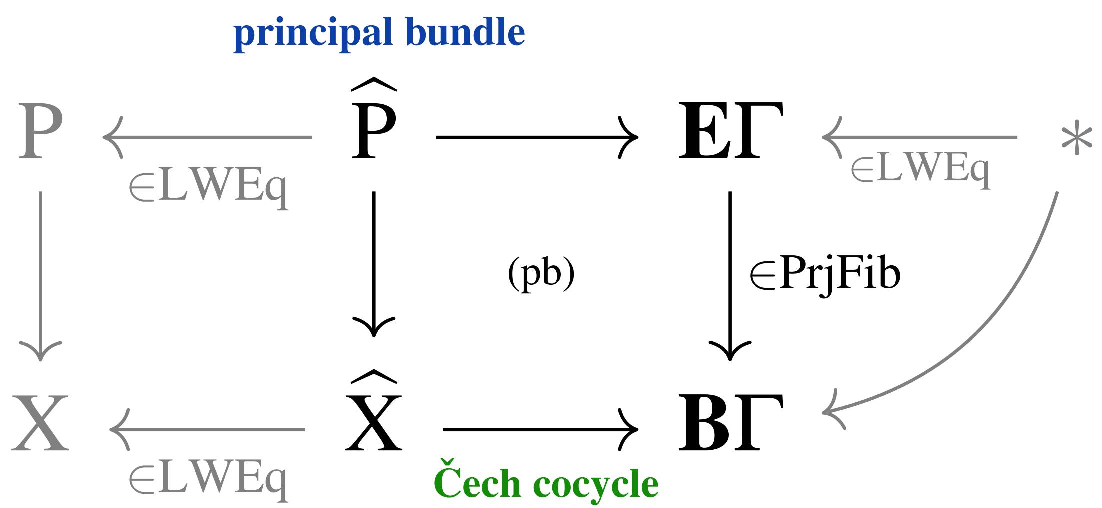
Principal bundles via smooth groupoids – Universal principal bundles.

The inclusion of the unique base point into $\mathbf{B}\Gamma$ has the following *fibrant resolution*:



Principal bundles via smooth groupoids.

The *homotopy fiber* of a 2-functor = Čech cocycle is equivalently
the principal bundle P *it classifies*:



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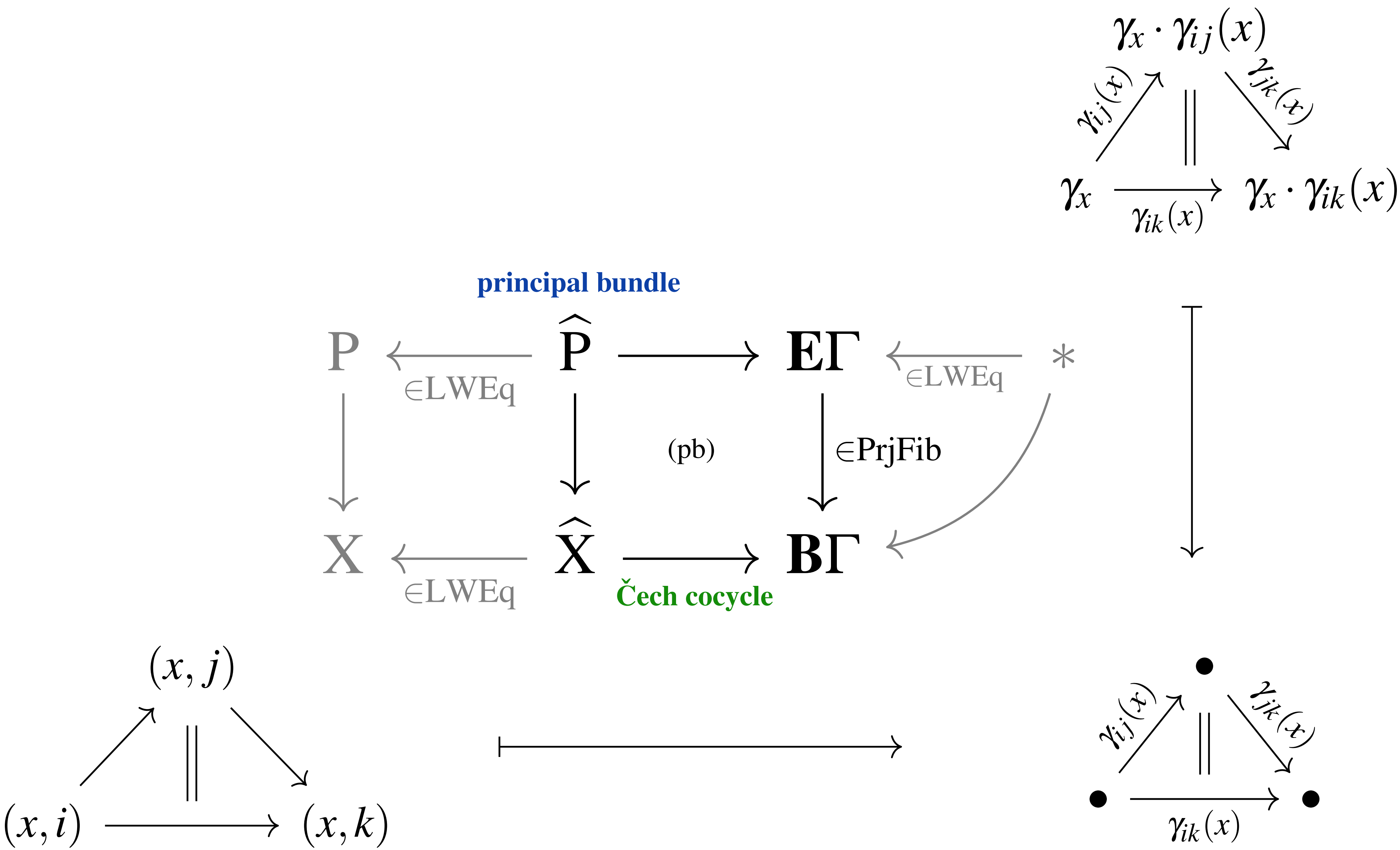

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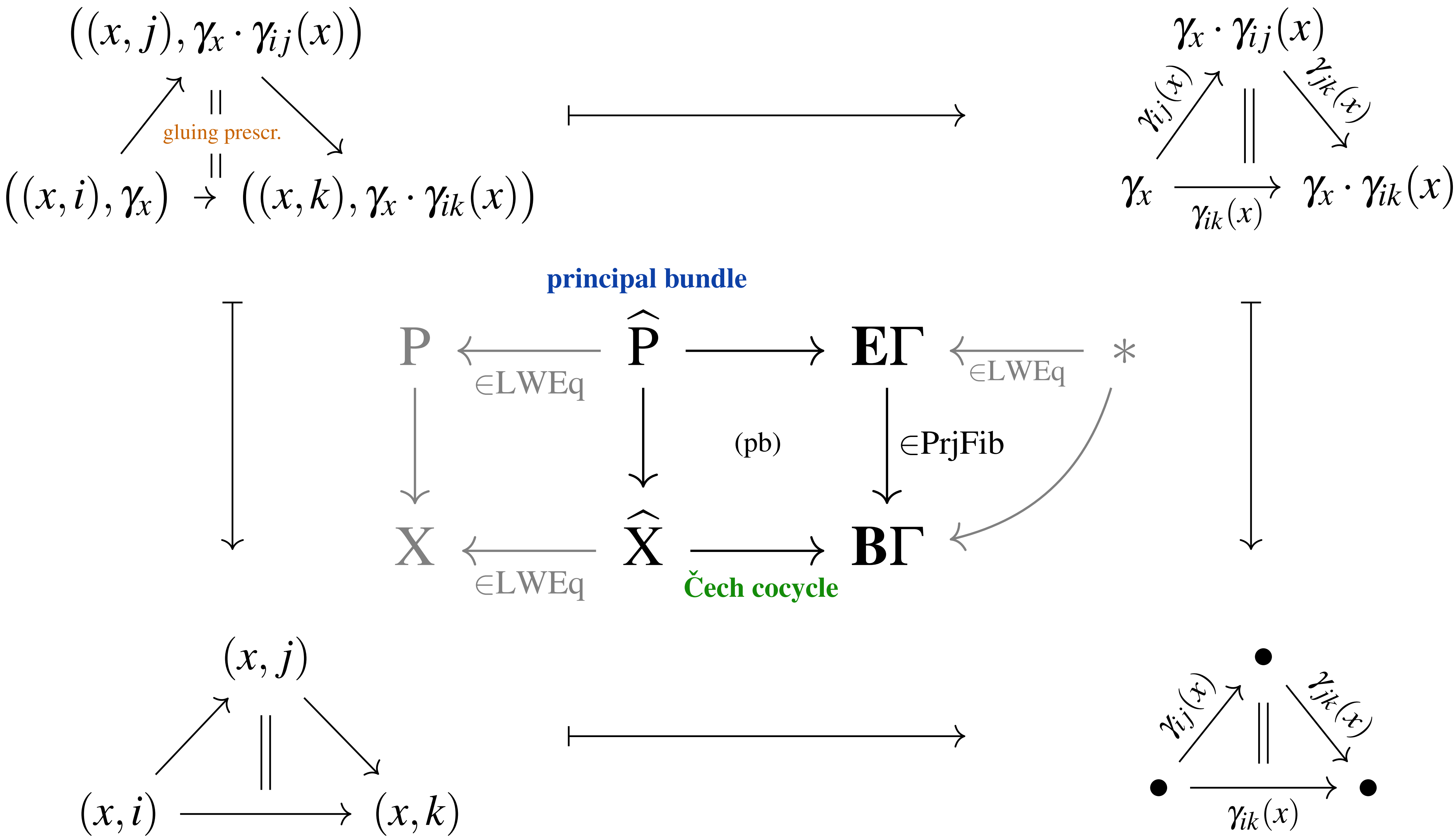

Principal 2-bundles via smooth 2-groupoids.

This neat formulation of ordinary principal bundles immediatly generalizes to give principal 2-bundles:

Principal 2-bundles via smooth 2-groupoids.

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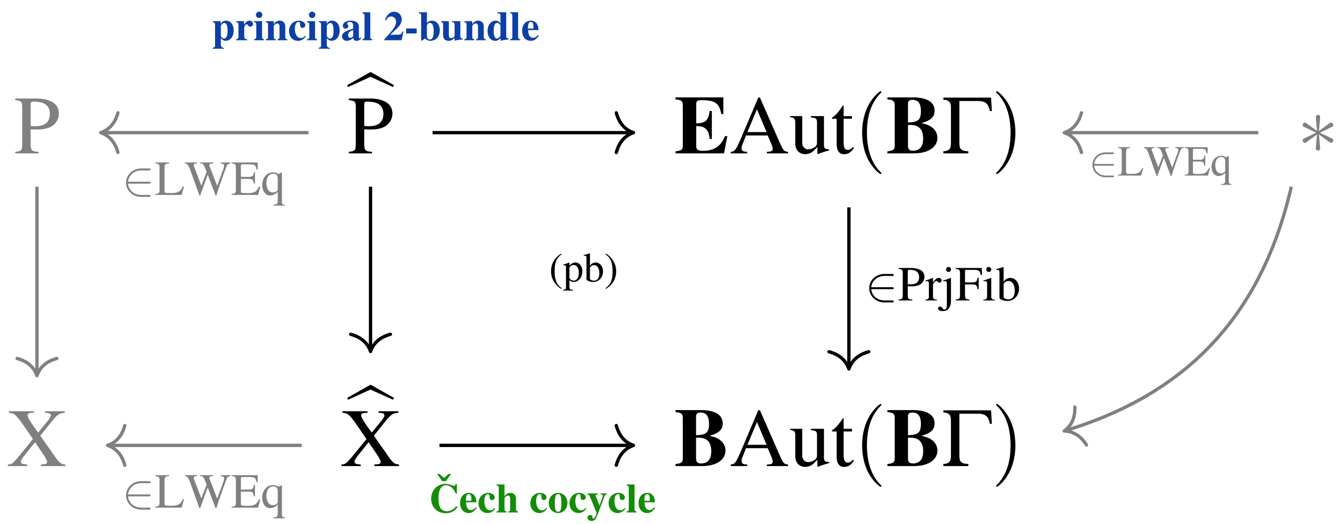
immediatly generalizes to give principal 2-bundles:

E.g. for the structure 2-group $\mathrm{Aut}(\mathbf{B}\Gamma)$

these are equivalently Giraud's *non-abelian gerbes*:



While it's tradition to be esoteric about this simple affair,

here to highlight that this is really about *twisted cohomology*: ⟶

Principal 2-bundles via smooth 2-groupoids – Example: Twisted cohomology.

For structure 2-group $\mathrm{Aut}(\mathbf{B}\mathbb{Z}) \simeq (\mathbf{B}\mathbb{Z}) \rtimes \mathbb{Z}_2$,
with $\mathbf{B}\mathrm{Aut}(\mathbf{B}\mathbb{Z}) \simeq (\mathbf{B}^2\mathbb{Z})/\!/\mathbb{Z}_2$ and $\widehat{\mathrm{X}} \xrightarrow{\sigma} \mathbf{B}\mathbb{Z}_2$ a double covering, then
2nd integral cohomology of X with local coefficients is smooth 2-functors:

$$\begin{array}{ccc} \widehat{\mathrm{X}} & \overset{\text{smooth 2-functor = Čech 2-cocycle}}{\dashrightarrow} & (\mathbf{B}^2\mathbb{Z})/\!/\mathbb{Z}_2 \\ & {}_{\sigma}\searrow \quad \swarrow & \\ & \mathbf{B}\mathbb{Z}_2 & \end{array}$$

