André Henriques: Addendum

Examples : Loop group nets

G compact simply connected Lie group

$$LG = \mathrm{Map}(S^1, G)$$

$$S^1 \longrightarrow \widetilde{LG} \longrightarrow LG$$

... get conformal net.

Lattices $\rightsquigarrow$ there's a variant like the Leech lattice which gives Moonshine net.

For every Integral lattice there's a net. Aut(Moonshine) = Monster.

Free fermion

Minimal model $\equiv$ Virasoro

Ultimately get functor

start with Hilbert space

$$\left\{ \begin{array}{c} \text{subintervals of } S^1, \\ \text{diffeos between} \\ \text{them} \end{array} \right\} \longrightarrow VN$$

Then you use the equivalence with subintervals of S^1.

OR ~~you~~ canonical approach where you don't start with a Hilbert space

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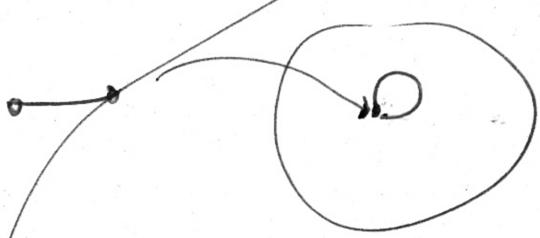
For loop groups: ~~they~~

let I be an interval.

$$I \longmapsto \mathrm{Maps}\big((I, \partial I), (G, \{e\}) \big)$$

i.e. based loops



has a central extension by S^1.

If $I \subset S^1$, then this is a subgroup of LG.

To I, we assign

$$I \longmapsto \mathbb{C}\left[\mathrm{Maps}\left((I, \partial I), (G, \{e\})\right)\right]$$

$A(I)$ is the completion of that algebra inside $B(H_0)$.

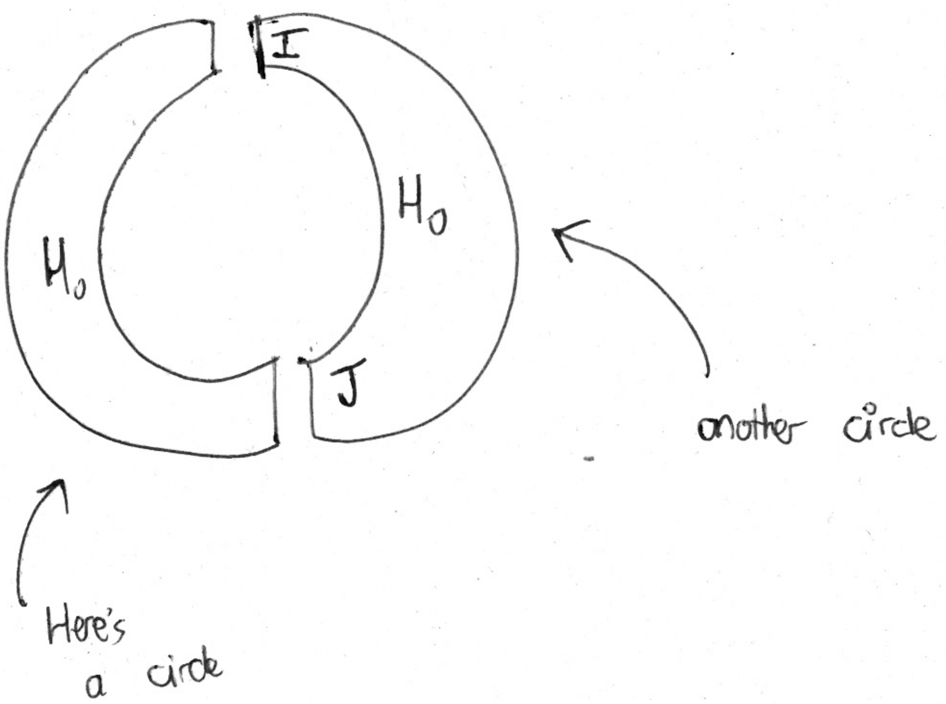
<u>Q</u>: What is the algebra assigned to the circle?

<u>A</u>: It's the direct sum of algebras acc. to each rep. of the loop group.

<u>Freed</u>: You should think of the intervals as thickened point.

These are all chiral theories.

How do you get the circle ?

