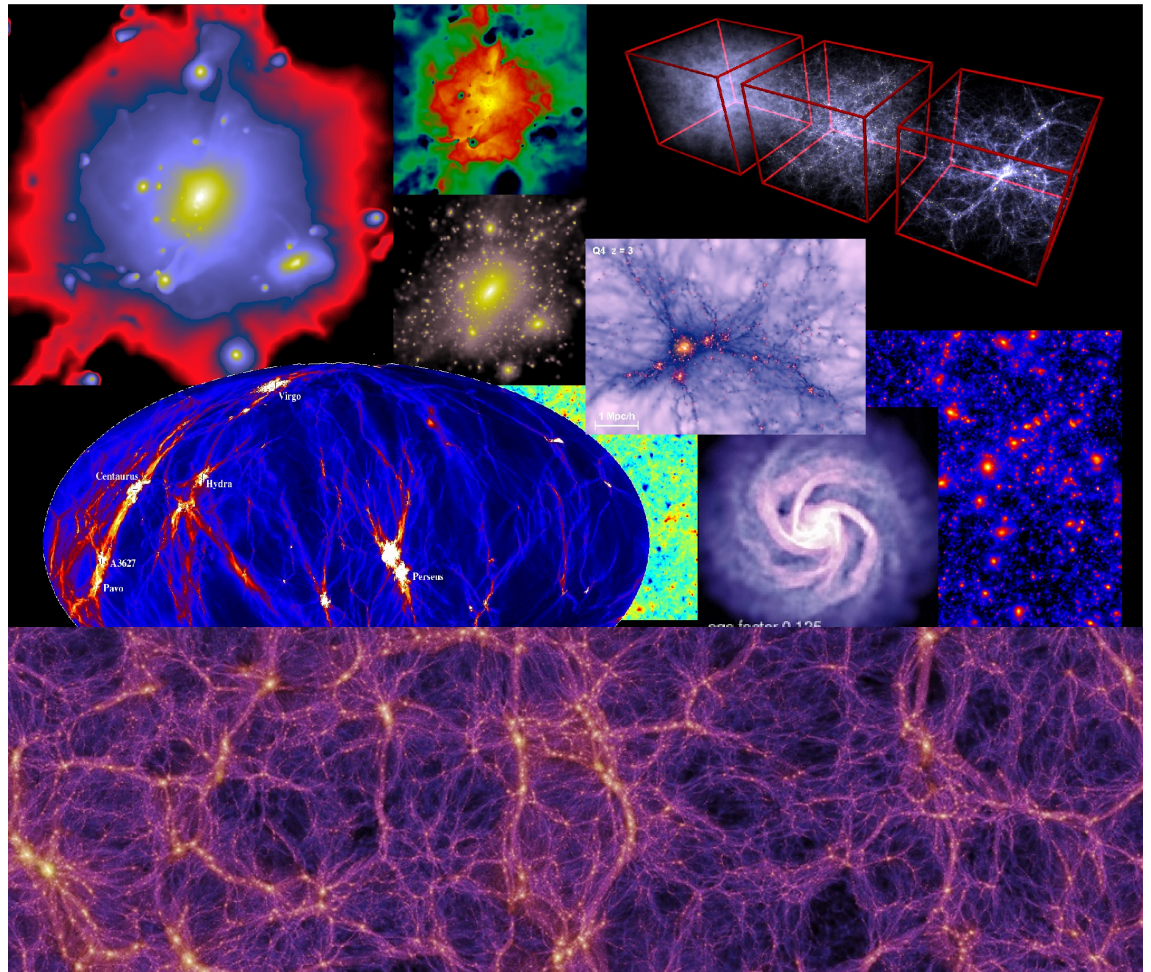


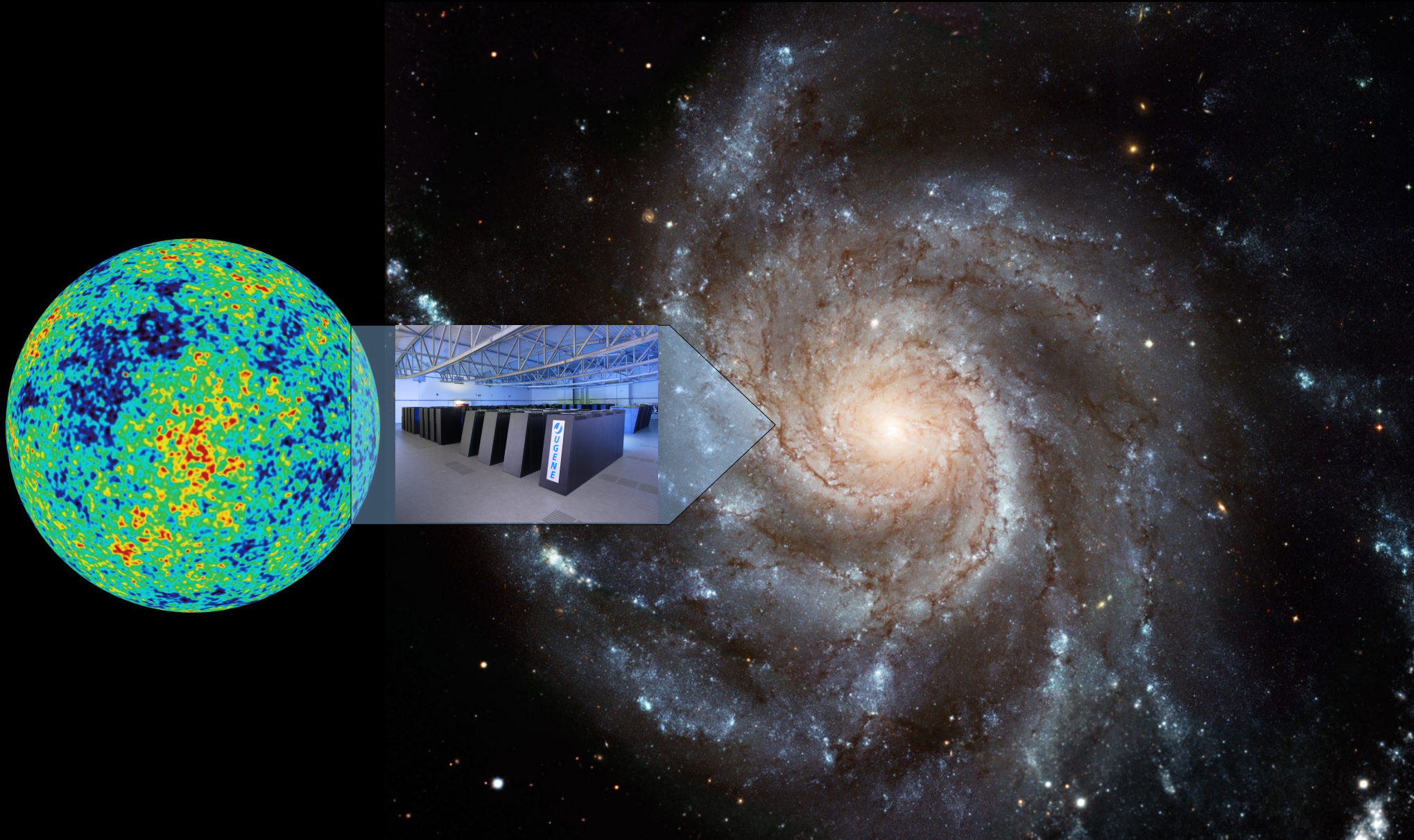
# Simulations of Cosmic Structure Formation

Volker Springel

- ▶ The role of simulations in cosmology
- ▶ High-resolution N-body simulations
- ▶ Millennium XXL
- ▶ Hydrodynamic simulations and recent results for galaxy formation



# Cosmological simulations aim to bridge 13.6 billion years of evolution



# Structure formation in the dark matter reduces to an N-body system

## BASIC EQUATIONS AND THEIR DISCRETIZATION

### Gravitation

General theory of relativity  
(Newtonian approximation in  
an expanding space-time)

Dark matter is collisionless



Monte-Carlo integration as  
**N-body System**



3N **coupled**, non-linear differential  
equations of second order

### Friedmann-Lemaitre model

$$H(a) = H_0 \sqrt{a^{-3}\Omega_0 + a^{-2}(1 - \Omega_0 - \Omega_\Lambda) + \Omega_\Lambda}$$

### Collisionless Boltzmann equation with self-gravity

$$\frac{df}{dt} \equiv \frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{r}} \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\nabla^2 \Phi(\mathbf{r}, t) = 4\pi G \int f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$$

### Hamiltonian dynamics in expanding space-time

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m_i a(t)^2} + \frac{1}{2} \sum_{ij} \frac{m_i m_j \varphi(\mathbf{x}_i - \mathbf{x}_j)}{a(t)}$$

$$\nabla^2 \varphi(\mathbf{x}) = 4\pi G \left[ -\frac{1}{L^3} + \sum_n \tilde{\delta}(\mathbf{x} - \mathbf{n}L) \right]$$



### Problems:

N is very large  
All equations are coupled  
with each other



**Currently the fastest supercomputers carry out about ~1 Petaflop, which are one million billion floating point operations per second**