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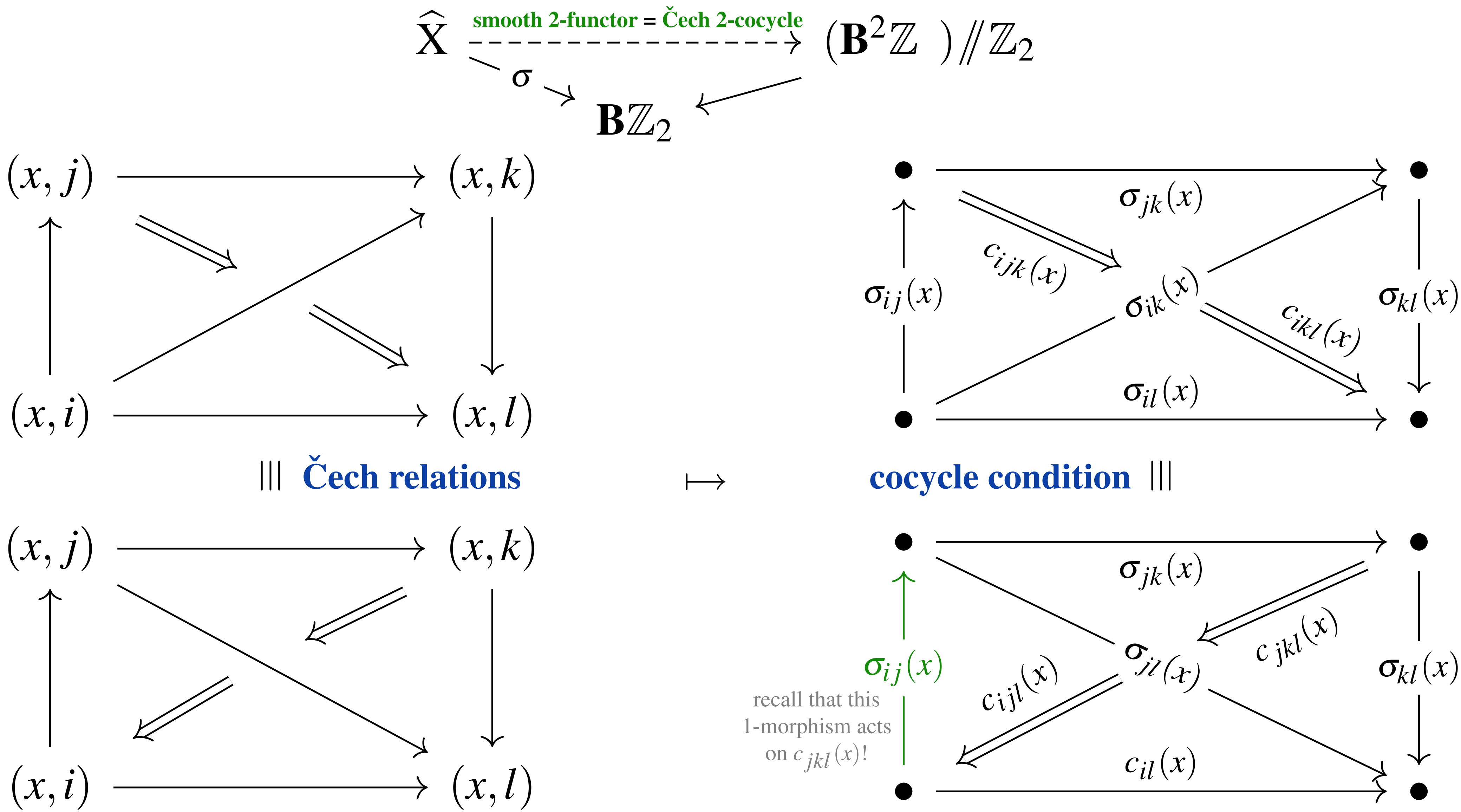
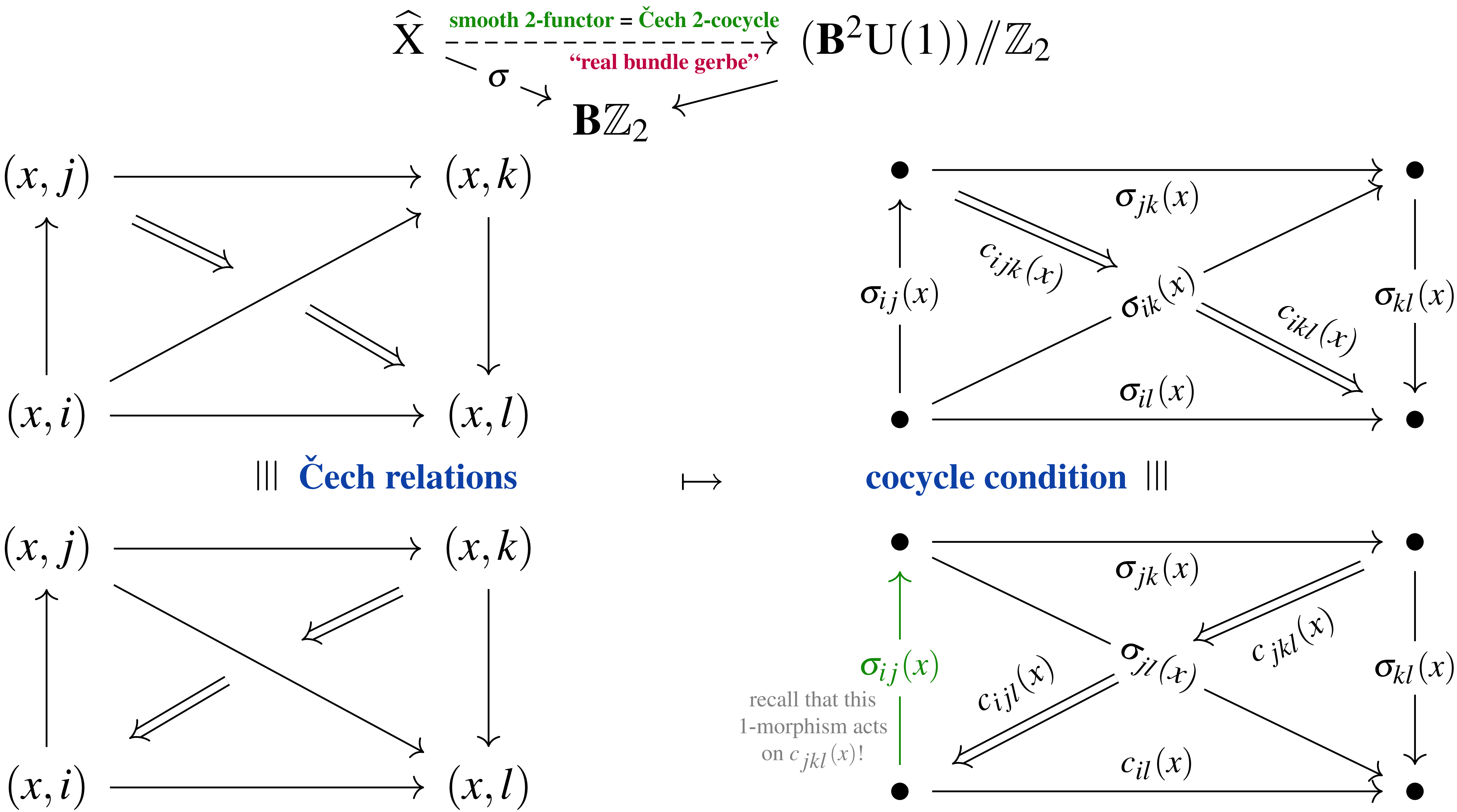


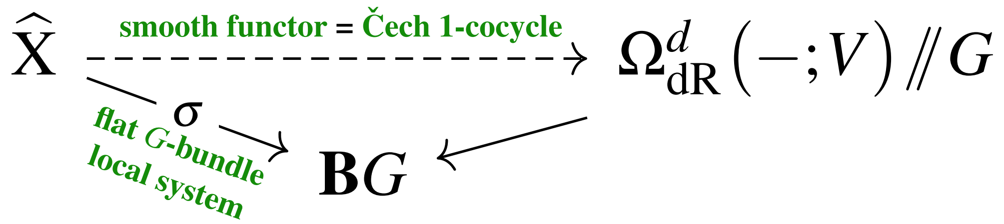
Principal 2-bundles via smooth 2-groupoids – Example: Jandl bundle gerbes.

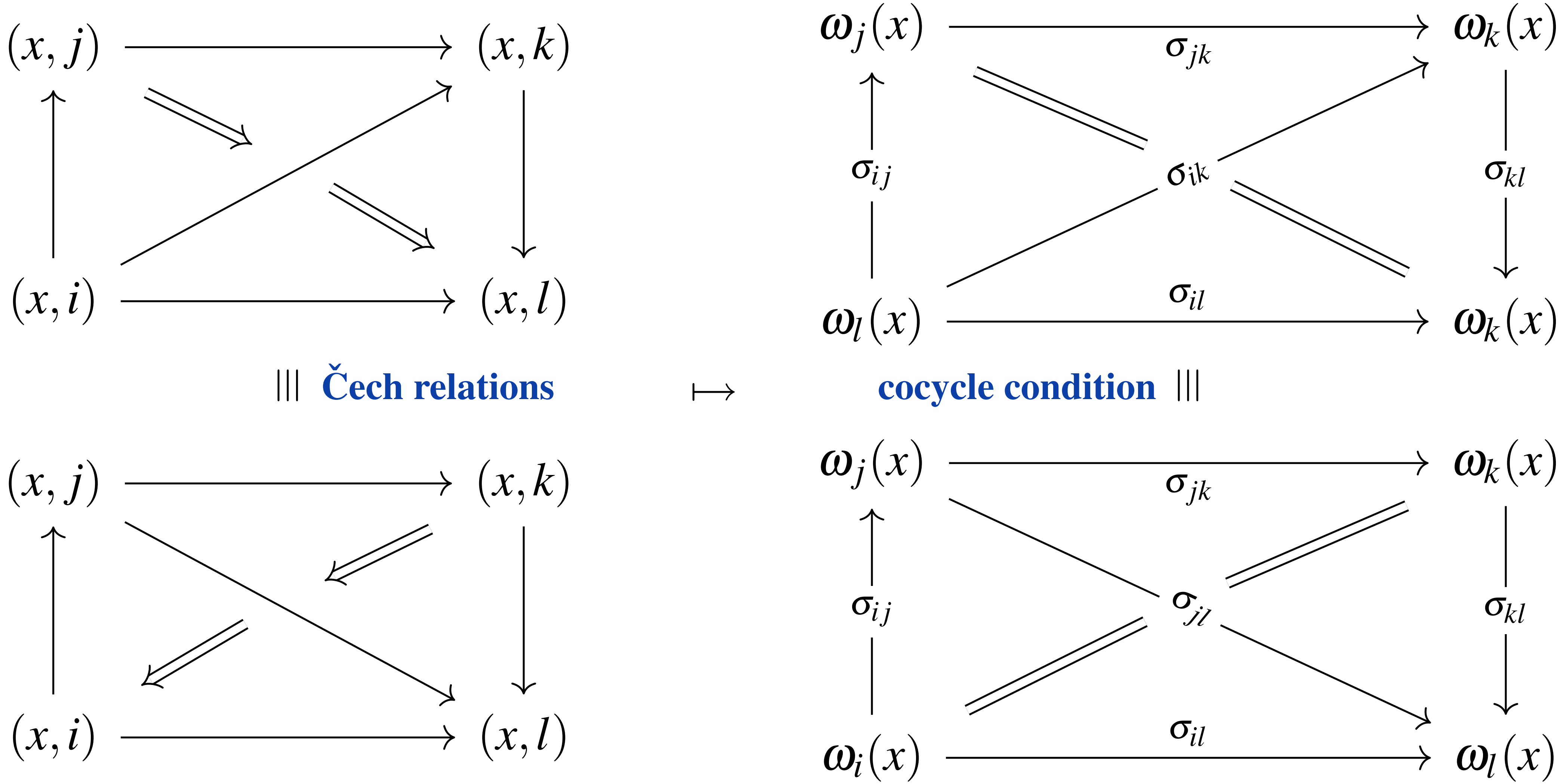
For structure 2-group $\mathrm{Aut}(\mathbf{B}\mathrm{U}(1)) \simeq (\mathbf{B}\mathrm{U}(1)) \rtimes \mathbb{Z}_2$,
with $\mathbf{B}\mathrm{Aut}(\mathbf{B}\mathrm{U}(1)) \simeq (\mathbf{B}^2\mathrm{U}(1))/\!\!/\mathbb{Z}_2$ and $\widehat{\mathrm{X}} \xrightarrow{\sigma} \mathbf{B}\mathbb{Z}_2$ a double covering, then
2nd $\mathrm{U}(1)$**-valued cohomology** of X with local coefficients is smooth 2-functors:



Principal 2-bundles via smooth 2-groupoids – Example: Twisted cohomology.

For equivariant de Rham coefficients with $G \circlearrowright V$ a representation of a finite group:





Principal 2-bundles via smooth 2-groupoids – Punchline.

So:

Non-abelian 1-cohomology is modulated by 1-stacks $\mathbf{B}\Gamma$,
abelian 2-cohomology is modulated by 2-stacks $\mathbf{B}^2 A$, etc.

Higher fiber/principal bundles are *bundles of such moduli stacks*,
hence are families of moduli stacks that vary over the base space,
hence locally modulate cohomology as before,
but now subject to global twists.

Principal 2-bundles via smooth 2-groupoids – Equivariance.

Finally, the "higher topos" of smooth 2-groupoids
has *equivariance* natively built into it: just let domain spaces be groupoids, too.

Principal 2-bundles via smooth 2-groupoids – Equivariance.

Finally, the "higher topos" of smooth 2-groupoids
has *equivariance* natively built into it: just let domain spaces be groupoids, too:

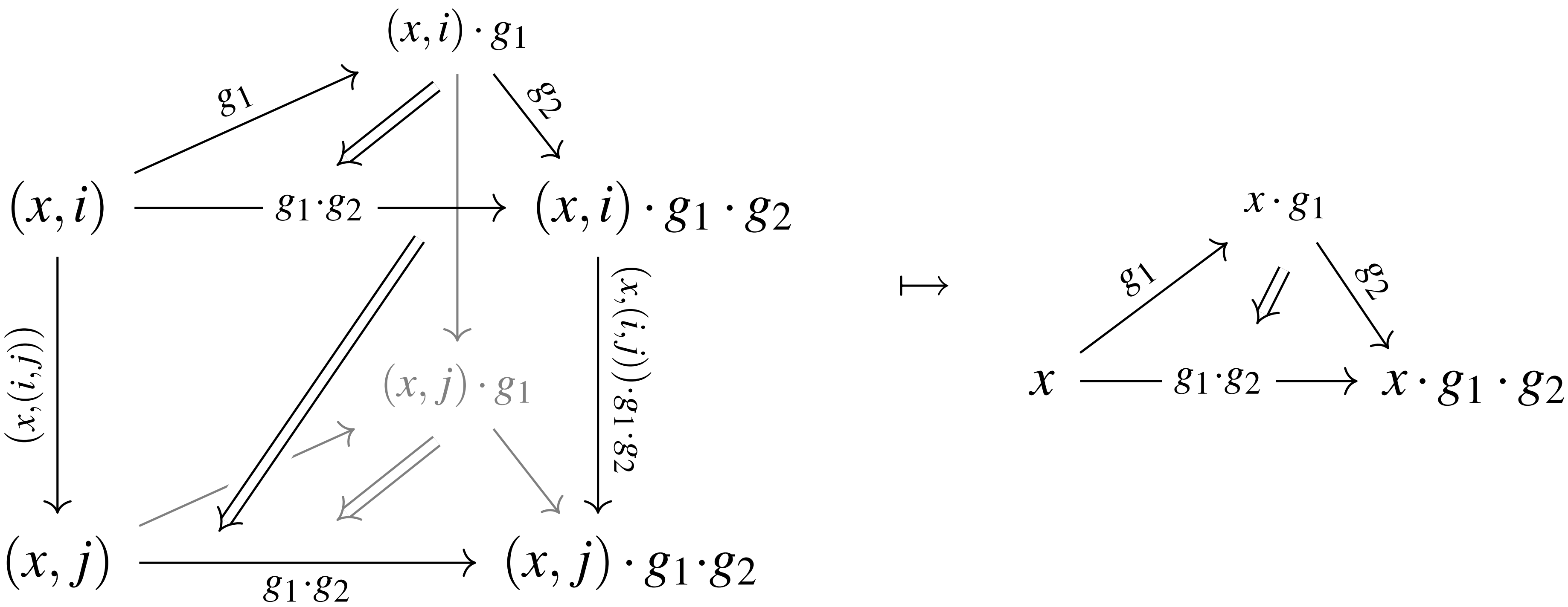
For $X \circlearrowleft G$ a smooth action of a finite group on a smooth manifold.

there exists an *equivariant good open cover*

$$\overset{\circlearrowleft G}{\coprod_{i \in I} U_i} \twoheadrightarrow \overset{\circlearrowleft G}{X}$$

and its *equivariant Čech groupoid*:

$$\varnothing \xrightarrow[\in \mathrm{PrjCof}]{} \widehat{X /\!\!/ G} \xrightarrow[\in \mathrm{LWEq}]{\text{cofibrant resolution}} \underset{\text{action groupoid}}{X /\!\!/ G}$$



Principal 2-bundles via smooth 2-groupoids – Equivariance.

For $\mathrm{X} \curvearrowleft G$ a smooth manifold and $\Gamma \curvearrowleft G$ a smooth 2-group
both equipped with smooth G-action, a
G-equivariant Γ-principal 2-bundle on X is modulated by a smooth 2-functor like this:

$$\begin{array}{ccc} \widehat{\mathrm{X} /\!\!/ G} & \overset{\text{modulating map}}{\dashrightarrow} & \mathbf{B}\Gamma /\!\!/ G \\ & \searrow \qquad \swarrow & \\ & \mathbf{B}G & \end{array}$$

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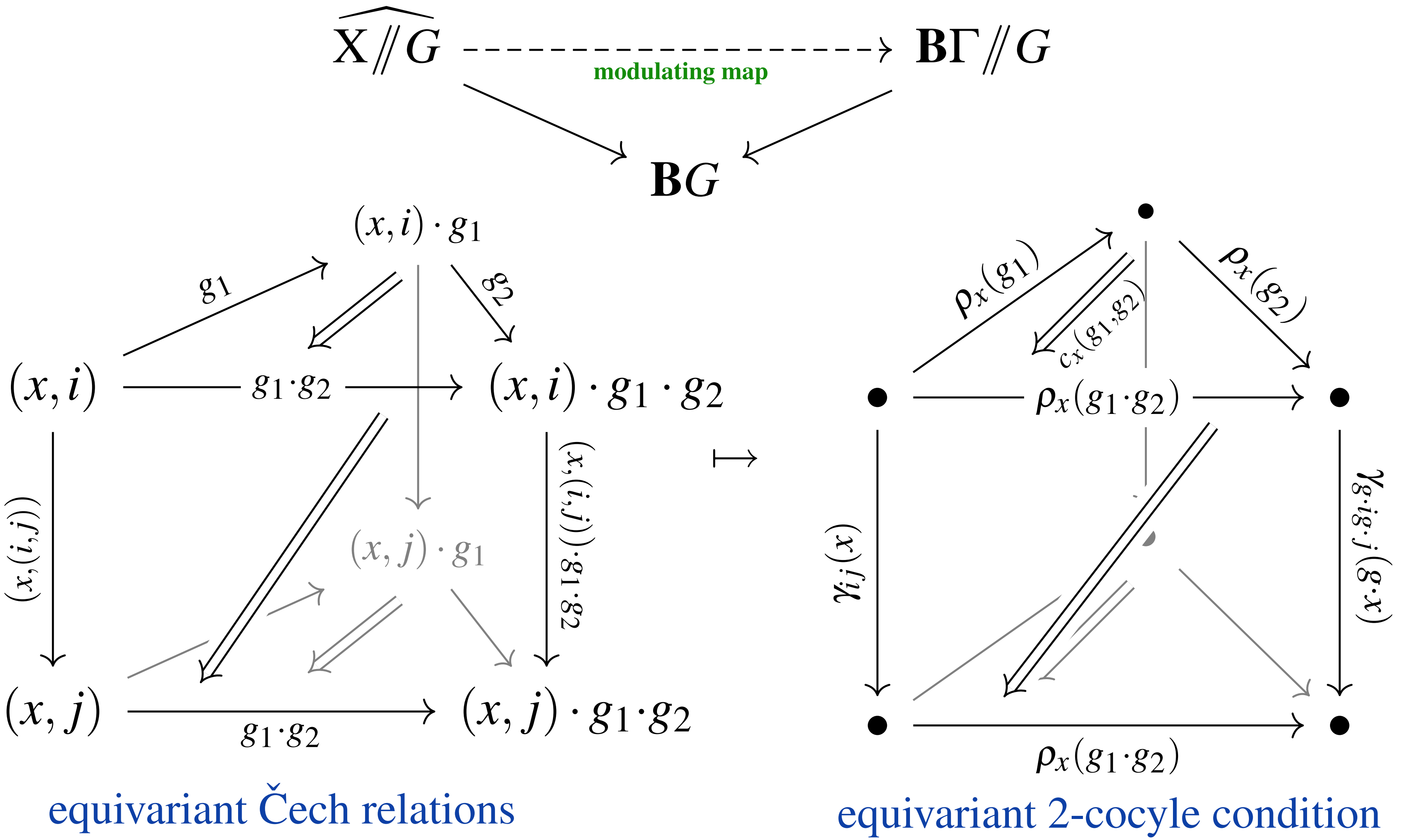

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$$
\begin{array}{ccc}
\widehat{\mathrm{X} /\!\!/ G} & \xrightarrow[\text{modulating map}]{} & \mathbf{B}\Gamma /\!\!/ G \simeq \mathbf{B}\big(\Gamma \rtimes G\big) \\
& \searrow \quad \swarrow & \\
& \mathbf{B}G &
\end{array}
$$

on the right we have equivalently the semidirect product 2-group.

