CFT and algebra in braided tensor categories

1. Modular tensor categories and RCFT's

1.1 ~~rational~~ semisimple conformal vertex algebra

$\downarrow$

representation category $\mathcal{C}$

$\rightsquigarrow$

conformal blocks (system of vector bundles over curves)

Gives $\mathcal{C}$ structure of a braided tensor category.

⋯> assume its a modular tensor category (what rational means).

Defn: Modular tensor category

- Abelian, $\mathbb{C}$-linear, semisimple tensor categories, noetherian (finitely many iso classes of simple objects),

  $\mathbb{1}$ simple, choose set I of ~~[illegible]~~ rep. simple objects

- ribbon category, in particular a braiding, ?∧ = ∨?
- nondegeneracy condition on the braiding must be iso.

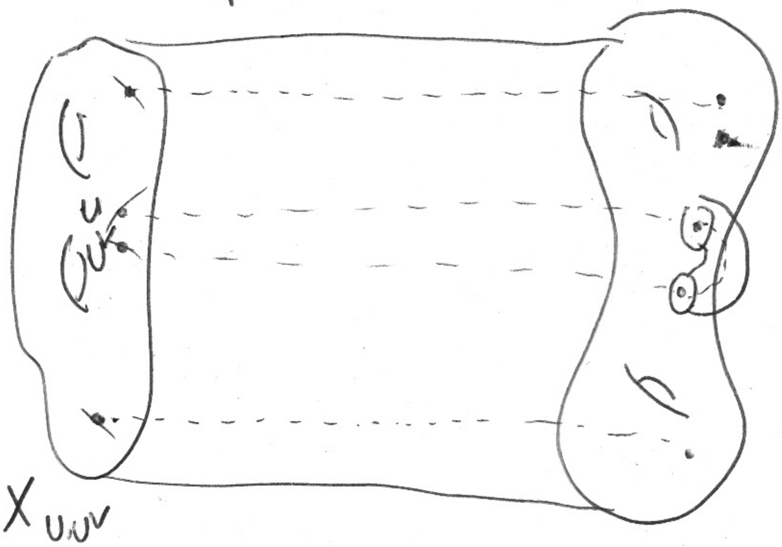
$$K_0(\mathcal{C}) \otimes_{\mathbb{Z}} \mathbb{C} \xrightarrow{\sim} \mathrm{End}(\mathrm{id}_{\mathcal{C}})$$

$$[U] \longmapsto \Big($$  $$= \alpha_V \in \mathrm{End}(V) \Big)$$

1.2. Fact [Reshetikhin, Turaev] For any modular tensor category $\mathcal{C}$, have tensor functor

$$tft_{\mathcal{C}} : \underset{\substack{\uparrow \\ \text{decorated cobordisms}}}{\mathrm{cobord}^{\mathcal{C}}_{3,2}} \longrightarrow \mathrm{vect}_f(\mathbb{C})$$

1. Factorization homomorphism



$X_{U,U^\vee}$

. apply TQFT functor

$$\mathrm{fact}_U \; (X_{U,U^\vee} \longrightarrow X).$$

Thm (Turaev)

(1) $\bigoplus_{i \in I} \mathrm{fact}_{U_i} : \bigoplus_{i \in I} tft(X_{U_i,U_i^\vee}) \xrightarrow{\sim} tft(X)$

(2) Representations of mapping class group $\mathrm{Map}(X)$ on $tft_{\mathcal{C}}(X)$.

2. CFT correlators

2.1. X a 2-dim ~~conformal~~ manifold → oriented / unoriented, w/ boundary.

"worldsheet"

~~conformal~~ ⇝ topological

2.2. Strategy – decorate X (till it forms part of cob. category)

– use tft to get appropriate "functors" for correlators

Holomorphic factorization

$X \rightsquigarrow$ oriented cover $\hat{X}$

$$\begin{array}{c} \hat{X} \circlearrowleft \sigma \\ \downarrow \pi \\ X \end{array}$$

fixed points = boundary

X	$\hat{X}$
disc	sphere
T^2	$T^2 \sqcup (-T^2)$
mob. band	torus

Goal: 1. find decoration for X s.t. $\hat{X} \in \mathrm{cobord}^{c}_{3,2}$.

2. $Cor(X) \in tft_c(\hat{X})$

a) $Cor(X)$ invariant under $Map(\hat{X})^{\sigma}$

"modular invariance"

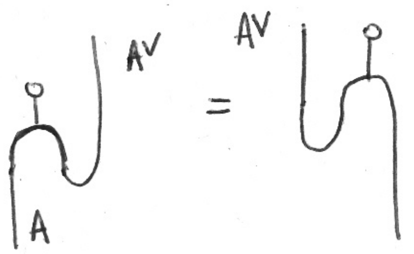
b) compatibility with factorization homomorphisms.

(links correlators with different topologies).

23 <u>Insight</u>: decoration furnished by bicategory of special symmetric Frobenius algebras in $\mathcal{C}$.