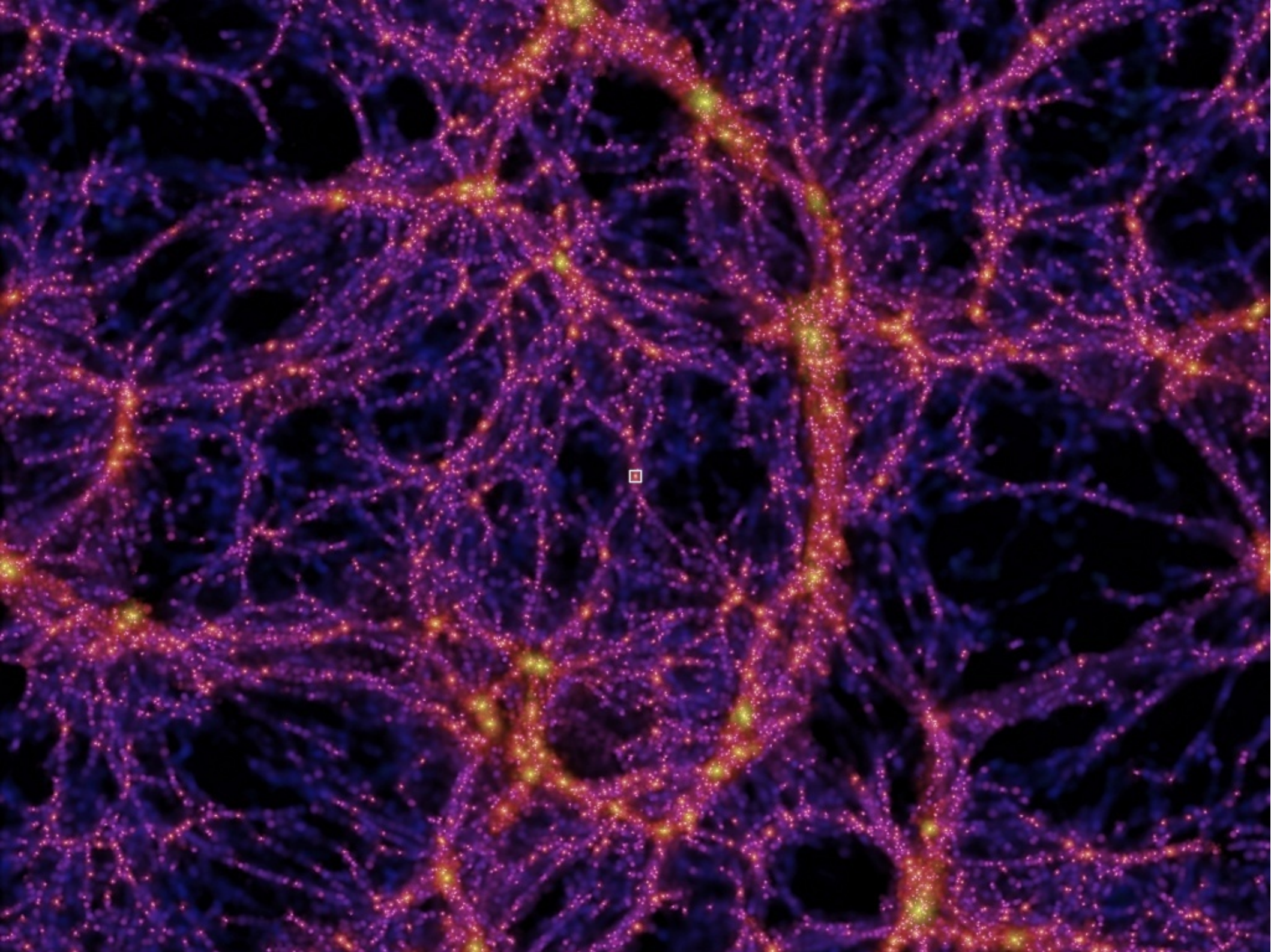
**JUGENE IN JUELICH**

$z = 48.1$

$T = 0.05 \text{ Gyr}$

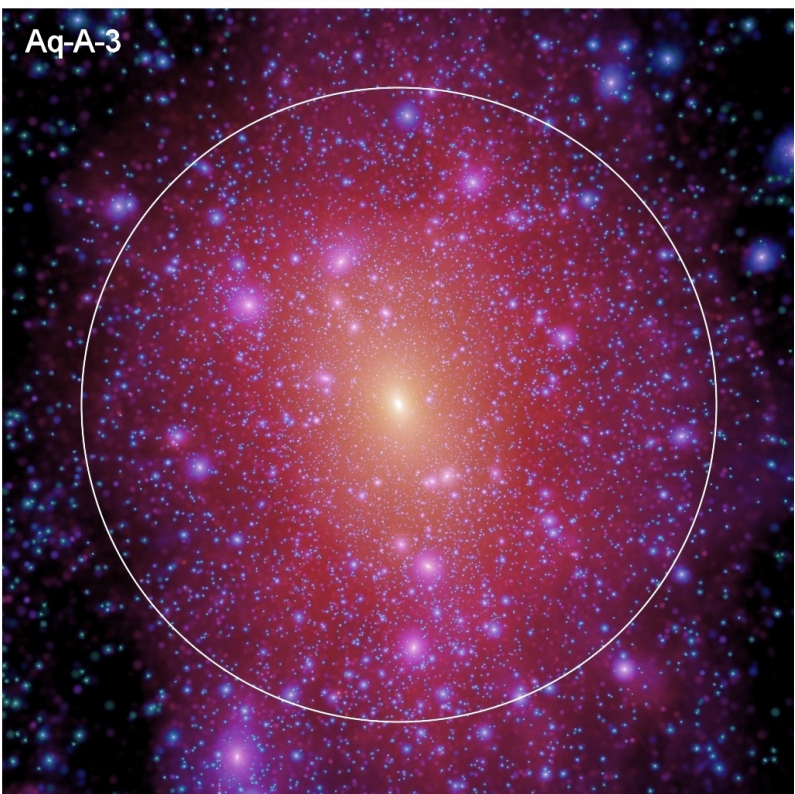
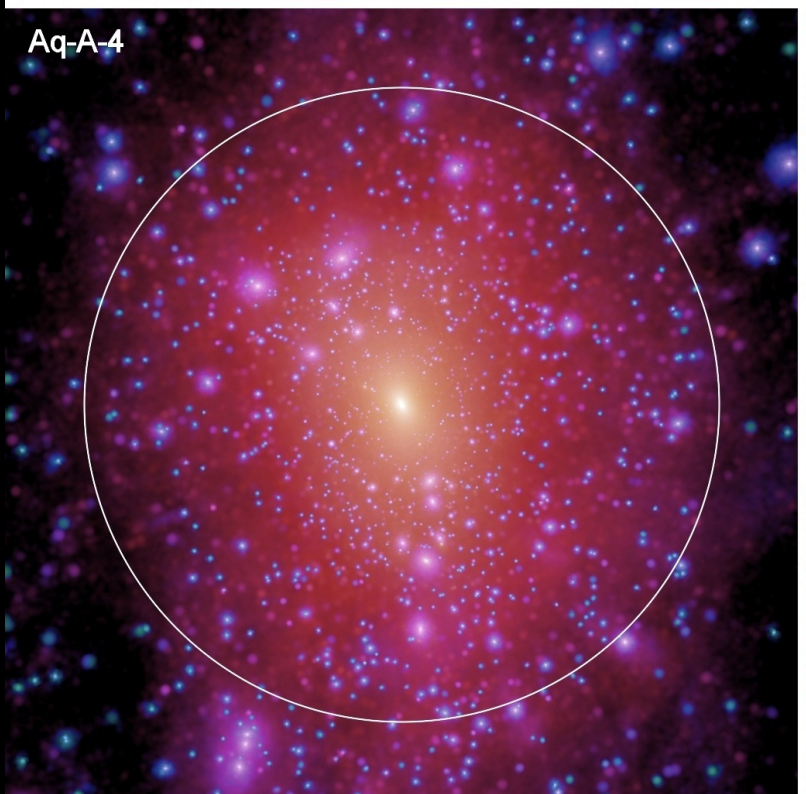
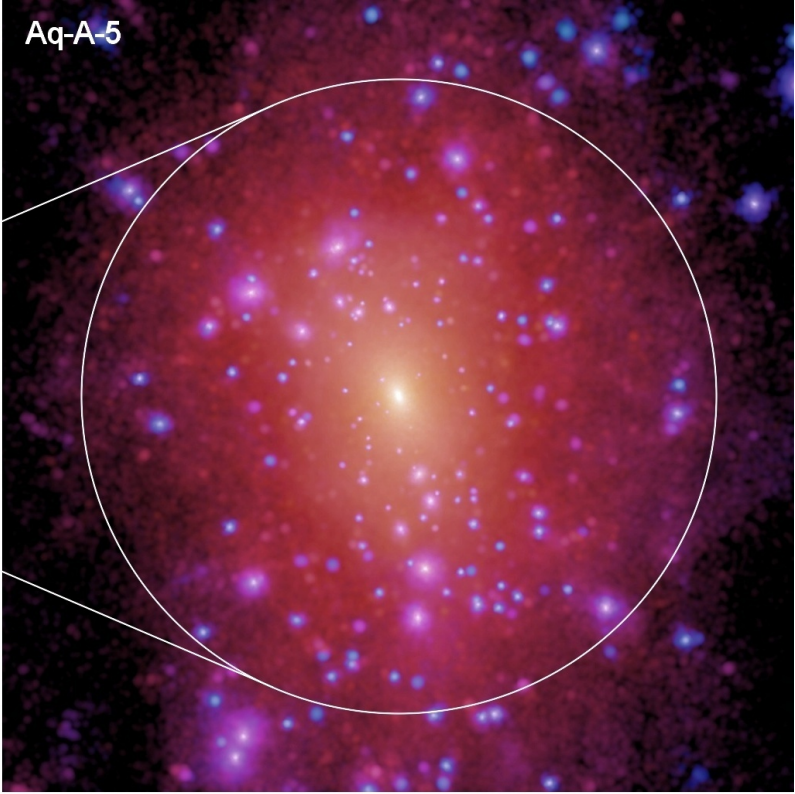
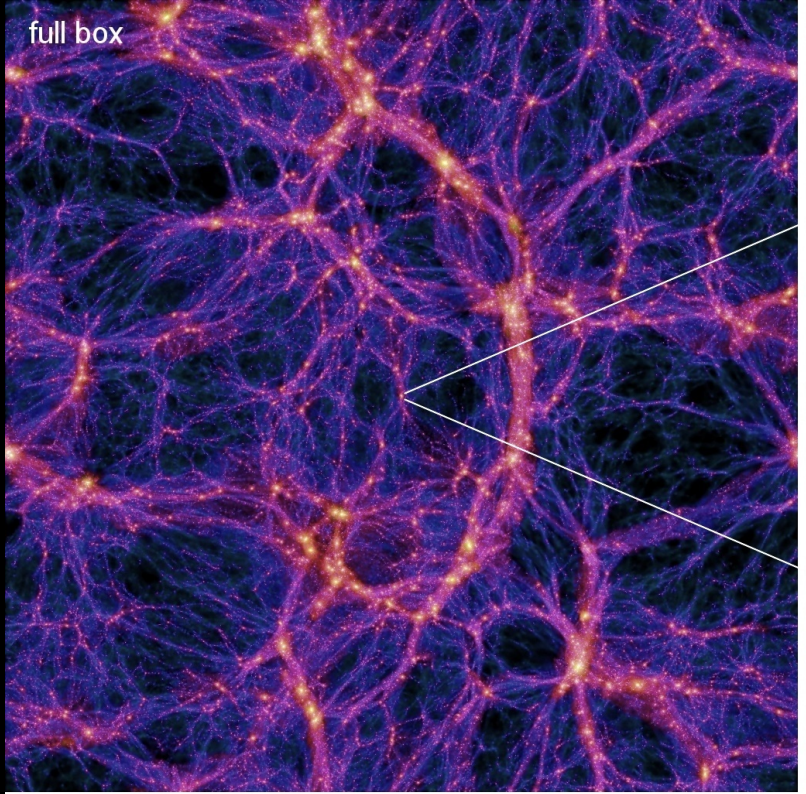


500 kpc



Zooming in on dark matter halos reveals a huge abundance of dark matter substructure

DARK MATTER DISTRIBUTION IN A MILKY WAY SIZED HALO AT DIFFERENT RESOLUTION



A visualization of the cosmic web from the Millennium simulation. The image shows a dense network of filaments and nodes, with colors ranging from dark purple to bright yellow. A horizontal scale bar is located in the upper middle part of the image, labeled "1 Gpc/h".