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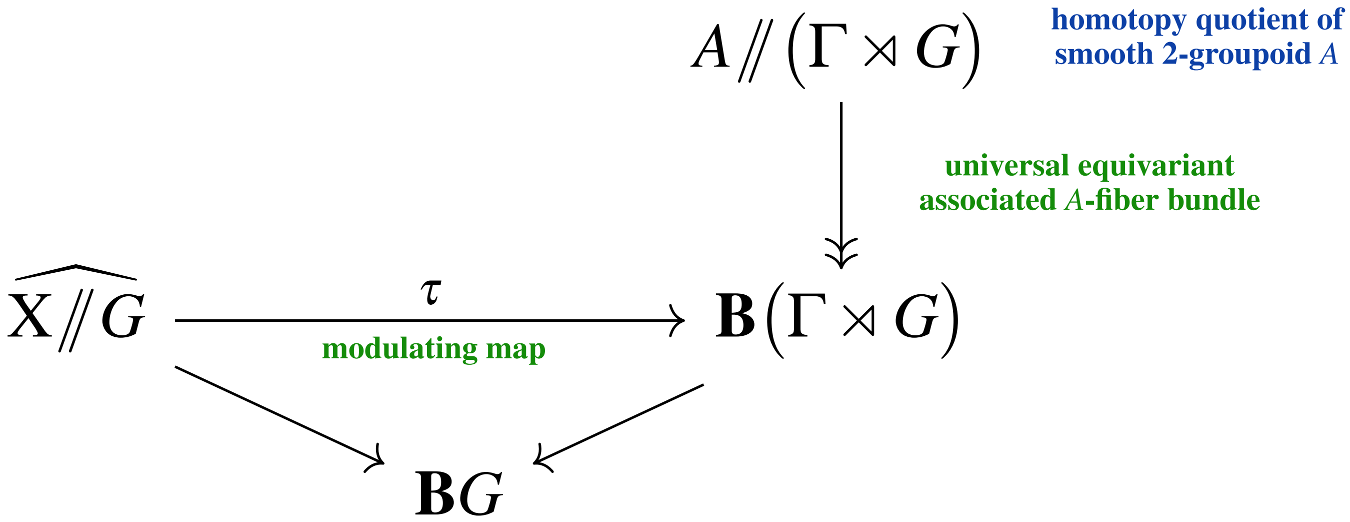
$$\begin{array}{ccc} \widehat{\mathrm{X} /\!\!/ G} & \xrightarrow[\text{modulating map}]{\text{eg. equivariant real bundle gerbe}} & \mathbf{B}^2 \mathrm{U}(1) /\!\!/ \mathbb{Z}_2 \simeq \mathbf{B}\big(\mathbf{B}\mathrm{U}(1) \rtimes \mathbb{Z}_2\big) \\ & \searrow \quad \swarrow & \\ & \mathbf{B}\mathbb{Z}_2 & \end{array}$$

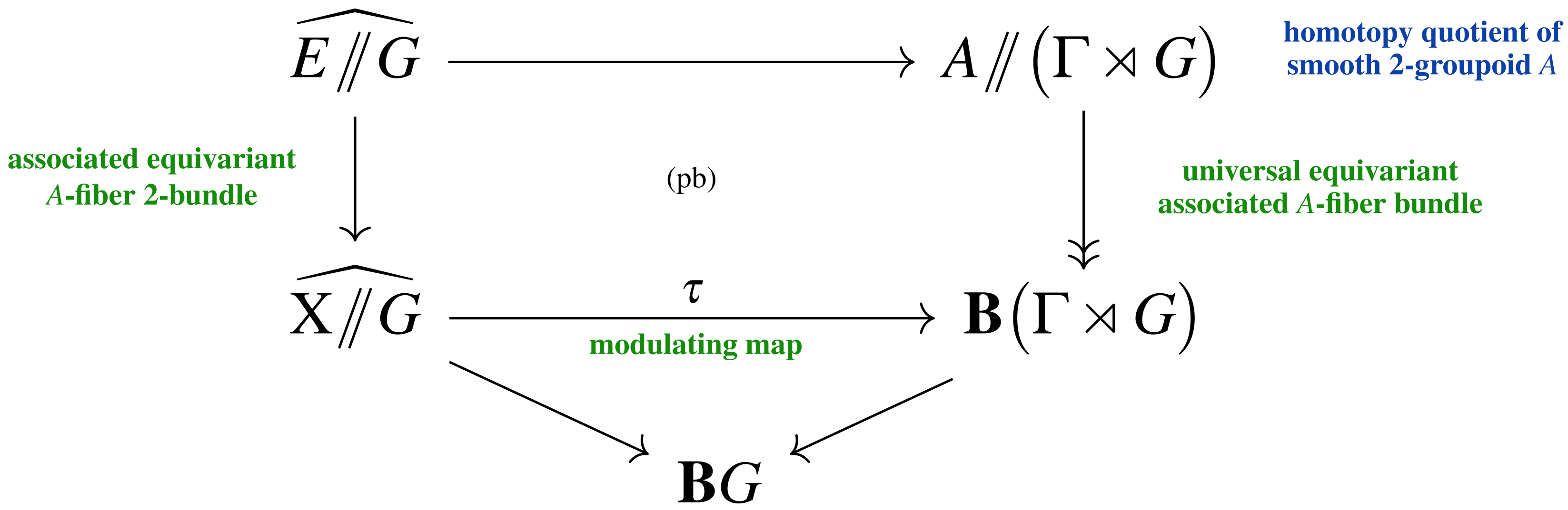
on the right we have equivalently the semidirect product 2-group.

Principal 2-bundles via smooth 2-groupoids – Associated 2-bundles.



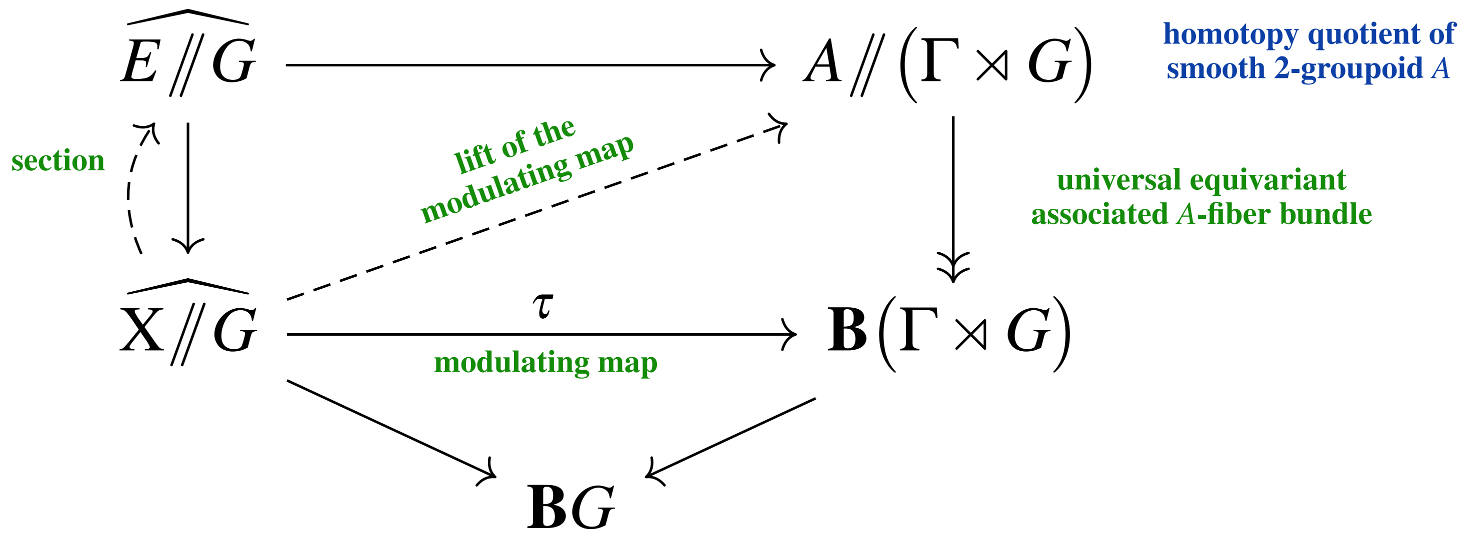
A fibration over that is equivalently
an equivariant ∞-action $(\Gamma \rtimes G) \circlearrowright A$
embodied by its universal *associated 2-bundle*.

Principal 2-bundles via smooth 2-groupoids – Associated 2-bundles.



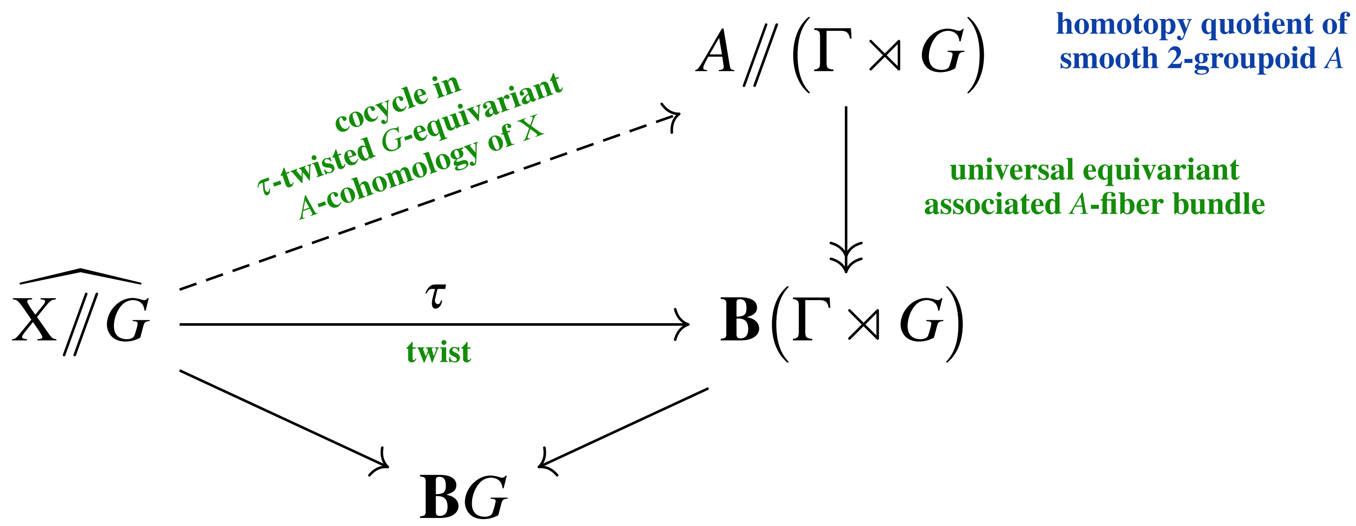
Its pullback is
the equivariant A-fiber 2-bundle
which is associated to
the given equivariant principal 2-bundle.

Principal 2-bundles via smooth 2-groupoids – Associated 2-bundles.



The equivariant sections are equivalently
the lifts of the modulating map.

Twisted equivariant non-abelian cohomology.

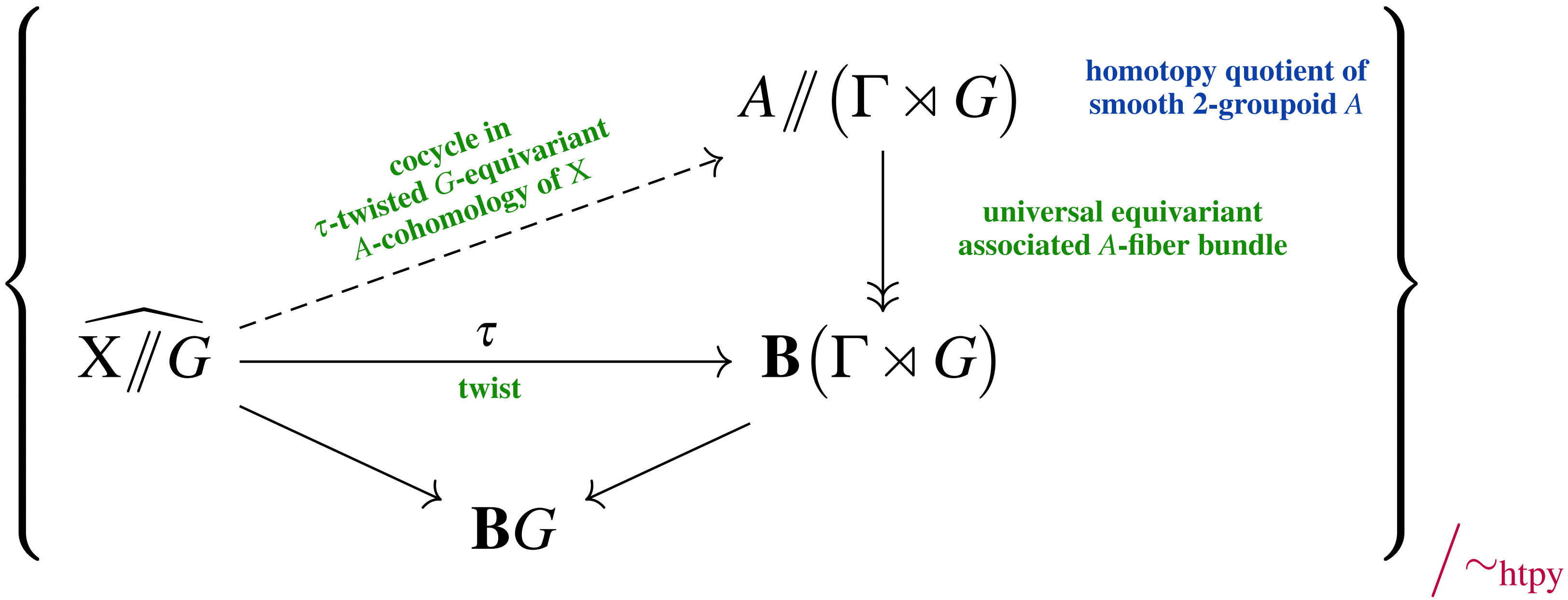


Equivalently, these are the *cocycles* of
τ-twisted G-equivariant A-cohomology.

