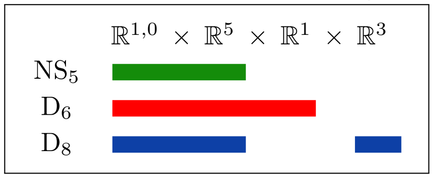
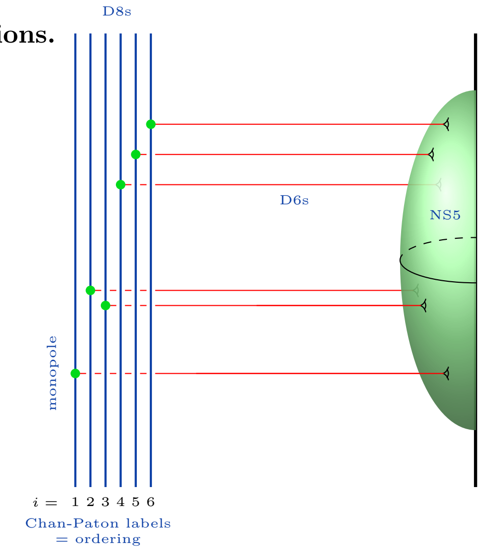
2.3.3 Quantum $\mathrm{D}_6 \perp \mathrm{D}_8$-branes via Fadell-Husseini's theorem

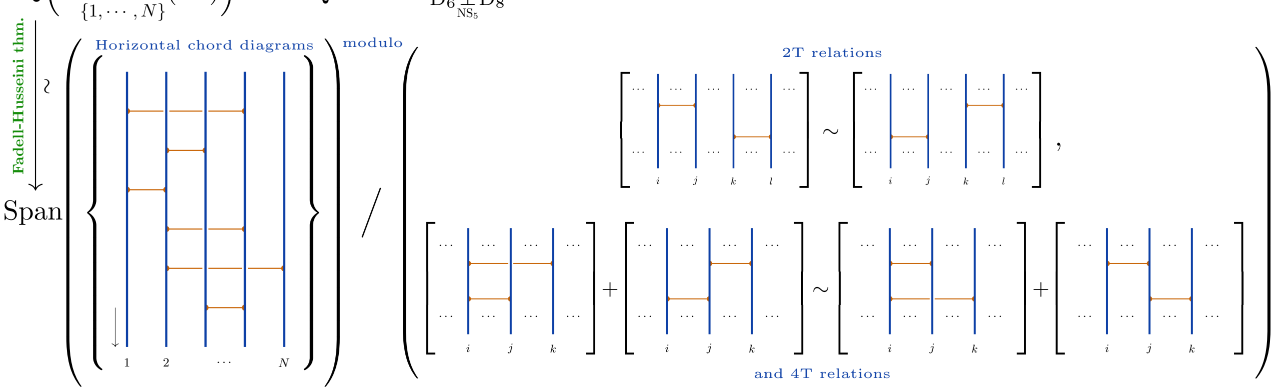
Cohomotopy Moduli of Hanany-Witten brane configurations. Consider the situation (119) for M-theory on S^1 with $\mathrm{D}_6 \perp \mathrm{D}_8$-intersections on NS_5-cores: hence for $n = 4,\ d = 9,\ p = 6.$ [SS22-Cnf, Rem. 2.14]



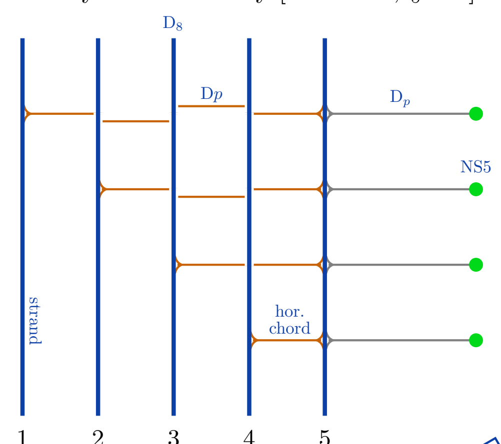


Quantum observables on Hanany-Witten configurations. It follows with the discussion in §2.2 that the light-cone quantum observables on these brane configurations form, for each number N of D_8-branes, the homology Pontrjagin-algebra of the based loop space of the ordered configuration space (119). Remarkably, by the Fadell-Husseini theorem [FH01, Thm. 2.2] this is isomorphic to the algebra of *horizontal chord diagrams* on N-strands modulo the "2T- and 4T-relations" [SS22-Cnf, Prop. 2.18]:

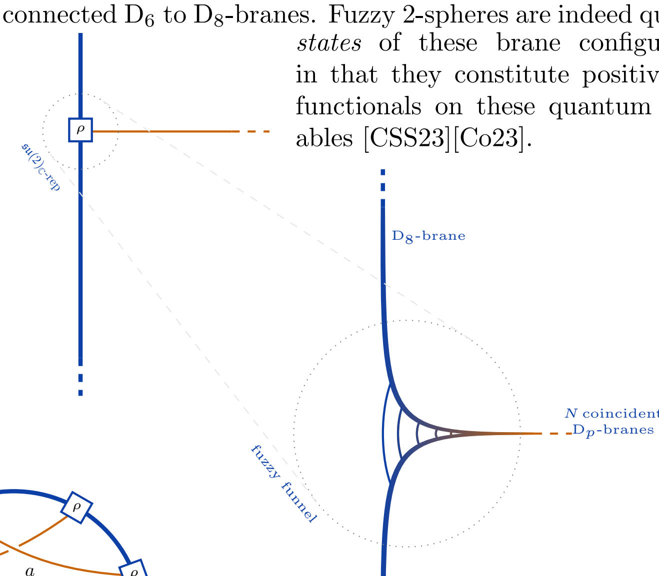
$$H_\bullet\Big(\Omega \underset{\{1,\cdots,N\}}{\mathrm{Conf}}(\mathbb{R}^3)\Big) \overset{(119)}{\simeq} \mathrm{QObsrvbs}_{\mathrm{D6}\underset{\mathrm{NS}_5}{\perp}\mathrm{D8}}$$



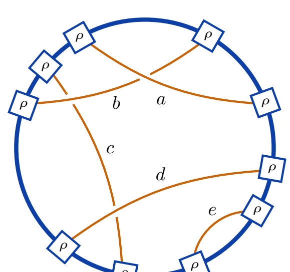
In their classical limit (graded-symmetric chord algebra) these relations match the **brane intersection rules** expected in Hanany-Witten theory [SS22-Cnf, §4.10]:



Generally, chord diagrams serve as observables on the geometry of fuzzy 2-spheres [SS22-Cnf, §4.2] as expected for **fuzzy funnels** connected D_6 to D_8-branes. Fuzzy 2-spheres are indeed quantum *states* of these brane configurations, in that they constitute positive linear functionals on these quantum observables [CSS23][Co23].



Typical value
of a quantum state
on chord diagrams
(a "weight system")
in Penrose notation.



$$= \quad \mathrm{Tr}\big(\rho_a \cdot \rho_b \cdot \rho_c \cdot \rho^b \cdot \rho_d \cdot \rho^c \cdot \rho_a \cdot \rho^e \cdot \rho^d\big)$$