1 Gpc/h

'Millennium' simulation

Springel et al. (2005)

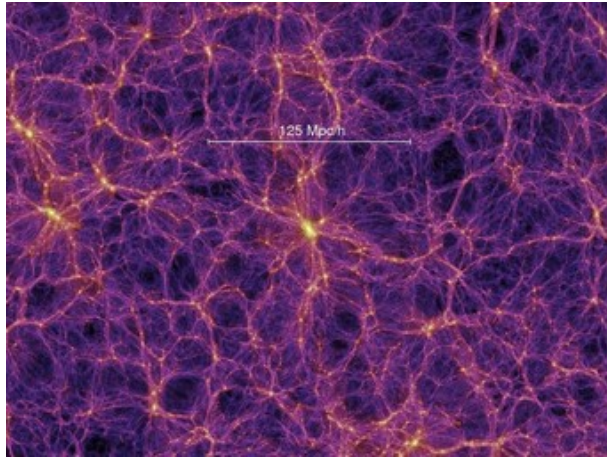
$\Lambda$ CDM

10,077,696,000 particles

$m=8.6 \times 10^8 M_{\odot}/h$



# Why are **cosmological simulations** of structure formation useful for studying the dark universe?



**Simulations are the theoretical tool of choice for calculations in the non-linear regime.**

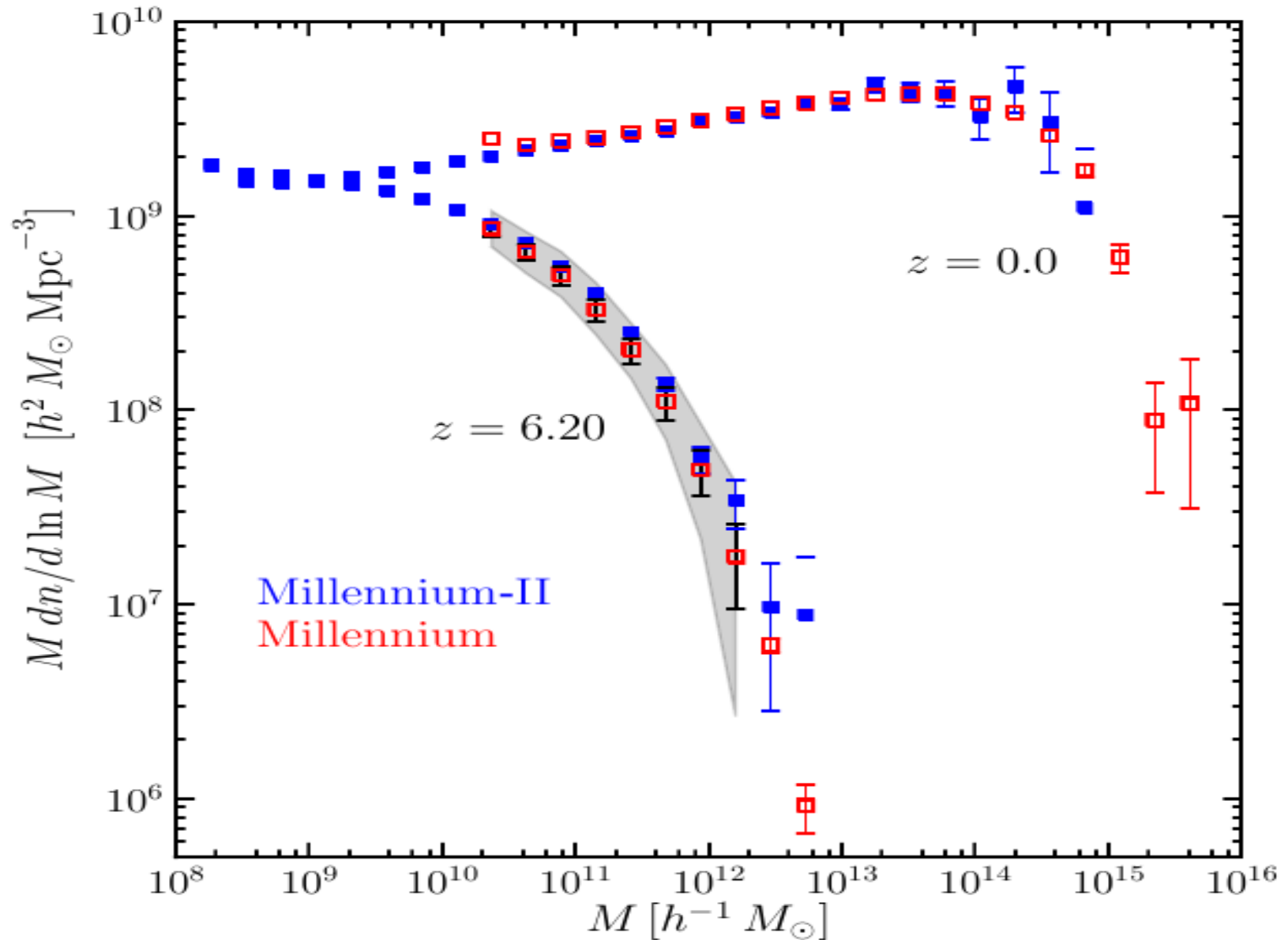
**They connect the (simple) cosmological initial conditions with the (complex) present-day universe.**

## **Predictions from N-body simulations:**

- Abundance of objects (as a function of mass and time)
- Their spatial distribution
- Internal structure of halos (e.g. density profiles, spin)
- Mean formation epochs
- Merger rates
- Detailed dark matter distribution on large *and* fairly small scales
- Galaxy formation models
- Gravitational lensing
- Baryonic acoustic oscillations in the matter distribution
- Integrated Sachs-Wolfe effect
- Dark matter annihilation rate
- Morphology of large-scale structure (“cosmic web”)
- ....