<u>Frobenius</u>: $\left(A, \eta, m, \Delta, \epsilon : A \otimes A \longrightarrow A \right)$

$\Delta : A \longrightarrow A \otimes A$ morphism of A bimodules

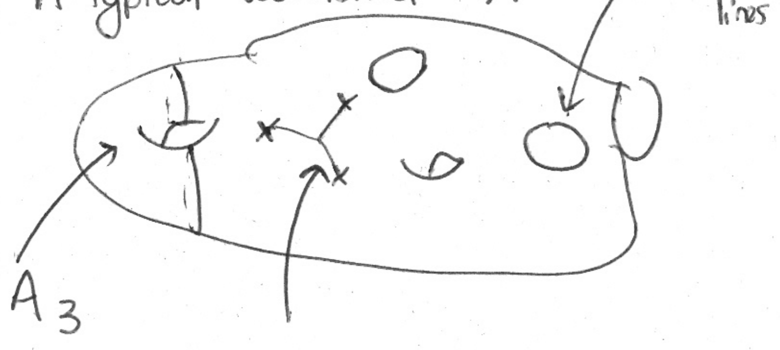
<u>symmetric</u>:  $\in \mathrm{Hom}(A, A^{\vee})$.

<u>special</u>: $m \circ \Delta =$  $= id_A$,  $= \epsilon \circ \eta = \dim A \; id_{1\!\!1}$.

could weaken with invertible scalar

objects are special symmetric Frob algebras in $\mathcal{C}$,
morphisms are bimodules
2-morphisms ...

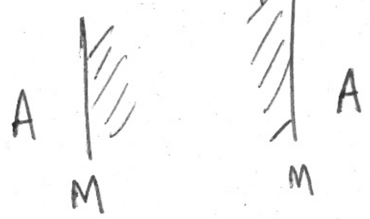
2.3. A typical worldsheet X



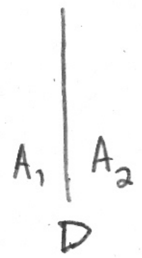
Decoration

2-dim cells $\rightsquigarrow$ s.s. Frob. algebra A

1-dim cells



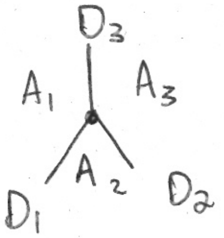
defect lines



on A_1-A_2 bimodule

0-dim

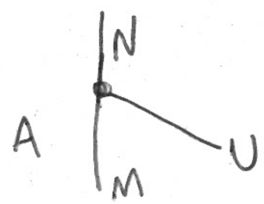
a)



$\mathrm{Hom}_{A_1|A_3}(D_1 \otimes_{A_2} D_2, D_3)$.

b) Field insertions.

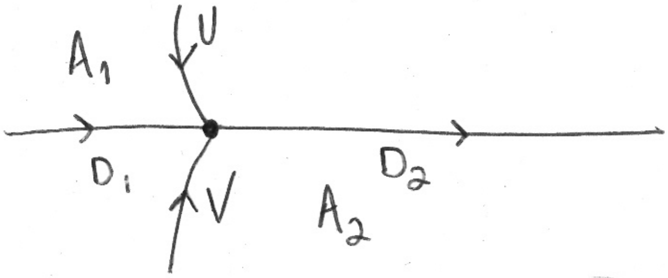
$x \in \partial X$ to $\pi_X^{-1}(x)$ attach $U \in \mathcal{C}$.



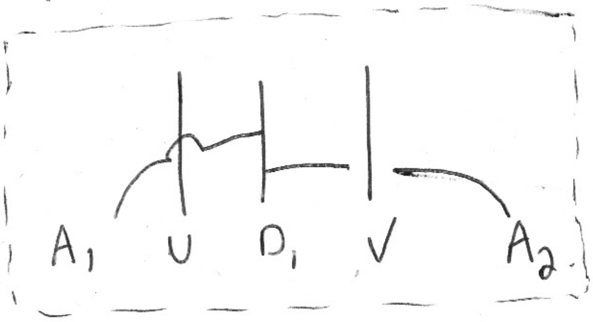
chiral insertion $\mathrm{Hom}_A(M \otimes U, N)$

c) $x \in \mathring{X}$ two preimages $\pi_X^{-1}(\hat{X})$ (U,V),

two objects



$$\mathrm{Hom}_{A_1|A_2}\left(U \otimes D_1 \otimes V,\ D_2\right)$$



2.4 Correlators from cobordisms

$$\emptyset \xrightarrow[M_X]{} \hat{X}$$

$$Cor(X) = tft_{\mathcal{C}}(M_X)\,1 \in tft_{\mathcal{C}}(\hat{X}).$$

decorated 3-manifold $\longrightarrow$ $M_X = \left(\hat{X} \times [-1,1]\right) / (\tilde{\sigma},\ t \mapsto -t)$

$$\partial M_X = \hat{X}, \quad X \hookrightarrow M_X.$$

$$x \longmapsto [\hat{x}, t=0]$$