Twisted equivariant non-abelian cohomology.

twisted equivariant cohomology set

$\mathrm{A}^{\tau}_{G}(\mathrm{X}) \; :=$



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I – Equivariant ∞-Bundles

II – TED-K-Theory

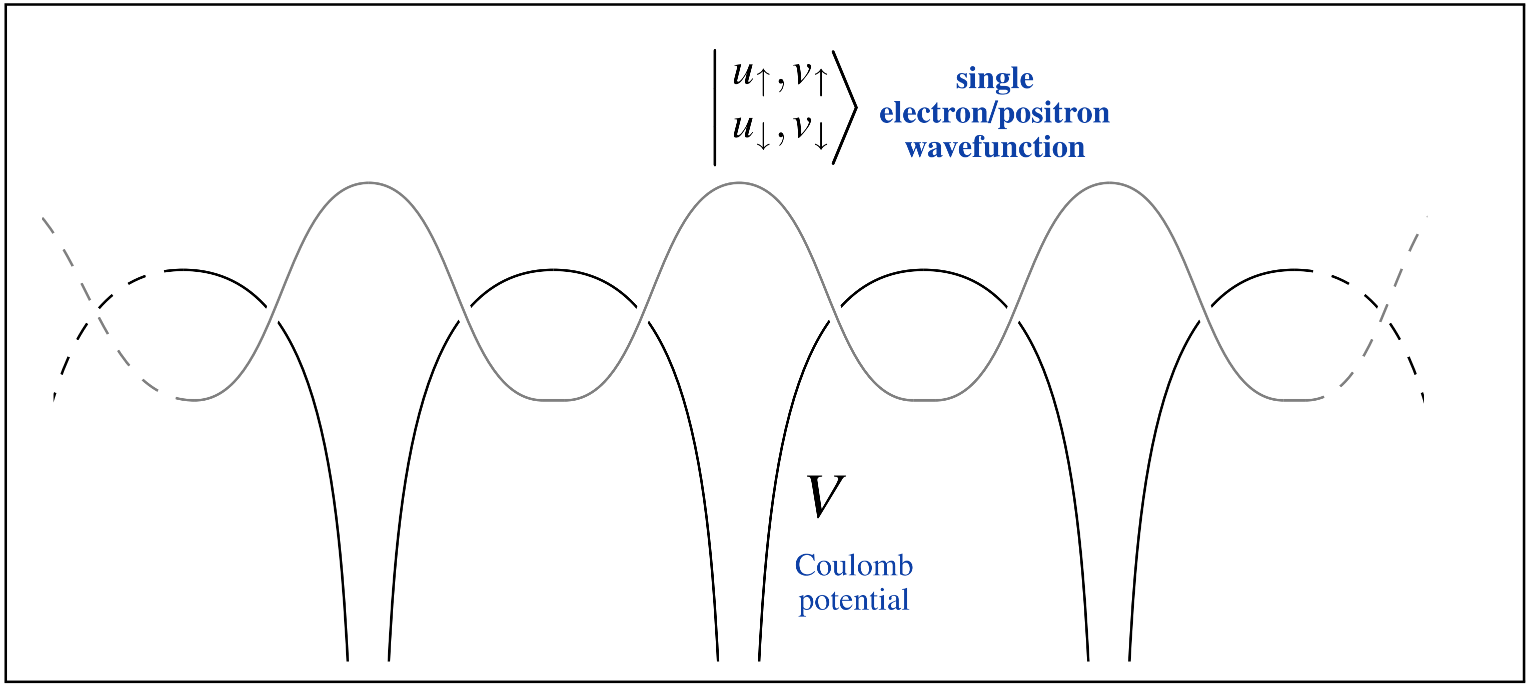
III – Anyonic Defect Branes

This part is a
quick motivation and exposition of
twisted equivariant KR-theory
following these articles:

Equivariant Principal ∞-bundles [arXiv:2112.13654]
Anyonic Defect Branes in TED-K-Theory [arXiv:2203.11838]
The TED character map (in preparation)

Vacua of electron/positron field in Coulomb background.

Fact ([KS77][CHO82]). The vacua of the free Dirac quantum field in a classical Coulomb background...



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finite-dimensional kernel | Fredholm operator | finite-dimensional cokernel

$$\overbrace{\left\{\psi \in \mathcal{H} \,\middle|\, \forall_\phi \,\langle \phi | F | \psi \rangle = 0\right\}}^{\text{ker}(F)} \hookrightarrow \mathcal{H} \xrightarrow[\text{bounded linear}]{F} \mathcal{H} \twoheadrightarrow \overbrace{\left\{\psi \in \mathcal{H} \,\middle|\, \forall_\phi \,\langle \psi | F | \phi \rangle = 0\right\}}^{\text{coker}(F)}$$

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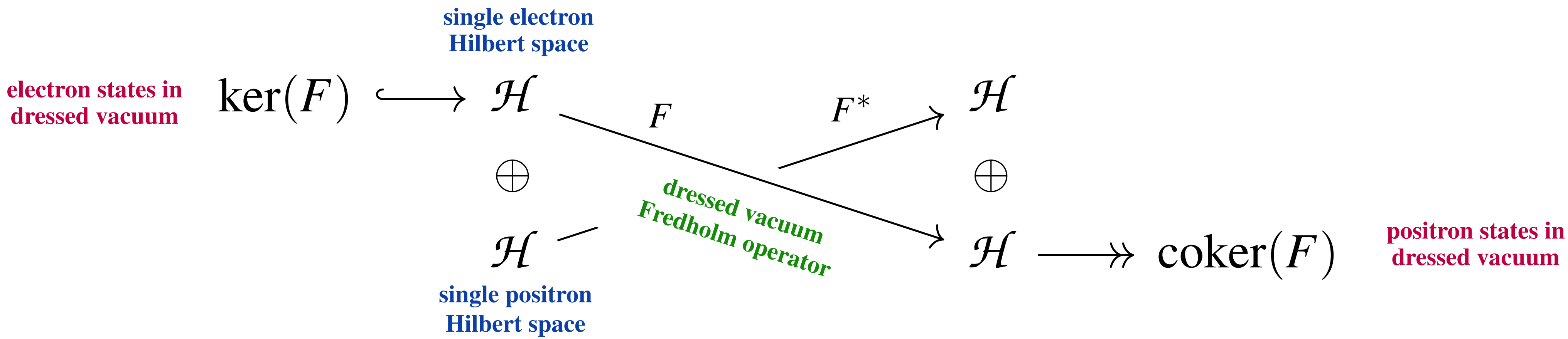
finite-dimensional kernel — Fredholm operator — finite-dimensional cokernel

$$\overbrace{\operatorname{ker}(F)}^{\text{finite-dimensional kernel}} \hookrightarrow \mathcal{H} \xrightarrow[\text{bounded linear}]{F} \mathcal{H} \twoheadrightarrow \overbrace{\operatorname{coker}(F)}^{\text{finite-dimensional cokernel}}$$

$$\operatorname{ker}(F) = \big\{ \psi \in \mathcal{H} \,\big|\, \forall_\phi \ \langle \phi | F | \psi \rangle = 0 \big\} \qquad \operatorname{coker}(F) = \big\{ \psi \in \mathcal{H} \,|\, \forall_\phi \ \langle \psi | F | \phi \rangle = 0 \big\}$$

on the single-electron/positron Hilbert space:



$$\underbrace{\operatorname{ind}(F)}_{\text{total charge in dressed vacuum}} = \underbrace{\dim\big(\operatorname{ker}(F)\big)}_{\text{number of electrons in dressed vacuum state}} - \underbrace{\dim\big(\operatorname{coker}(F)\big)}_{\text{number of positrons in dressed vacuum state}}$$

$$= \dim\big(\operatorname{coker}(F^*)\big) - \dim\big(\operatorname{ker}(F^*)\big)$$

Quantum symmetries.

On these dressed vacua of electron/positron states
the following *CPT-twisted projective group*

even projective unitary group

grading involution

complex conjugation

$$\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \Big(\underbrace{\mathbb{Z}_2}_{\{\mathrm{e}, P\}} \times \underbrace{\mathbb{Z}_2}_{\{\mathrm{e}, T\}} \Big)$$

group of quantum symmetries

$$C := PT\,, \quad P \cdot \big[U_+, U_-\big] := \big[U_-, U_+\big] \cdot P\,, \qquad T \cdot \big[U_+, U_-\big] := \big[\overline{U}_+, \overline{U}_-\big] \cdot T$$

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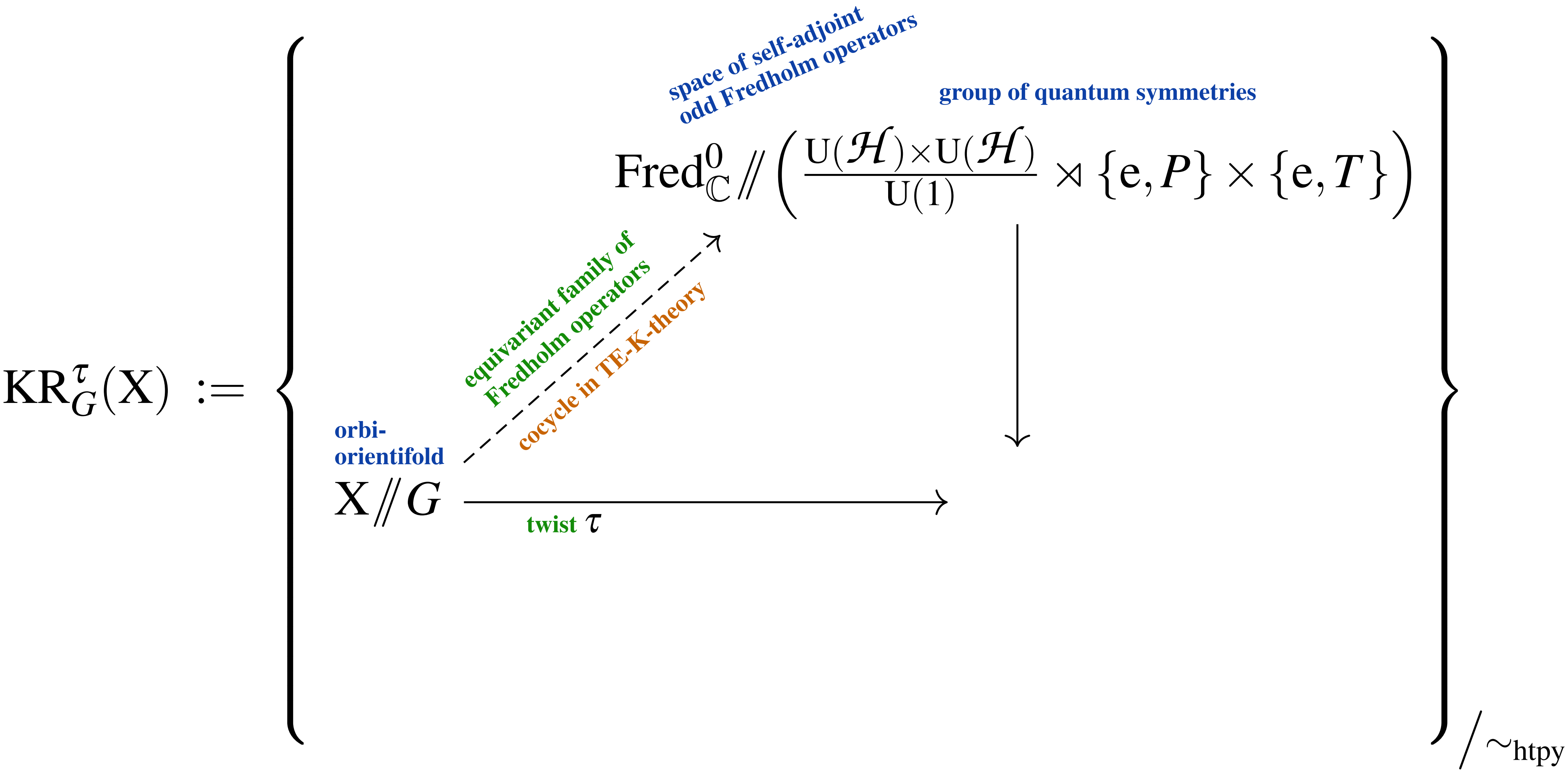
naturally acts by conjugation:

$$\begin{array}{rcccl}
[U_+, U_-] & : & F & \longmapsto & U_+^{-1} \circ F \circ U_- \\
C \cdot [U_+, U_-] & : & F & \longmapsto & U_-^{-1} \circ F^t \circ U_+ \\
P \cdot [U_+, U_-] & : & F & \longmapsto & U_-^{-1} \circ F^* \circ U_+ \\
T \cdot [U_+, U_-] & : & F & \longmapsto & U_+^{-1} \circ \overline{F} \circ U_-
\end{array}$$

Twisted equivariant KR-theory – As a single diagram of smooth groupoids.

Homotopy classes of quantum-symmetry equivariant families
of such self-adjoint odd Fredholm operators
constitute *twisted equivariant* KR-*cohomology*:



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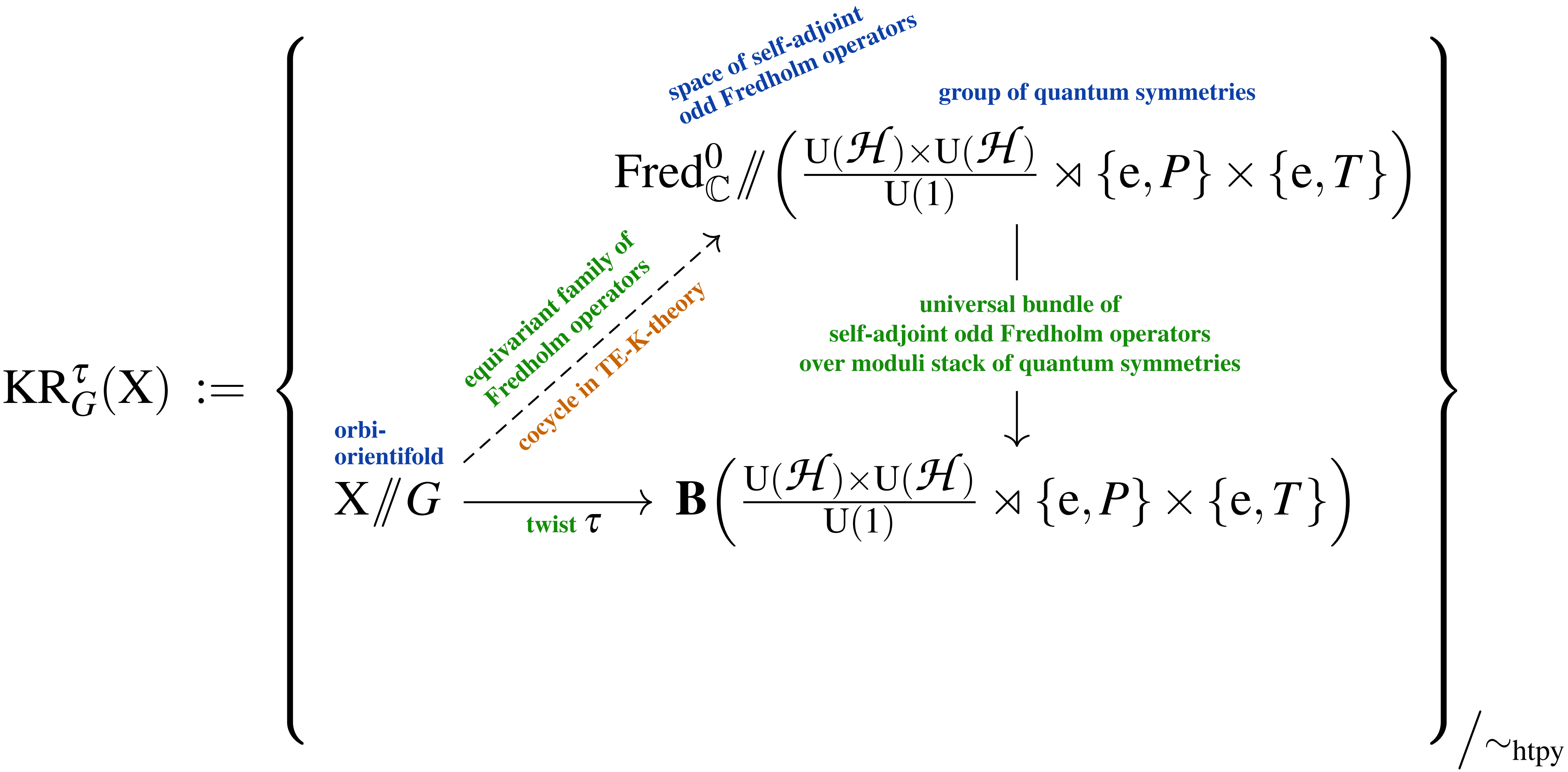