Simulations provide accurate measurements for halo abundance as a function of time

### CONVERGENCE RESULTS FOR HALO ABUNDANCE

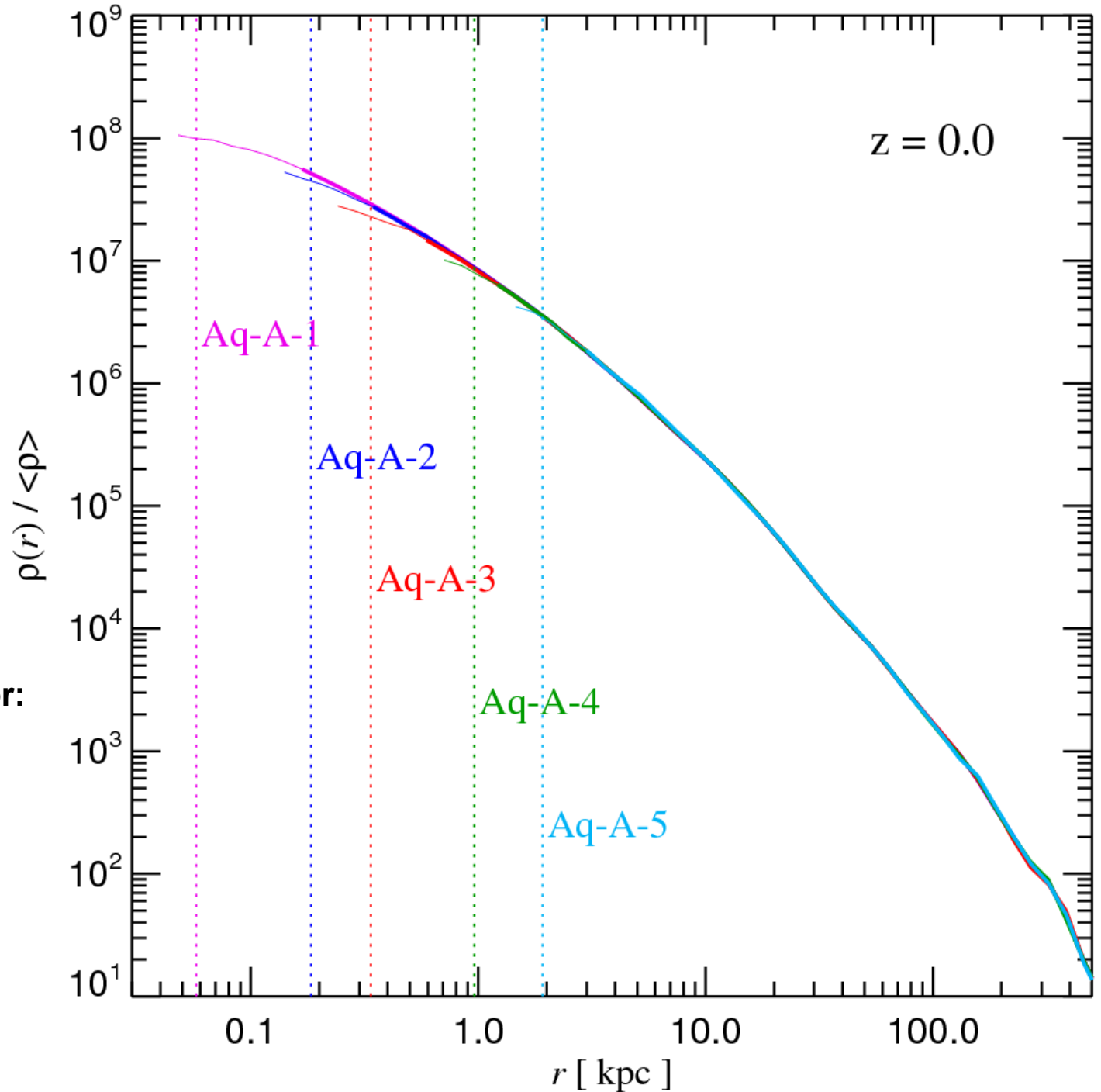


Spherically averaged density profiles of dark matter halos have a nearly universal shape

