Double dimensional reduction of Hypothesis H. [FSS18-T, §3][BMSS19, §2.2][SS23-Cyc, p. 6][SV23].

Cyclicfication of classifying spaces. The free loop space of a (classifying space) carries a canonical S^1-action by rotation of loops, its homotopy quotient is the *cyclification*.

cyclified free loop space

$$\mathrm{Cyc}(\mathcal{A}) \;:=\; \mathrm{Maps}\big(S^1,\, \mathcal{A}\big) /\!\!/ S^1$$

Topological KK-Reduction. For a principal circle bundle $X^{10}_{\mathrm{M}} \to X^9_{\mathrm{IIA}}$, the moduli of topological $\mathcal{A}$-fields on X^{10}_{M} are equivalent to those of topological $\mathrm{Cyc}(\mathcal{A})$-fields on X^9_{IIA} sliced over BS^1

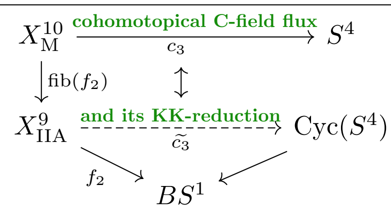
$$\mathrm{Maps}\big(X^{11}, \mathcal{A}\big) \underset{\text{KK-oxidation}}{\overset{\text{KK-reduction}}{\simeq}} \mathrm{Maps}_{/BS^1}\big(X^{10},\; \mathrm{Cyc}(\mathcal{A})\big)$$

This is **double dimensional** reduction in that with the domain space dimension also the degree of the fluxes is reduced, if they "wrap" the KK-fiber.

e.g. $\mathrm{Cyc}\big(B^{n+1}\mathbb{Z}\big) \;\simeq\; \big(B^n\mathbb{Z} \times B^{n+1}\mathbb{Z}\big) /\!\!/ S^1$

wrapped fluxes; non-wrapped fluxes; D0-WZ term

Hence for flat X^{10}_{M} (eg. a torus bundle over a Euclidean space with a point at infinity), **Hypothesis H** implies that fluxes in type IIA string theory are quantized in $\mathrm{Cyc}(S^4)$-cohomologly.



Rationally this cyclification is indeed like twisted K-theory, but without the "Romans mass" term F_0 sourced by singular D_8-branes (we find *solitonic* D_8-branes in hupf).
Hence in IIA, Hypothesis H predicts a non-abelian **modification of the traditional Hypothesis K**.

$$\Omega_{\mathrm{dR}}\Big(X^9_{\mathrm{IIA}}, \mathfrak{l}\mathrm{Cyc}(S^4)\Big)_{\mathrm{clsd}} \overset{\text{[FSS17-Sph, Ex. 3.3]}}{=}$$

$$\left\{ \begin{array}{lcl} \mathrm{d}F_2 &=& 0 \\ \mathrm{d}F_4 &=& H_3 \wedge F_2 \\ \mathrm{d}F_6 &=& H_3 \wedge F_4 \end{array} \quad \begin{array}{lcl} \mathrm{d}H_3 &=& 0 \\ \mathrm{d}H_7 &=& F_2 \wedge F_6 - \tfrac{1}{2} F_4 \wedge F_4 \end{array} \right\}$$

U-Duality. This process of double dimensional reduction by cyclification of the 4-sphere coefficients continues to yield, rationally, the expected U-duality symmetries of M-theory [SV23, p. 5]:

D	k	Type of E_k	Lie algebra $\mathfrak{g}$	del Pezzo	Model	Maximal Split Torus
11	0	A_{-1}	$\mathfrak{sl}_0 = \varnothing$	$\mathbb{CP}^2$	S^4	$\mathbb{G}_m$
10	1	A_0	$\mathfrak{sl}_1 = 0$	$\mathbb{B}_1$	$\mathscr{L}_c S^4$	$\mathbb{G}_m \times \mathbb{G}_m$
10	1	A_1	$\mathfrak{sl}_2$	$\mathbb{CP}^1 \times \mathbb{CP}^1$	*IIB*	$\mathbb{G}_m \times \mathbb{G}_m$
9	2	A_1	$\mathfrak{sl}_2$	$\mathbb{B}_2$	$\mathscr{L}_c^2 S^4$	$\mathbb{G}_m^2 \times \mathbb{G}_m$
8	3	$A_2 \times A_1$	$\mathfrak{sl}_3 \oplus \mathfrak{sl}_2$	$\mathbb{B}_3$	$\mathscr{L}_c^3 S^4$	$\mathbb{G}_m^3 \times \mathbb{G}_m$
7	4	A_4	$\mathfrak{sl}_5$	$\mathbb{B}_4$	$\mathscr{L}_c^4 S^4$	$\mathbb{G}_m^4 \times \mathbb{G}_m$
6	5	D_5	$\mathfrak{so}_{10}$	$\mathbb{B}_5$	$\mathscr{L}_c^5 S^4$	$\mathbb{G}_m^5 \times \mathbb{G}_m$
5	6	E_6	$\mathfrak{e}_6$	$\mathbb{B}_6$	$\mathscr{L}_c^6 S^4$	$\mathbb{G}_m^6 \times \mathbb{G}_m$
4	7	E_7	$\mathfrak{e}_7$	$\mathbb{B}_7$	$\mathscr{L}_c^7 S^4$	$\mathbb{G}_m^7 \times \mathbb{G}_m$
3	8	E_8	$\mathfrak{e}_8$	$\mathbb{B}_8$	$\mathscr{L}_c^8 S^4$	$\mathbb{G}_m^8 \times \mathbb{G}_m$

D	k	Type of E_k	Kac-Moody algebra $\mathfrak{g}$	Non-Fano Surface	Model	Maximal Split Torus
2	9	$E_9 = \widehat{E}_8$	affine $\mathfrak{e}_9 = \widehat{\mathfrak{e}}_8$	$\mathbb{B}_9$	$\mathscr{L}_c^9 S^4$	$\mathbb{G}_m^9 \times \mathbb{G}_m$
1	10	E_{10}	hyperbolic $\mathfrak{e}_{10}$	$\mathbb{B}_{10}$	$\mathscr{L}_c^{10} S^4$	$\mathbb{G}_m^{10} \times \mathbb{G}_m$
0	11	E_{11}	Lorentzian $\mathfrak{e}_{11}$	$\mathbb{B}_{11}$	$\mathscr{L}_c^{11} S^4$	$\mathbb{G}_m^{11} \times \mathbb{G}_m$