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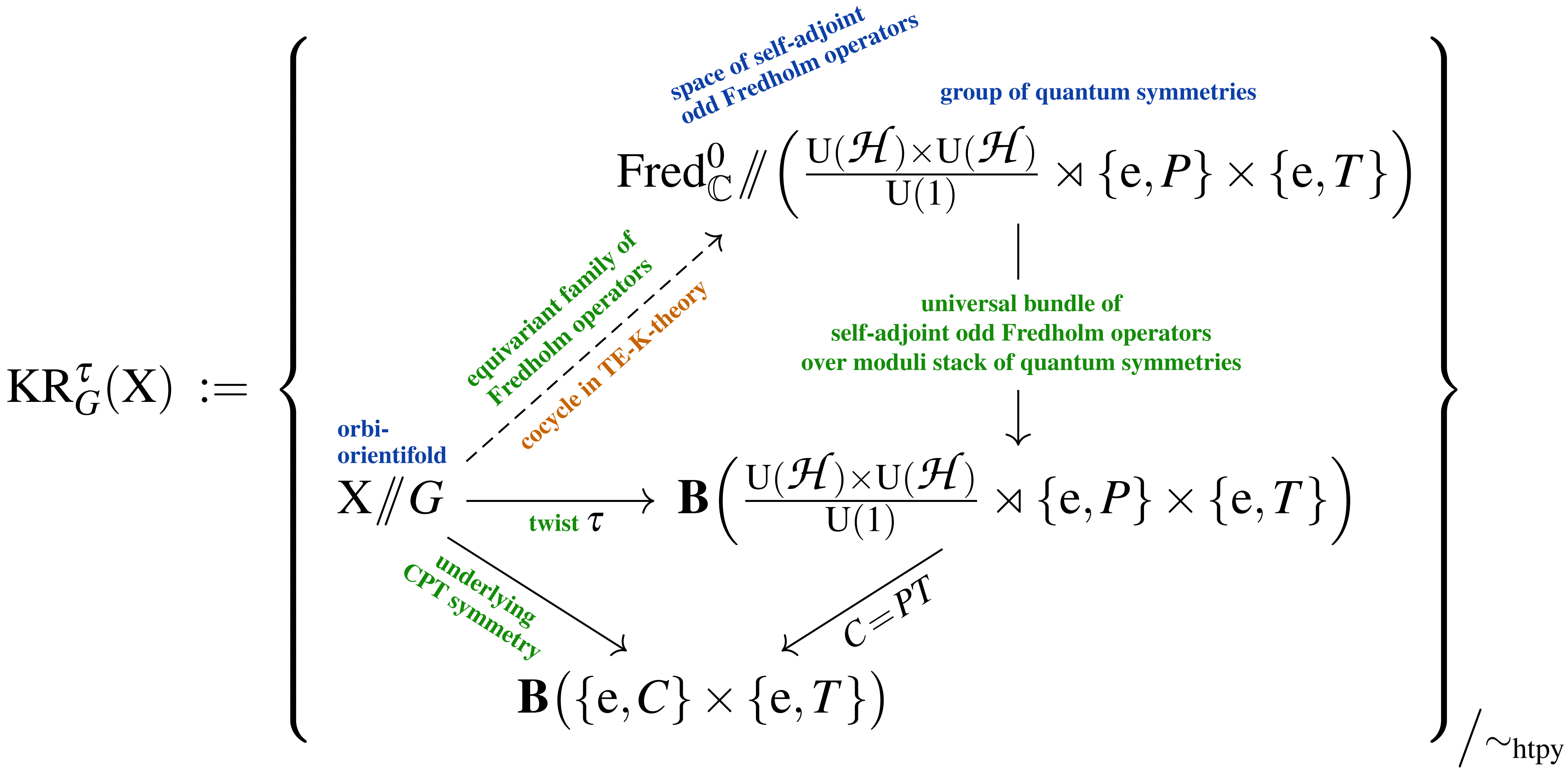


CPT Quantum symmetries.

$$
\begin{array}{ccccc}
\mathbf{B}\big(\{\mathrm{e}, T\}\big) & \xrightarrow[\;]{\substack{\textbf{pure quantum T-symmetry} \\ T \longmapsto \widehat{T}}} & \mathbf{B}\left(\dfrac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \{\mathrm{e}, T\}\right) & \longrightarrow & \mathbf{B}\big(\mathbf{B}\mathrm{U}(1) \rtimes \{\mathrm{e}, T\}\big) \\
 & \searrow \quad \swarrow & & & \\
 & \mathbf{B}\big(\{\mathrm{e}, P\} \times \{\mathrm{e}, T\}\big) & & &
\end{array}
$$

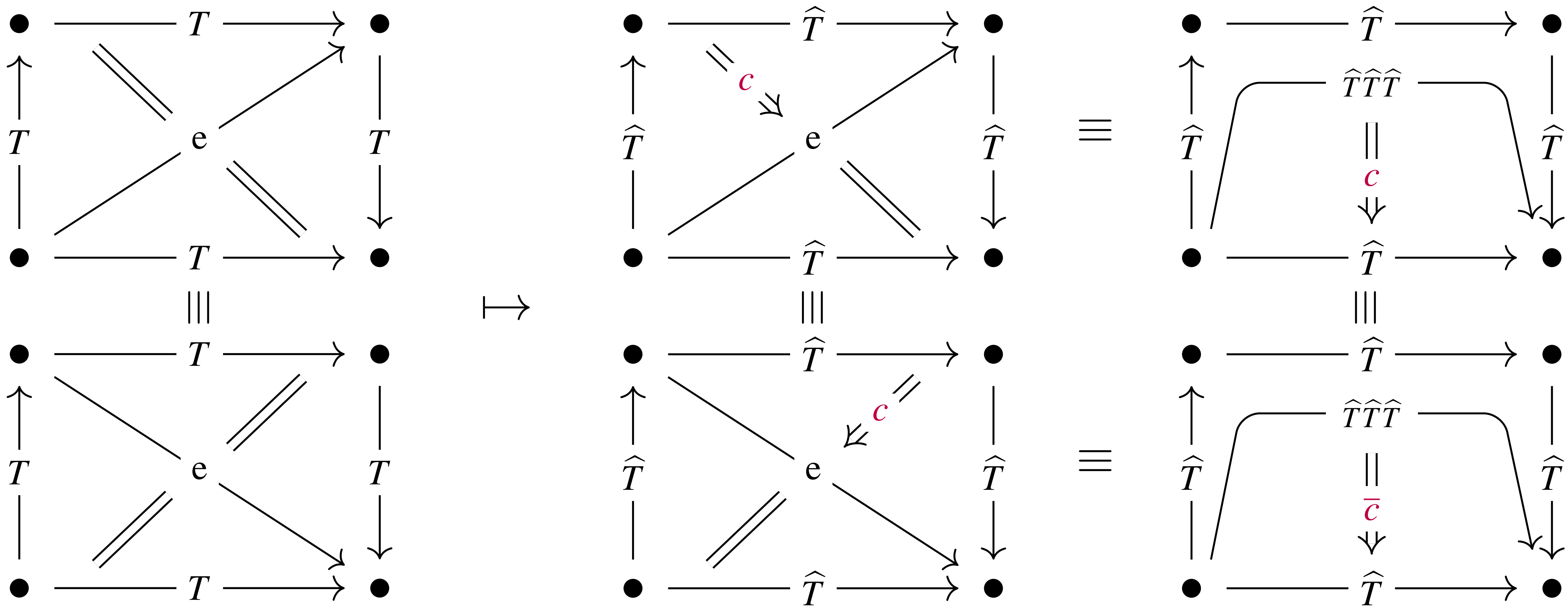
Let’s use the previous machinery to compute the possible quantum T-symmetries...

CPT Quantum symmetries.

pure quantum T-symmetry

$$\mathbf{B}\big(\{\mathrm{e},T\}\big) \xrightarrow{\;T \,\longmapsto\, \widehat{T}\;} \mathbf{B}\left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \{\mathrm{e},T\} \right) \longrightarrow \mathbf{B}\big(\mathbf{B}\mathrm{U}(1) \rtimes \{\mathrm{e},T\}\big)$$

$$\mathbf{B}\big(\{\mathrm{e},T\}\big) \hookrightarrow \mathbf{B}\big(\{\mathrm{e},P\} \times \{\mathrm{e},T\}\big) \longleftarrow \mathbf{B}\left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \{\mathrm{e},T\} \right)$$

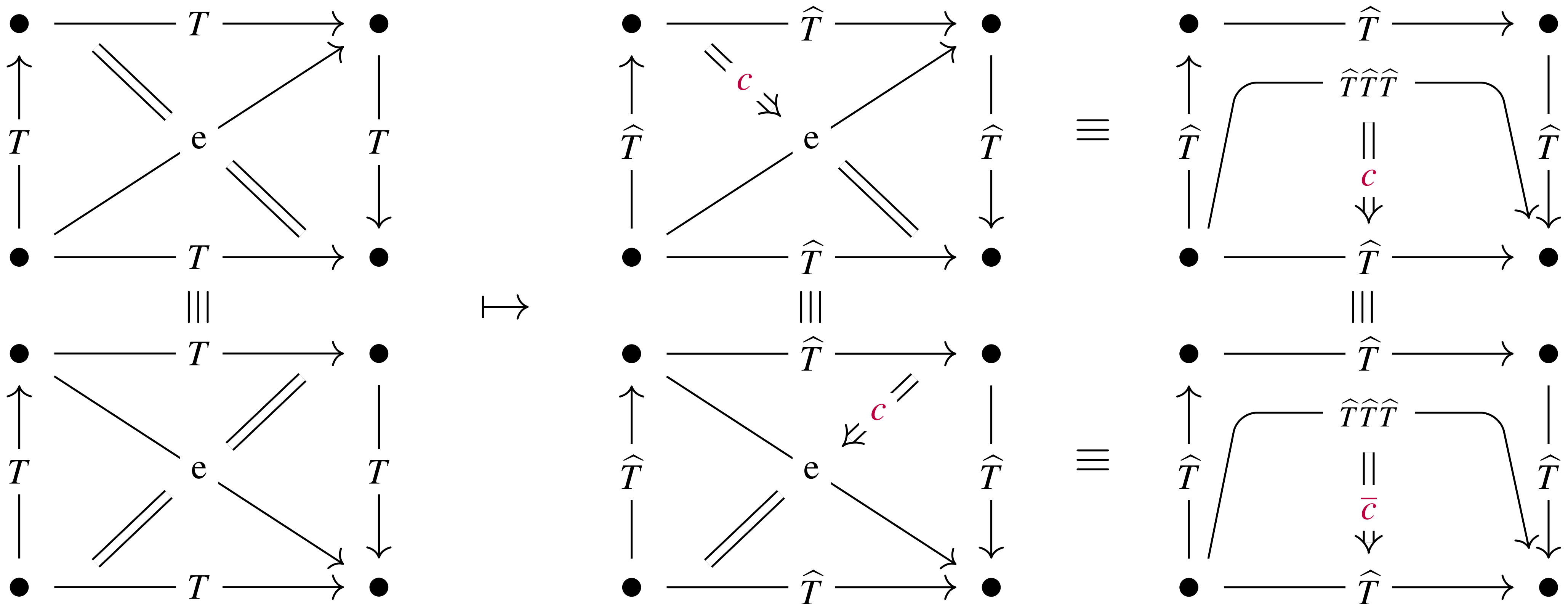


CPT Quantum symmetries.

$$\mathbf{B}\big(\{\mathrm{e}, T\}\big) \xrightarrow[T \longmapsto \widehat{T}]{\text{pure quantum T-symmetry}} \mathbf{B}\left(\frac{\mathrm{U}(\mathcal{H}) \times \mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \{\mathrm{e}, T\} \right) \longrightarrow \mathbf{B}\big(\mathbf{B}\mathrm{U}(1) \rtimes \{\mathrm{e}, T\}\big)$$

$$\searrow \qquad \swarrow$$

$$\mathbf{B}\big(\{\mathrm{e}, P\} \times \{\mathrm{e}, T\}\big)$$



So $\overline{c} = c$ and hence there are two choices for quantum T-symmetry, up to homotopy:

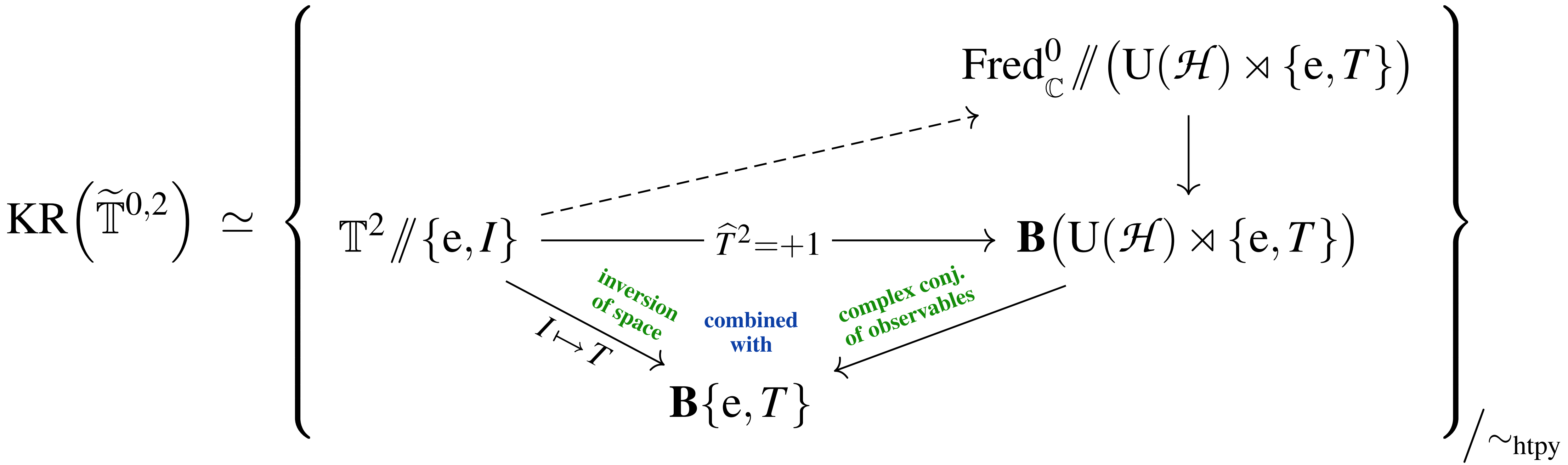
$$\widehat{T}^2 = \pm 1 \quad \text{and similarly} \quad \widehat{C}^2 = \pm 1 \,.$$

Example – Orientifold KR-theory

Let I be *I*nversion action on 2-torus $\widetilde{\mathbb{T}}^2 \simeq \mathbb{R}^2/\mathbb{Z}^2$ and trivial action on observables

$$\begin{array}{ccc} \mathbb{T}^2 & \xrightarrow{\;\;I\;\;} & \mathbb{T}^2 \\ k & \longmapsto & -k\,, \end{array} \qquad\qquad \begin{array}{ccc} \mathrm{Fred}^0_{\mathbb{C}} & \xrightarrow{\;\;I\;\;} & \mathrm{Fred}^0_{\mathbb{C}} \\ F & \longmapsto & F\,. \end{array}$$

If T acts as I on $\mathbb{T}^2$, then $\mathrm{KR}^{\widehat{T}^2 = +1}$ is *Atiyah's Real K-theory* aka *orienti-fold* K-theory:



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$$\mathrm{KR}\left(\widetilde{\mathbb{T}}^{0,2}\right) \simeq \left\{ \begin{array}{ccc} & & \mathrm{Fred}^0_{\mathbb{C}} /\!\!/ \big(\mathrm{U}(\mathcal{H}) \rtimes \{\mathrm{e}, T\}\big) \\ & \nearrow & \downarrow \\ \mathbb{T}^2 /\!\!/ \{\mathrm{e}, I\} & \xrightarrow{\;\widehat{T}^2 = +1\;} & \mathbf{B}\big(\mathrm{U}(\mathcal{H}) \rtimes \{\mathrm{e}, T\}\big) \\ & {\scriptstyle I \mapsto T}\searrow \qquad \swarrow & \\ & \mathbf{B}\{\mathrm{e}, T\} & \end{array} \right\}_{\big/\sim_{\mathrm{htpy}}}$$

inversion of space

combined with

complex conj. of observables

But what happens on I-fixed loci i.e. on "orientifolds" ? $\longrightarrow$

CPT Quantum symmetries – 10 global choices. (following [FM12, Prop. 6.4])