**DENSITY PROFILE AS A FUNCTION OF RADIUS**

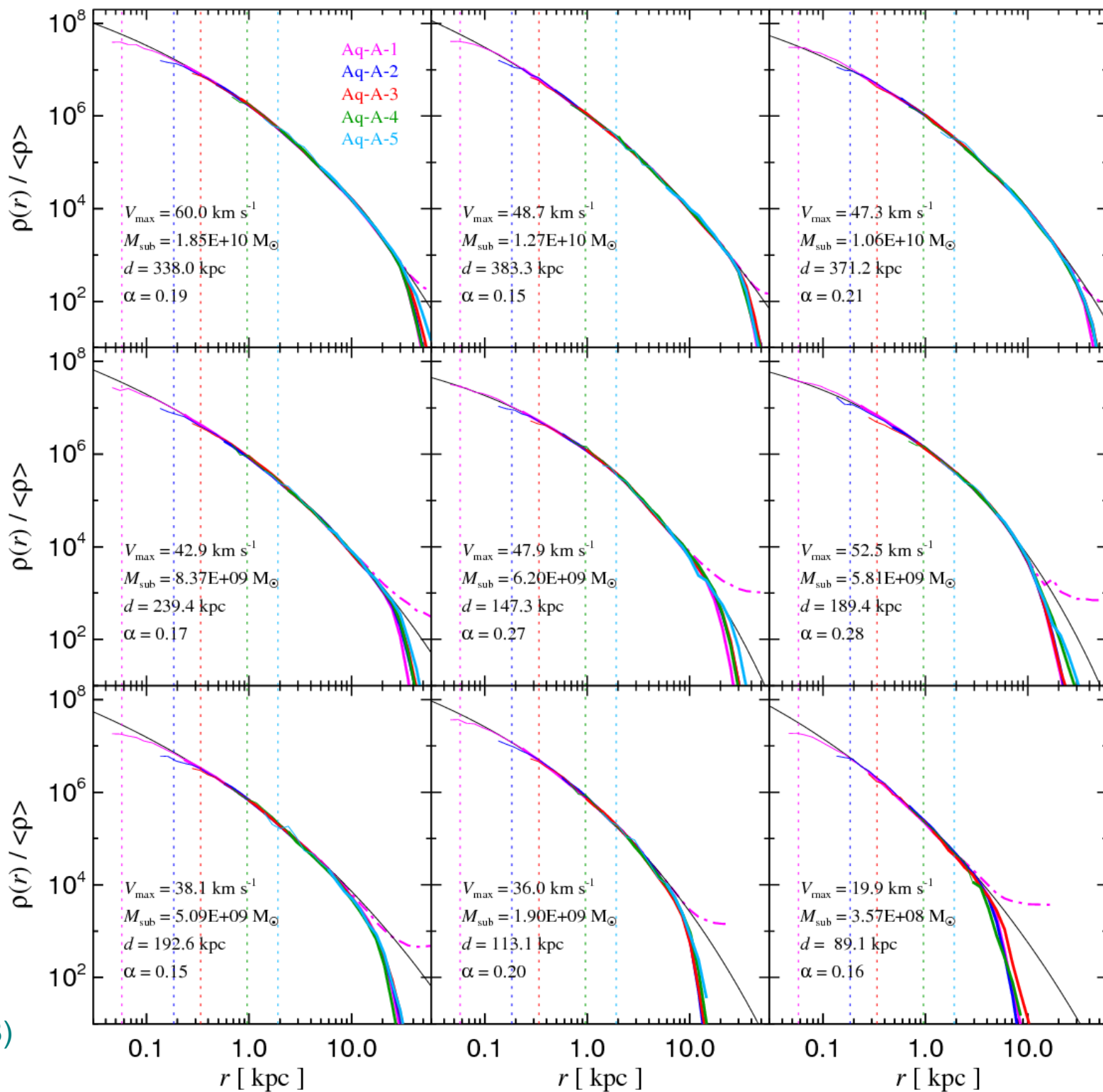
**Fundamental importance for:**

- Rotation curve of galaxies
- Internal structure of galaxy clusters
- Gravitational lensing
- DM annihilation
- Galaxy mergers