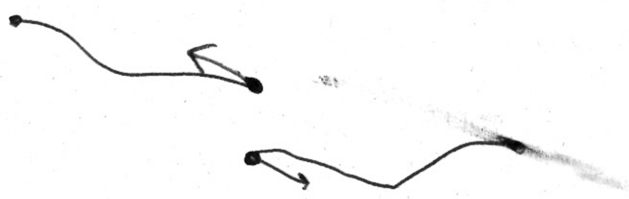


Form Connes fusion

$$H_0 \boxtimes_{A(I) \otimes A(J)} H_0$$

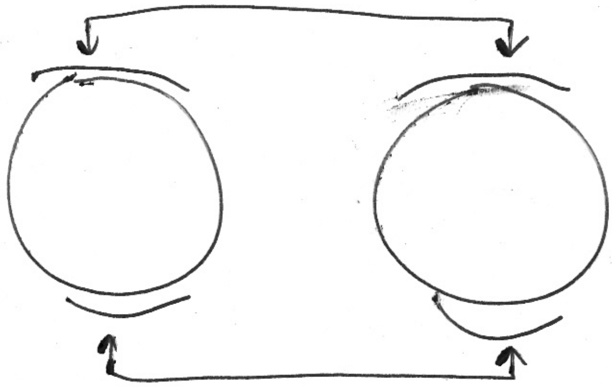
"... this is the Hilbert space in which I will complete.

How to glue smoothly ?



<u>Claim</u> $I_1 \cup_{\{pt\}} I_2$ aquires a C^1 or C^2 structure.

<u>or</u>



$$(A \otimes A^{op})(I)$$

category $\mathbb{C}$ $\cong$ category of 1d lines

But smooth functors

~~R~~ $M \longrightarrow \mathbb{C}$

equivalent
$\not\equiv$

smooth functors

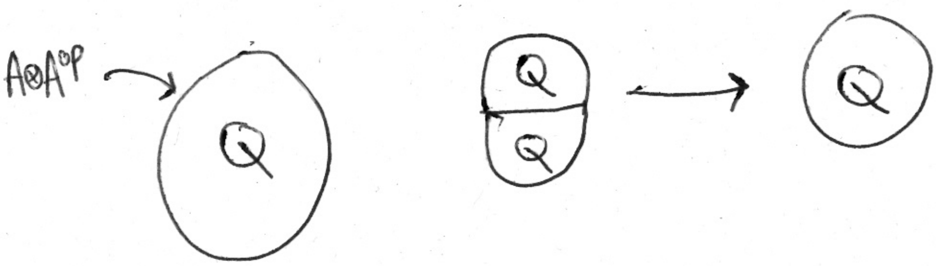
$M \longrightarrow$ 1d lines

Ingo: We are classifying these things.

Note: $$\mathrm{Rep}A = \left\{ \begin{tikzcd} A \arrow[r, bend left=50, "1_A"{name=U}] \arrow[r, bend right=50, "1_A"'{name=D}] & A \arrow[from=U, to=D, phantom, "\Downarrow"] \end{tikzcd} \right\} = \mathrm{End}(\mathrm{id}_A)$$

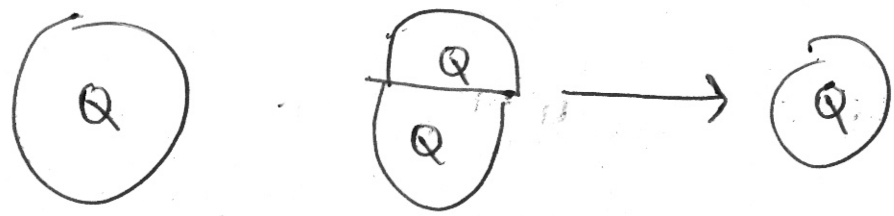
a braided monoidal category.

Let Q be a Frob. algebra in $\mathrm{Rep}(A \otimes A^{op})$.



<u>Quest</u>: What is the state space of a circle?

<u>A</u>:

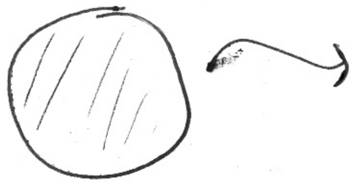


$B(\cap) \in C$

$A(\cup)' =: C$

<u>Quest</u>. What is the vector associated to a surface?

<u>Ans</u>: I don't know. For the disc, it's



vector = invariant vector in $SL(2,\mathbb{R}) \subset \mathrm{Diff}(S^1)$ acting on Hilbert space.

<u>Lemma</u> (He) If I have a rep of $SL(2,\mathbb{R})$ on a Hilbert space,

$SU(2)$, 1 | 2 3 5

1 2 3