Equivariance group	$G =$	$\{\mathrm{e}\}$	$\{\mathrm{e}, P\}$	$\{\mathrm{e}, T\}$		$\{\mathrm{e}, C\}$		$\{\mathrm{e}, T\} \times \{\mathrm{e}, C\}$			
Realization as quantum symmetry τ :	$\widehat{T}^2 =$			$+1$	-1			$+1$	-1	-1	$+1$
	$\widehat{C}^2 =$					$+1$	-1	$+1$	$+1$	-1	-1
Maximal induced Clifford action anticommuting with all G-invariant odd Fredholm operators	$E_{-3} =$								$\mathrm{i}\widehat{T}\widehat{C}\beta$		
	$E_{-2} =$					$\mathrm{i}\widehat{C}\beta$			$\mathrm{i}\widehat{C}\beta$		
	$E_{-1} =$		$\widehat{P}\beta$			$\widehat{C}\beta$		$\widehat{C}\beta$	$\widehat{C}\beta$		
	$E_{+0} =$	β	β	β	$\begin{pmatrix} \beta & 0 \\ 0 & -\beta \end{pmatrix}$	β	β	β	β	β	β
	$E_{+1} =$				$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$		$\widehat{C}\beta$			$\widehat{C}\beta$	$\widehat{C}\beta$
	$E_{+2} =$				$\begin{pmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$		$\mathrm{i}\widehat{C}\beta$			$\mathrm{i}\widehat{C}\beta$	
	$E_{+3} =$				$\begin{pmatrix} 0 & -\widehat{T} \\ \widehat{T} & 0 \end{pmatrix}$					$\mathrm{i}\widehat{T}\widehat{C}\beta$	
	$E_{+4} =$				$\begin{pmatrix} 0 & \mathrm{i}\widehat{T} \\ \mathrm{i}\widehat{T} & 0 \end{pmatrix}$						
τ-twisted G-equivariant KR-theory of fixed loci	$\mathrm{KR}^{\tau} =$	KU^0	KU^1	KO^0	KO^4	KO^2	KO^6	KO^1	KO^3	KO^5	KO^7

$$
\left\{
\begin{array}{lll}
\text{bounded opers.} & \widehat{F} : \mathcal{H}^2 \xrightarrow[\mathbb{K}-\text{linear}]{\text{bounded}} \mathcal{H}^2 \\
\text{self-adjoint} & \widehat{F}^* = \widehat{F} := F + F^* \\
\text{Fredholm} & \dim\big(\ker(\widehat{F})\big) < \infty
\end{array}
\;\middle|\;
\begin{array}{l}
\text{graded comm.} \\
E_i \circ \widehat{F} = -\widehat{F} \circ E_i
\end{array}
\text{ with }
\begin{array}{ll}
\text{bounded oper.} & E_0, \cdots, E_p : \mathcal{H}^2 \xrightarrow[\mathbb{K}-\text{linear}]{\text{bounded}} \mathcal{H}^2 \\
\text{(anti-)self-adjoint} & (E_i)^* = \mathrm{sgn}_i \cdot E_i \\
\text{Clifford gen.} & E_i \circ E_j + E_j \circ E_i = 2\mathrm{sgn}_i \cdot \delta_{ij}
\end{array}
\right\}
$$

$$
=: \mathrm{Fred}_{\mathbb{C}}^{-p}
$$

$$
\text{[Karoubi 70]:} \quad \left\{ X \xrightarrow[\text{cnts}]{} \mathrm{Fred}^p_{\mathbb{K}} \right\} \big/_{\sim_{\text{htpy}}} \;=\; \left\{
\begin{array}{lcl}
\mathrm{KU}^p(X) = \mathrm{KU}^{p+2}(X) & | & \mathbb{K} = \mathbb{C} \\
\mathrm{KO}^p(X) = \mathrm{KO}^{p+8}(X) & | & \mathbb{K} = \mathbb{R}
\end{array}
\right.
$$

Maximal induced Clifford action anticommuting with all G-invariant odd Fredholm operators	$E_{-3} =$								$\mathrm{i}\widehat{T}\widehat{C}\beta$		
	$E_{-2} =$					$\mathrm{i}\widehat{C}\beta$			$\mathrm{i}\widehat{C}\beta$		
	$E_{-1} =$		$\widehat{P}\beta$			$\widehat{C}\beta$		$\widehat{C}\beta$	$\widehat{C}\beta$		
	$E_{+0} =$	β	β	β	$\begin{pmatrix} \beta & 0 \\ 0 & -\beta \end{pmatrix}$	β	β	β	β	β	β
	$E_{+1} =$				$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$		$\widehat{C}\beta$			$\widehat{C}\beta$	$\widehat{C}\beta$
	$E_{+2} =$				$\begin{pmatrix} 0 & \mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$		$\mathrm{i}\widehat{C}\beta$			$\mathrm{i}\widehat{C}\beta$	
	$E_{+3} =$				$\begin{pmatrix} 0 & -\widehat{T} \\ \widehat{T} & 0 \end{pmatrix}$					$\mathrm{i}\widehat{T}\widehat{C}\beta$	
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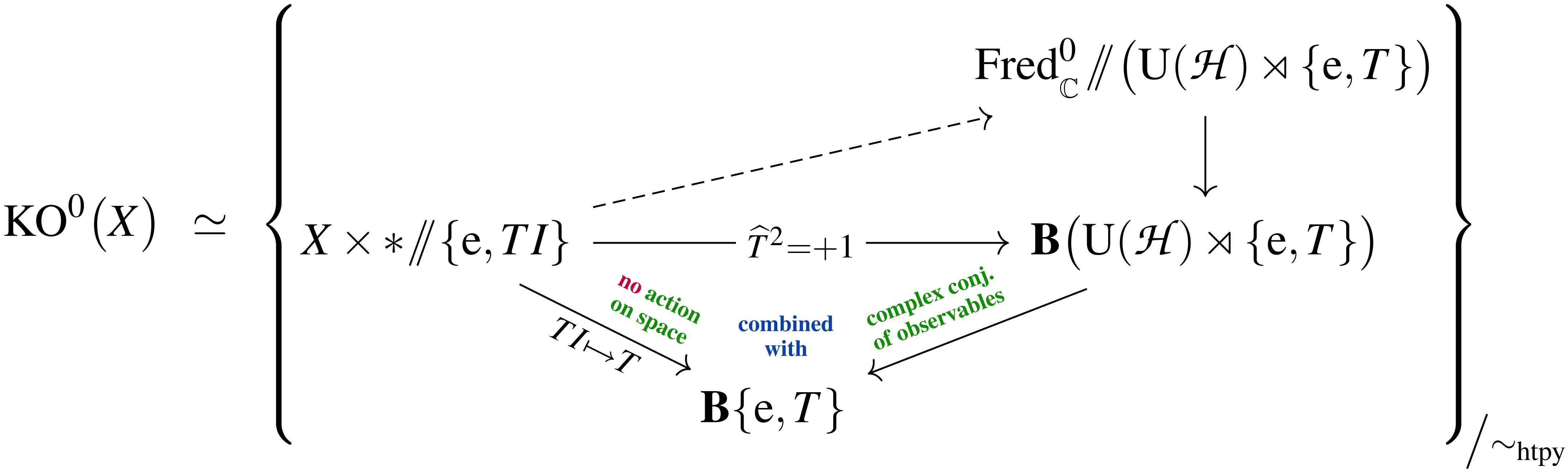
τ-twisted G-equivariant KR-theory of fixed loci	$\mathrm{KR}^\tau =$	KU^0	KU^1	KO^0	KO^4	KO^2	KO^6	KO^1	KO^3	KO^5	KO^7

Example – TI-equivariant KR-theory is KO^0-theory.

The combination $T \cdot I$ acts trivially on the domain space and by complex conjugation on observables.

Hence $(T \cdot I)$-equivariant $(\widehat{T}^2 = +1)$-twisted KR-theory is KO^0-theory:

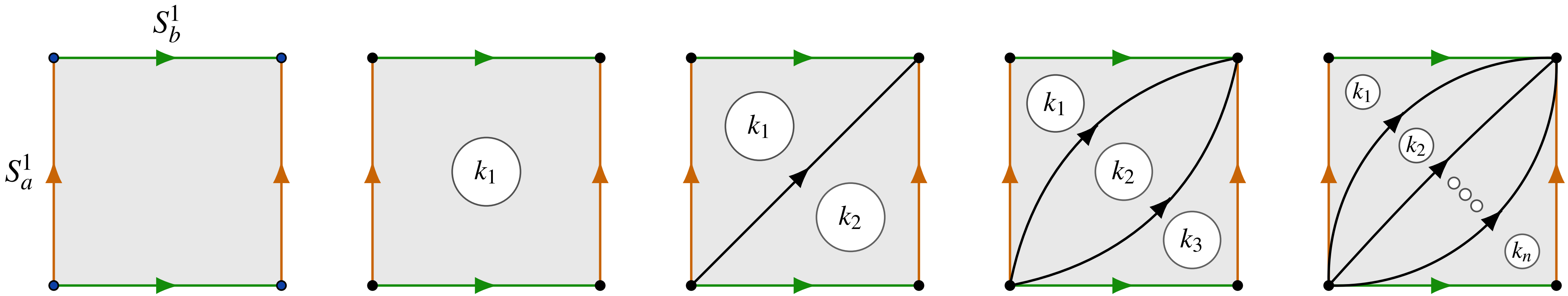


$n =$	0	1	2	3	4	5	6	7	8	9	$\cdots$
$\mathrm{KO}^0(S^n_*) =$	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	$\mathbb{Z}_2$	$\cdots$

Example – TI-equivariant KR-theory of punctured torus.

So the TI-equivariant $(\widehat{T}^2 = +1)$-twisted KR-theory of the N-punctured torus is

$$\begin{aligned} &\mathrm{KR}^{(\widehat{T}^2=+1)}\big(\widetilde{\mathbb{T}}^2 \setminus \{k_1, \cdots, k_N\}\big) \\ &\simeq \mathrm{KO}^0\big(\widetilde{\mathbb{T}}^2 \setminus \{k_1, \cdots, k_N\}\big) \\ &\simeq \mathrm{KO}^0\Big(\bigvee_{1+N} S^1_*\Big) \qquad (N \geq 1) \\ &\simeq \bigoplus_{1+N} \mathbb{Z}_2 \end{aligned}$$



$\widetilde{\mathbb{T}}^2 \underset{\text{stable}}{\simeq} S^1_a \vee S^1_b \vee S^2_{\text{bulk}}$

$\widetilde{\mathbb{T}}^2 \setminus \{k_1\} \underset{\text{htpy}}{\simeq} S^1_a \vee S^1_b$

$\widetilde{\mathbb{T}}^2 \setminus \{k_1, k_2\} \underset{\text{htpy}}{\simeq} S^1_a \vee S^1_b \vee S^1$

$\widetilde{\mathbb{T}}^2 \setminus \{k_1, k_2, k_3\} \underset{\text{htpy}}{\simeq} S^1_a \vee S^1_b \vee S^1 \vee S^1$

$\widetilde{\mathbb{T}}^2 \setminus \{k_1, \cdots, k_n\} \underset{\text{htpy}}{\simeq} \bigvee_{1+n} S^1$

The B-field twist.

Besides these CPT-quantum symmetries,
K-theory generically admits the famous *twisting by a B-field*:

The homotopy fiber sequence of 2-stacks discussed before

$$\mathbf{B}\mathrm{U}(\mathcal{H}) \longrightarrow \mathbf{B}\big(\mathrm{U}(\mathcal{H})/\mathrm{U}(1)\big) \xrightarrow[\text{}]{\overset{\textbf{universal Dixmier-Douady class}}{\mathrm{DD}}} \mathbf{B}^2\mathrm{U}(1)$$

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induces a surjection of equivalence classes of equivariant higher bundles

equivariant projective bundles — **equivariant bundle gerbes**

$$\pi_0 \; \mathrm{Map}\Big(\widehat{\mathrm{X}/\!\!/G}, \mathbf{B}\big(\mathrm{U}(\mathcal{H})/\mathrm{U}(1)\big)\Big) \xrightarrow{\mathrm{DD}_*} \!\!\!\!\!\rightarrow \pi_0 \, \mathrm{Map}\Big(\widehat{\mathrm{X}/\!\!/G}, \mathbf{B}^2\mathrm{U}(1)\Big)$$

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which has a natural section:

"stable twists"

$$\pi_0\mathrm{Map}\big(\widehat{\mathrm{X}/\!\!/G}, \mathbf{B}^2\mathrm{U}(1)\big) \hookrightarrow \pi_0\,\mathrm{Map}\Big(\widehat{\mathrm{X}/\!\!/G}, \mathbf{B}\Big(\tfrac{\mathrm{U}(\mathcal{H})\times\mathrm{U}(\mathcal{H})}{\mathrm{U}(1)} \rtimes \big(\{\mathrm{e},C\}\times\{\mathrm{e},P\}\big)\Big)\Big).$$

equivariant bundle gerbes — full quantum-symmetry twists

The B-field twist – Inner local systems.

On fixed loci (orbi-singularities)

$$\mathrm{X}/\!\!/G \;\simeq\; \mathrm{X} \times * /\!\!/G \;=\; \mathrm{X} \times \mathbf{B}G$$

the B-field twist induces *secondary* twists by "inner local systems":

stable twists over fixed locus

$$\mathrm{Map}\big(\mathrm{X} \times * /\!\!/G,\, \mathbf{B}^2\mathrm{U}(1)\big) \simeq \mathrm{Map}\big(\mathrm{X} \times \mathbf{B}G,\, \mathbf{B}^2\mathrm{U}(1)\big)$$

$$\simeq\; \mathrm{Map}\big(\mathrm{X},\, \mathrm{Map}(\mathbf{B}G,\, \mathbf{B}^2\mathrm{U}(1))\big)$$

The B-field twist – Inner local systems.

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$$\begin{aligned}\mathrm{Map}\big(\mathrm{X} \times * /\!\!/ G, \mathbf{B}^2\mathrm{U}(1)\big) &\simeq \mathrm{Map}\big(\mathrm{X} \times \mathbf{B}G, \mathbf{B}^2\mathrm{U}(1)\big) \\ &\simeq \mathrm{Map}\big(\mathrm{X}, \mathrm{Map}(\mathbf{B}G, \mathbf{B}^2\mathrm{U}(1))\big) \\ &\simeq \mathrm{Map}\big(\mathrm{X}, \mathbf{B}G^* \times \mathbf{B}^2\mathrm{U}(1)\big)\end{aligned}$$

Here we are assuming $G \underset{\mathrm{fin}}{\subset} \mathrm{SU}(2)$ so that $H^2_{\mathrm{Grp}}\big(G, \mathrm{U}(1)\big) = 0$.

And $G^* := \mathrm{Hom}\big(G, \mathrm{U}(1)\big)$ denotes the Pontrjagin-dual group.

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$$\begin{aligned}
\overset{\text{stable twists over fixed locus}}{\mathrm{Map}\big(\mathrm{X} \times * /\!\!/ G,\, \mathbf{B}^2\mathrm{U}(1)\big)} &\simeq \mathrm{Map}\big(\mathrm{X} \times \mathbf{B}G,\, \mathbf{B}^2\mathrm{U}(1)\big) \\
&\simeq \mathrm{Map}\big(\mathrm{X},\, \mathrm{Map}(\mathbf{B}G,\, \mathbf{B}^2\mathrm{U}(1))\big) \\
&\simeq \mathrm{Map}\big(\mathrm{X},\, \mathbf{B}G^* \times \mathbf{B}^2\mathrm{U}(1)\big) \\
&\simeq \underbrace{\mathrm{Map}\big(\mathrm{X},\, \mathbf{B}G^*\big)}_{\text{inner local systems}} \times \underbrace{\mathrm{Map}\big(\mathrm{X},\, \mathbf{B}^2\mathrm{U}(1)\big)}_{\text{bundle gerbes}}
\end{aligned}$$

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The B-field twist – Inner local systems – The diagrammatics.

Hence the

inner local system-twisted KU-*cohomology*
of a G-orbi-singularity of shape X

arises as follows:

$$
\mathrm{KU}_G^{n+[\omega_1]}(X) \;=\; \left\{ \begin{array}{ccc} & & \mathrm{Fred}^n_{\mathbb{C}} /\!\!/ \mathrm{PU}(\mathcal{H}) \\ & \overset{\text{cocycle}}{\nearrow} & \big\downarrow \\ X \times * /\!\!/ G & \underset{\text{inner local system twist}}{\xrightarrow{\;\;\tau\;\;}} & \mathbf{B}\mathrm{PU}(\mathcal{H}) \end{array} \right\}_{\Big/ \sim_{\mathrm{htpy}}}
$$

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X & \underset{\textbf{inner local system}}{\xrightarrow{\;\;\omega_1\;\;}} & \mathbf{B}G^{*} & \underset{\substack{\textbf{automorphisms of} \\ \textbf{univ. stable equiv. twist}}}{\longrightarrow} & \mathrm{Map}\big(\mathbf{B}G,\, \mathbf{B}\mathrm{PU}(\mathcal{H})\big)
\end{array}
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The B-field twist – Inner local systems – The diagrammatics.

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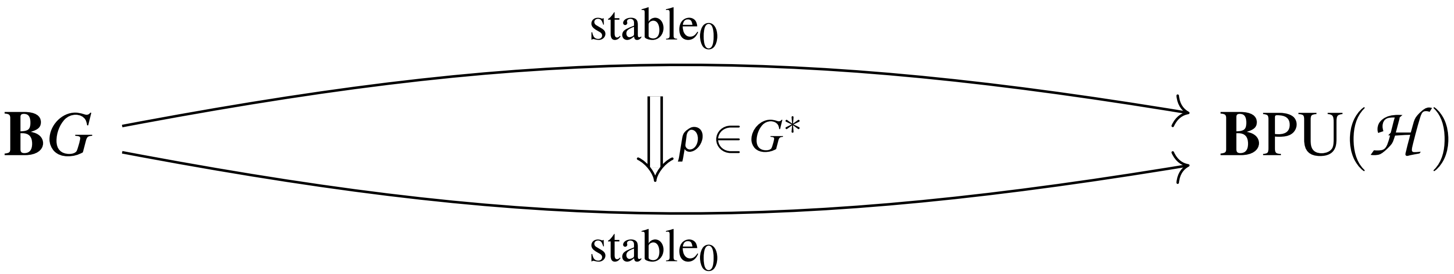
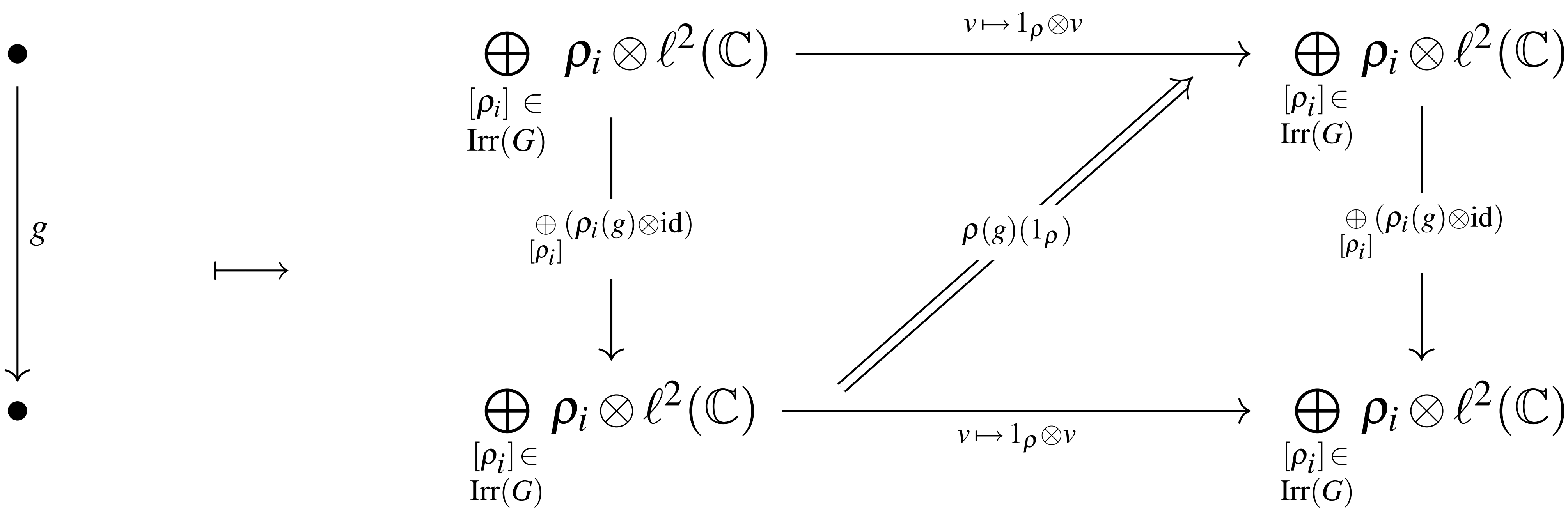
 & & \big(\mathrm{Fred}^0_{\mathbb{C}}\big)^G /\!\!/ G^\ast & \to & \mathrm{Map}\big(\mathbf{B}G, \mathrm{Fred}^n_{\mathbb{C}} /\!\!/ \mathrm{PU}(\mathcal{H})\big) \\
 & \overset{\text{cocycle}}{\nearrow} & \Big\downarrow & \text{(pb)} & \Big\downarrow \\
X & \underset{\text{inner local system}}{\xrightarrow{\;\;\omega_1\;\;}} & \mathbf{B}G^\ast & \underset{\substack{\text{automorphisms of} \\ \text{univ. stable equiv. twist}}}{\xrightarrow{\qquad\qquad}} & \mathrm{Map}\big(\mathbf{B}G, \mathbf{B}\mathrm{PU}(\mathcal{H})\big)
\end{array}
\right\} \Big/ \sim_{\mathrm{htpy}}
$$

The B-field twist – Inner local systems – The proof.

For the proof we consider the following diagram [SS22-Bun, Ex. 4.1.56][SS22, §3]:





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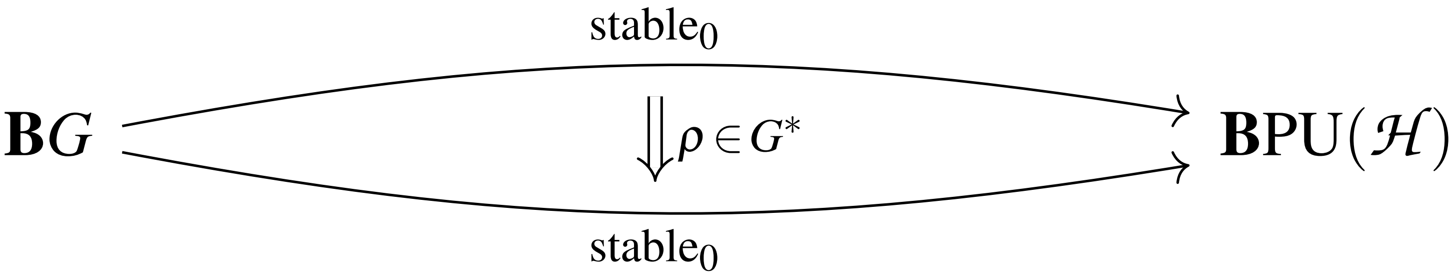

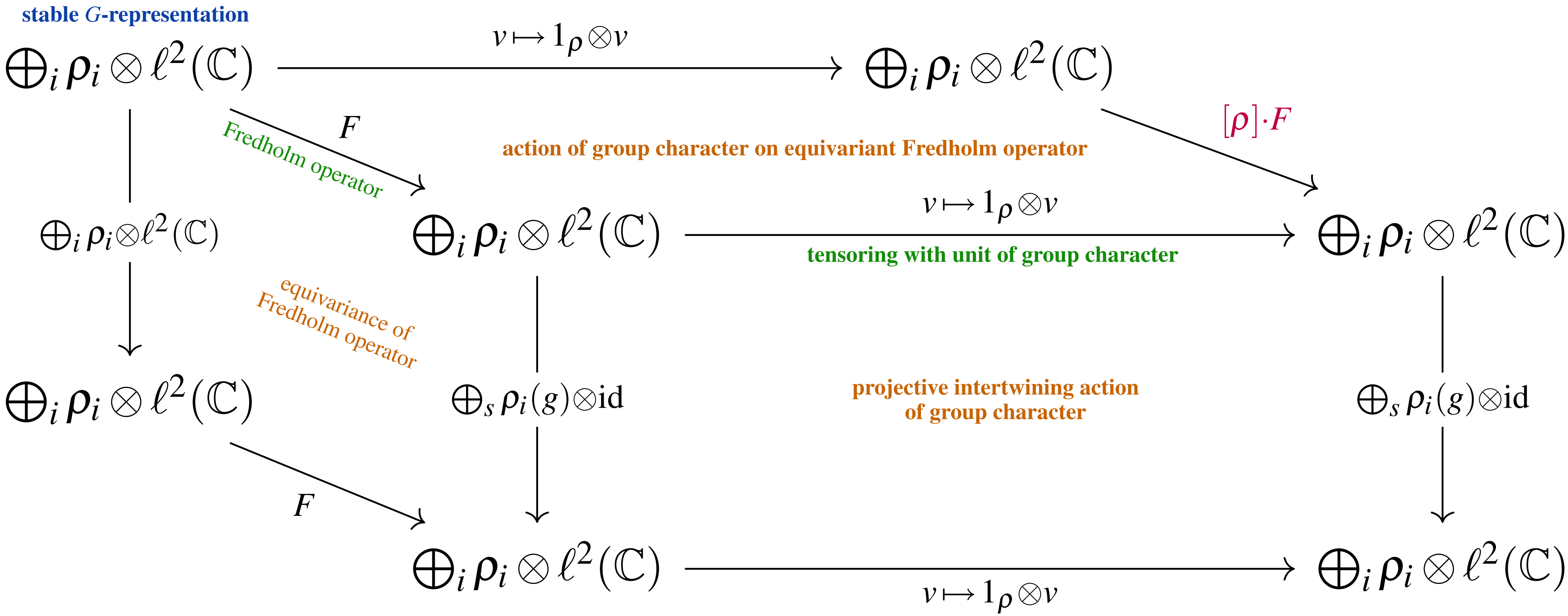

The B-field twist – Inner local systems – Chern character.

One aspect of these twistings becomes transparent under the *Chern character*:

complex K-theory

periodic de Rham cohomology

$$\mathrm{KU}^0(\mathrm{X}) \xrightarrow[\text{Chern character}]{} \mathrm{KU}^0(\mathrm{X};\mathbb{C}) \;\simeq\; \bigoplus_{d\in\mathbb{N}} H^{2d}\Big(\Omega^\bullet_{\mathrm{dR}}\big(\mathrm{X};\mathbb{C}\big),\mathrm{d}\Big)$$

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For twist by B-field $\widehat{B}_2$ there is a closed differential 3-form H_3 such that:

plain B-field
-twisted K-theory

3-twisted periodic de Rham cohomology

$$\mathrm{KU}^{n+\widehat{B}_2}(\mathrm{X}) \xrightarrow[\text{Chern character}]{\text{twisted}} \mathrm{KU}^{\widehat{B}_2}(\mathrm{X};\mathbb{C}) \simeq \bigoplus_{d\in\mathbb{Z}} H^{n+2d}\Big(\Omega^\bullet_{\mathrm{dR}}(\mathrm{X};\mathbb{C}),\mathrm{d}+H_3\wedge\Big)$$

The B-field twist – Inner local systems – Chern character.

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complex K-theory — periodic de Rham cohomology

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plain B-field -twisted K-theory — 3-twisted periodic de Rham cohomology

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For twist by inner C_κ-local system, there is closed 1-form ω_1 with holon. in $C_\kappa \subset \mathrm{U}(1)$ such that:

inner local system -twisted K-theory of A-type singularity — 1-twisted periodic de Rham cohomology

$$\mathrm{KU}^{n+[\omega_1]}_{C_\kappa}(\mathrm{X}) \xrightarrow[\text{Chern character}]{\text{twisted equivariant}} \bigoplus_{\substack{d\in\mathbb{Z}\\ 1\leq r\leq\kappa}} H^{n+2d}\Big(\Omega^\bullet_{\mathrm{dR}}(\mathrm{X};\mathbb{C}),\mathrm{d}+r\cdot\omega_1\wedge\Big)$$

The B-field twist – Inner local systems – Chern character.

One aspect of these twistings becomes transparent under the Chern character:

This is the hidden 1-twisting in TED-K – that we will next relate to anyons. $\longrightarrow$

inner local system
-twisted K-theory

1-twisted periodic de Rham cohomology

$$\mathrm{KU}^{n+[\omega_1]}_{C_\kappa}(\mathrm{X}) \xrightarrow[\text{Chern character}]{\text{twisted equivariant}} \bigoplus_{\substack{d \in \mathbb{Z} \\ 1 \leq r \leq \kappa}} H^{n+2d}\Big(\Omega^\bullet_{\mathrm{dR}}(\mathrm{X};\mathbb{C}), \mathrm{d} + r \cdot \omega_1 \wedge\Big)$$

of A-type singularity

I – Equivariant ∞-Bundles

II – TED-K-Theory

III –Anyonic Defect Branes

This part is a brief indication
of a few aspects discussed in:

Anyonic Defect Branes in TED-K-Theory [arXiv:2203.11838]

K-Theory classifies non-perturbative vacua.

Solid state physics	K-theory	String theory

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Single electron state	Line bundle	Single D-brane
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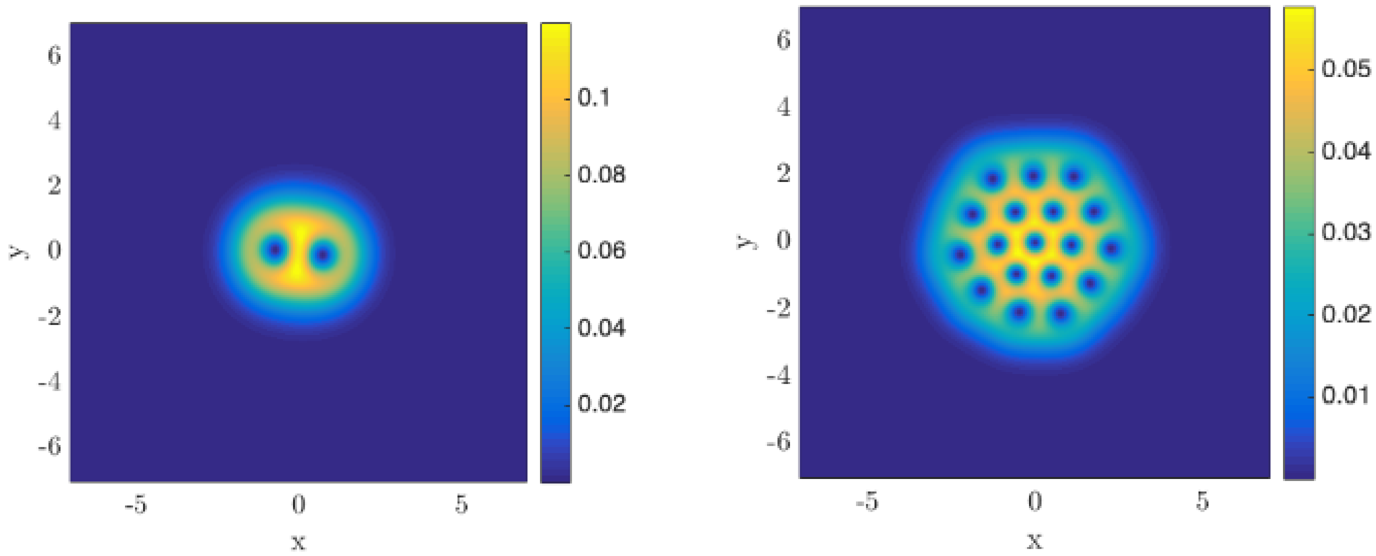
Anyons	Punctures	Defect branes

Anyons in condensed matter & string theory.

In solid state physics

anyons are presumed pointlike defects in gapped topological phases of effectively 2-dimensional materials whose adiabatic dynamics is that of Wilson lines in $\mathfrak{su}(2)$-CS theory.



(numerical simulation from `arXiv:1901.10739`)

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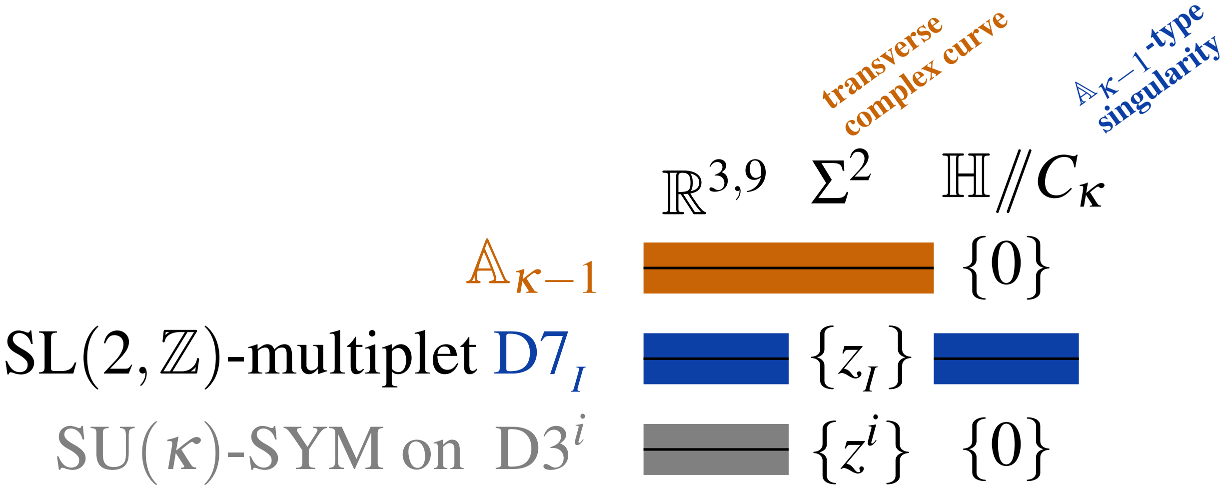

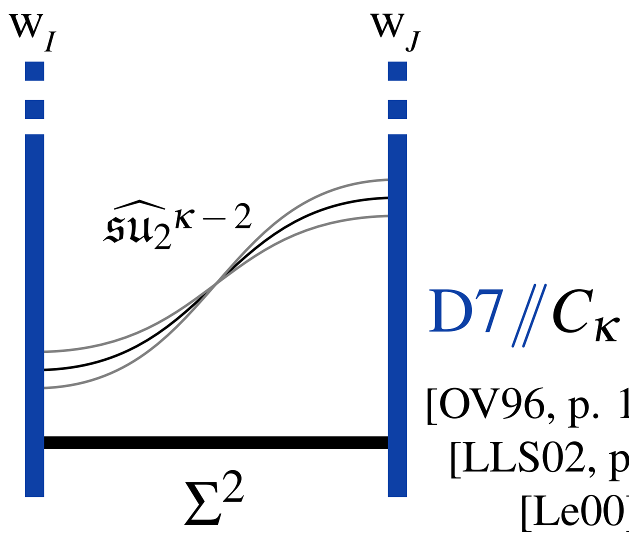

[OV96, p. 10-12]
[LLS02, p. 4s]
[Le00]

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Yes! $\longrightarrow$

TED-Cohomological incarnation of Conformal blocks.

Consider

κ	$:=$	$k+2$	"level"
w_I	$\in$	$\{0,\cdots,k\}$	"weights"
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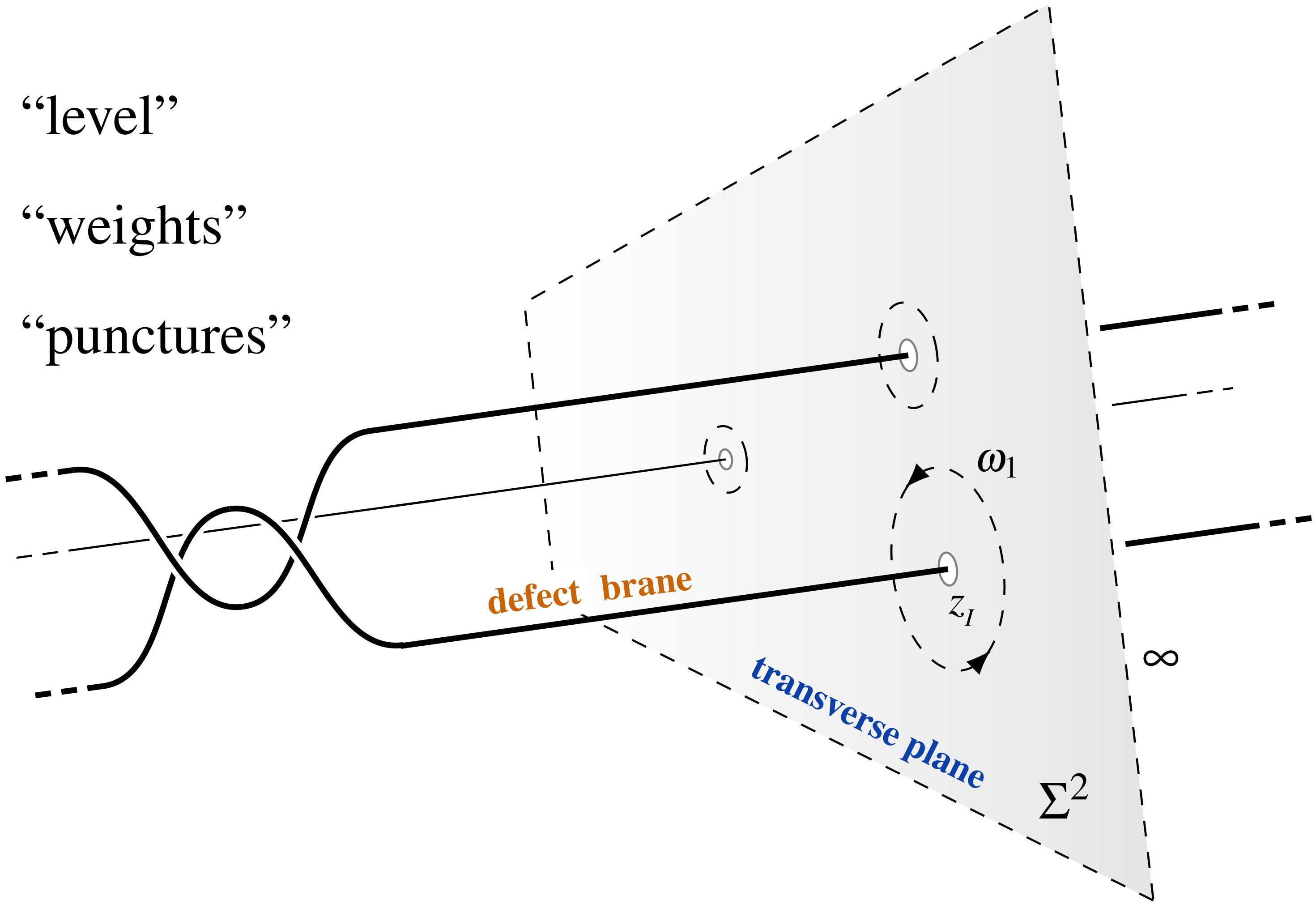
ω_1	$:=$	$\sum_I -\frac{\mathrm{w}_I}{\kappa}\frac{\mathrm{d}z}{z-z_I}$	



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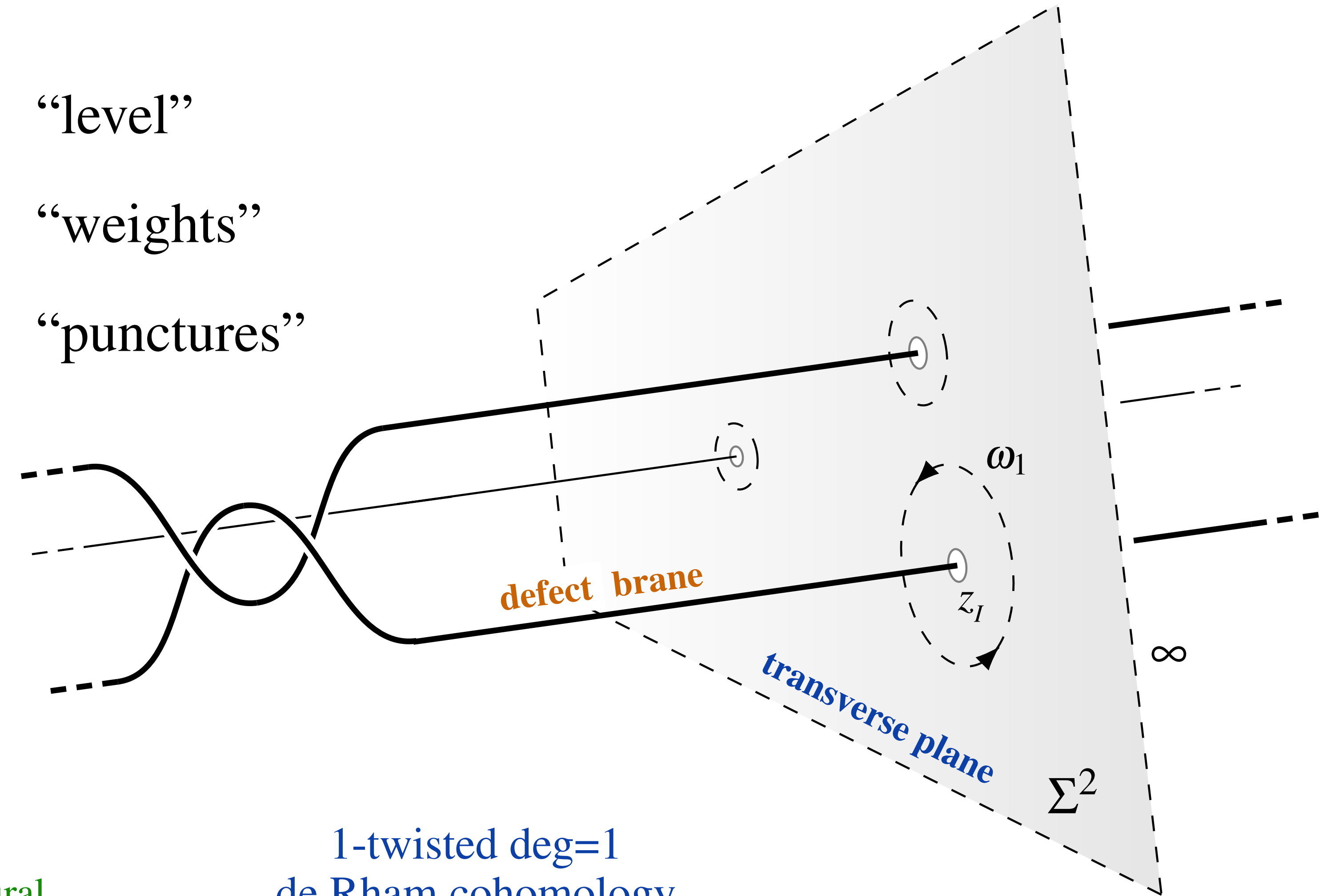

$\mathfrak{su}(2)$-affine deg=1 conformal blocks $\xrightarrow{\text{natural inclusion}}$ 1-twisted deg=1 de Rham cohomology

$$\mathrm{CnfBlck}^1_{\widehat{\mathfrak{sl}_2}^k}(\vec{\mathrm{w}}, \vec{z}) \hookrightarrow H^1\Big(\Omega^\bullet_{\mathrm{dR}}\big(\mathbb{C}\setminus\{\vec{z}\}\big), \mathrm{d} + \omega_1 \wedge\Big)$$

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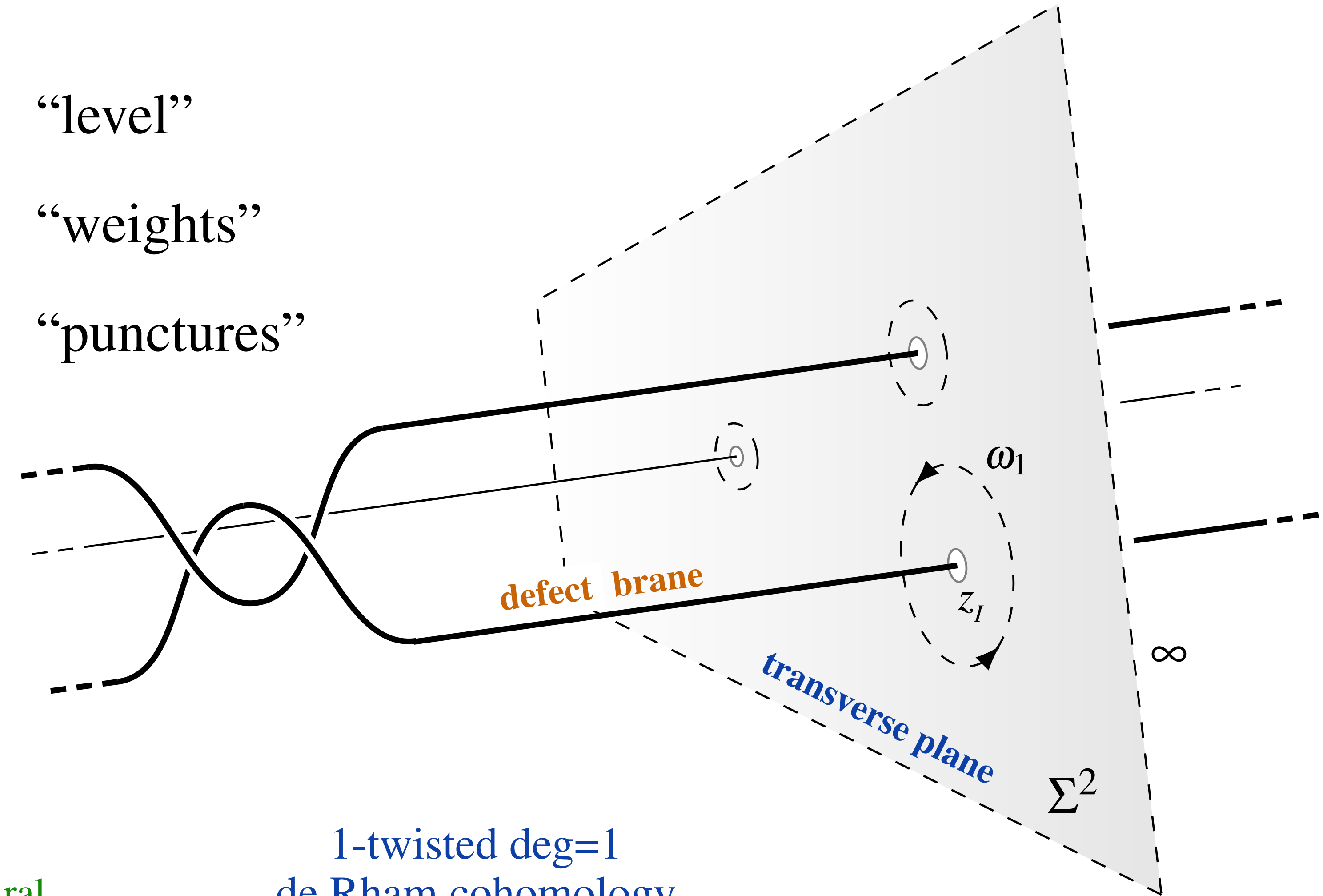

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$$\xhookrightarrow{\text{natural inclusion}} \mathrm{KU}^{1+\omega_1}\Big(\big(\mathbb{C}\setminus\{\vec{z}\}\big)\times * /\!\!/ C_\kappa;\, \mathbb{C}\Big) \qquad \text{[SS22, Prop. 2.16]}$$

inner local system-twisted deg=1 K-theory of $\mathbb{A}_{\kappa-1}$-singularity (as explained above)

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Generally, consider *configuration spaces of points* (e.g. [SS19, §2.2])

$$\underset{\{1,\cdots,n\}}{\mathrm{Conf}}(\mathrm{X}) \;:=\; \Big\{ z^1, \cdots, z^n \in \mathrm{X} \;\Big|\; \underset{i<j}{\forall}\; z^i \neq z^j \Big\}.$$

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The previous statement is subsumed since $\underset{\{1\}}{\mathrm{Conf}}(\mathrm{X}) = \mathrm{X}$.

Conclusion.

The commonly expected $\widehat{\mathfrak{su}_2}^k$-charges of anyons and defect branes *are* reflected in the TED-K-theory of *configuration spaces of points* in 2-dimensional transverse spaces *inside* $\mathbb{A}_{k+1}$*-orbi-singularities*.

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Configuration space of ordered points in the plane

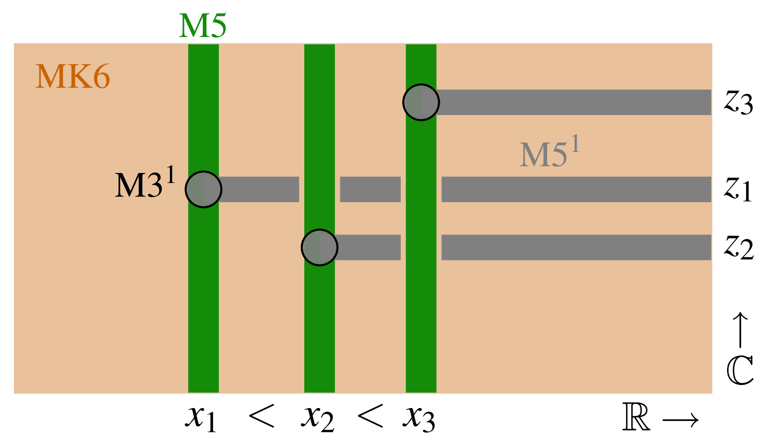
3-Cohomotopy cocycle space for codim=1 branes

3-Cohomotopy cocycle space for codim-2 branes

$$\coprod_n \underset{\{1,\cdots,n\}}{\mathrm{Conf}}(\mathbb{C}) \;\simeq\; \overbrace{\coprod_n \mathrm{Conf}_n\big(\mathbb{C};\,\mathbb{R}_{\mathrm{cpt}}\big)}^{\mathrm{Map}^*(\mathbb{R}_+\wedge\mathbb{C}_{\mathrm{cpt}},\,S^3)\,\simeq} \;\underset{\coprod_n \mathrm{Conf}_n\left(*;\,(\mathbb{R}\times\mathbb{C})_{\mathrm{cpt}}\right)}{\times}\; \overbrace{\coprod_n \mathrm{Conf}_n\big(\mathbb{R};\,\mathbb{C}_{\mathrm{cpt}}\big)}^{\mathrm{Map}^*(\mathbb{R}_{\mathrm{cpt}}\wedge\mathbb{C}_+,\,S^3)\,\simeq}$$

Fiber product of respective configuration spaces (of un-ordered points escaping to transverse infinity) reflecting the brane intersections

e.g.: $\underset{\{1,\cdots,3\}}{\mathrm{Conf}}(\mathbb{C}) \;\simeq\; \Big\{$



$\Big\}$

The moduli space of flat M3-branes according to *Hypothesis H* is the configuration space of ordered points in their transverse plane.

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Anyonic Defect Branes in TED-K-Theory

Urs Schreiber on joint work with Hisham Sati

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Thanks!

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slides and pointers at: ncatlab.org/schreiber/show/Anyonic+defect+branes+in+TED-K-theory