Simulations allow us to study the convergence of subhalo density profiles

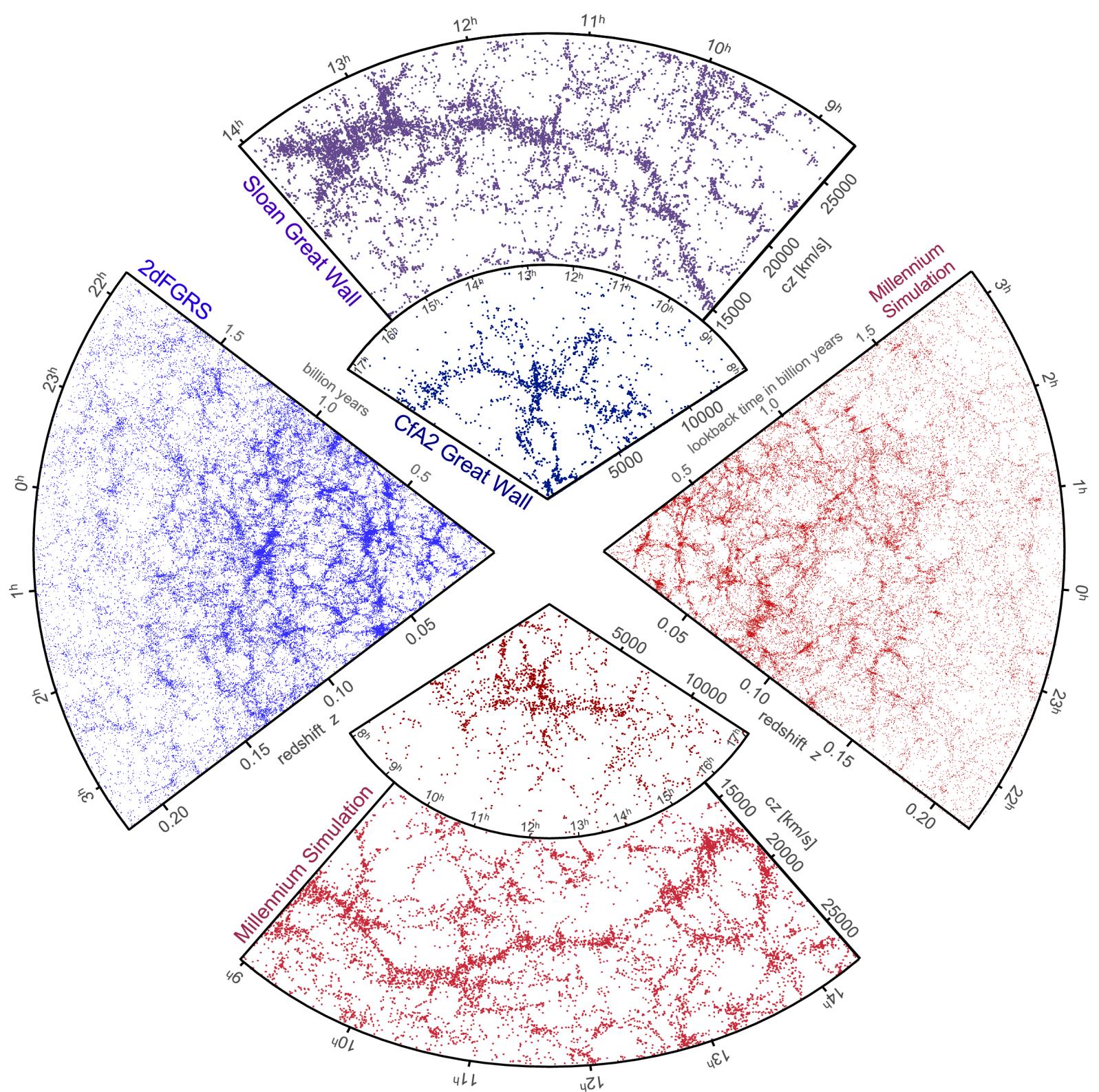
SPHERICALLY AVERAGED DENSITY PROFILES IN THE AQ-A HALO AT DIFFERENT RESOLUTION



Springel et al. (2008)

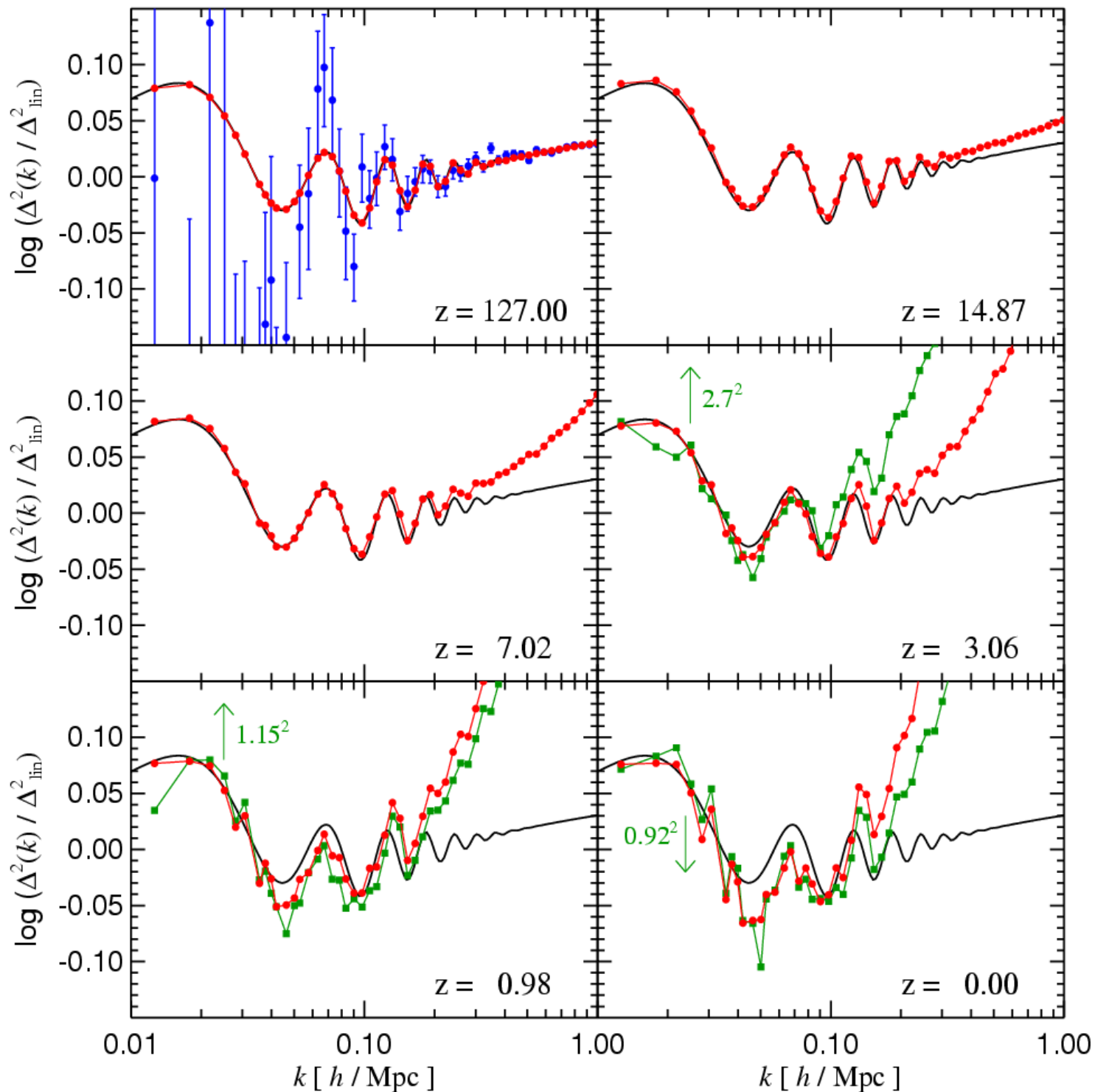
# Simulated and observed large-scale structure in the galaxy distribution

## MOCK PIE DIAGRAMS COMPARED TO SDSS, 2DFGRS, AND CFA-2



The baryonic wiggles remain visible in the galaxy distribution down to low redshift and may serve as a "standard ruler" to constrain dark energy

**DARK MATTER AND GALAXY POWER SPECTRA FROM THE MILLENNIUM SIMULATION IN THE REGION OF THE WIGGLES**



# Millennium-XXL

Largest  
high-resolution  
N-body simulation

**303 billion particles**

$L = 3 \text{ Gpc}/h$

~700 million halos  
at  $z=0$

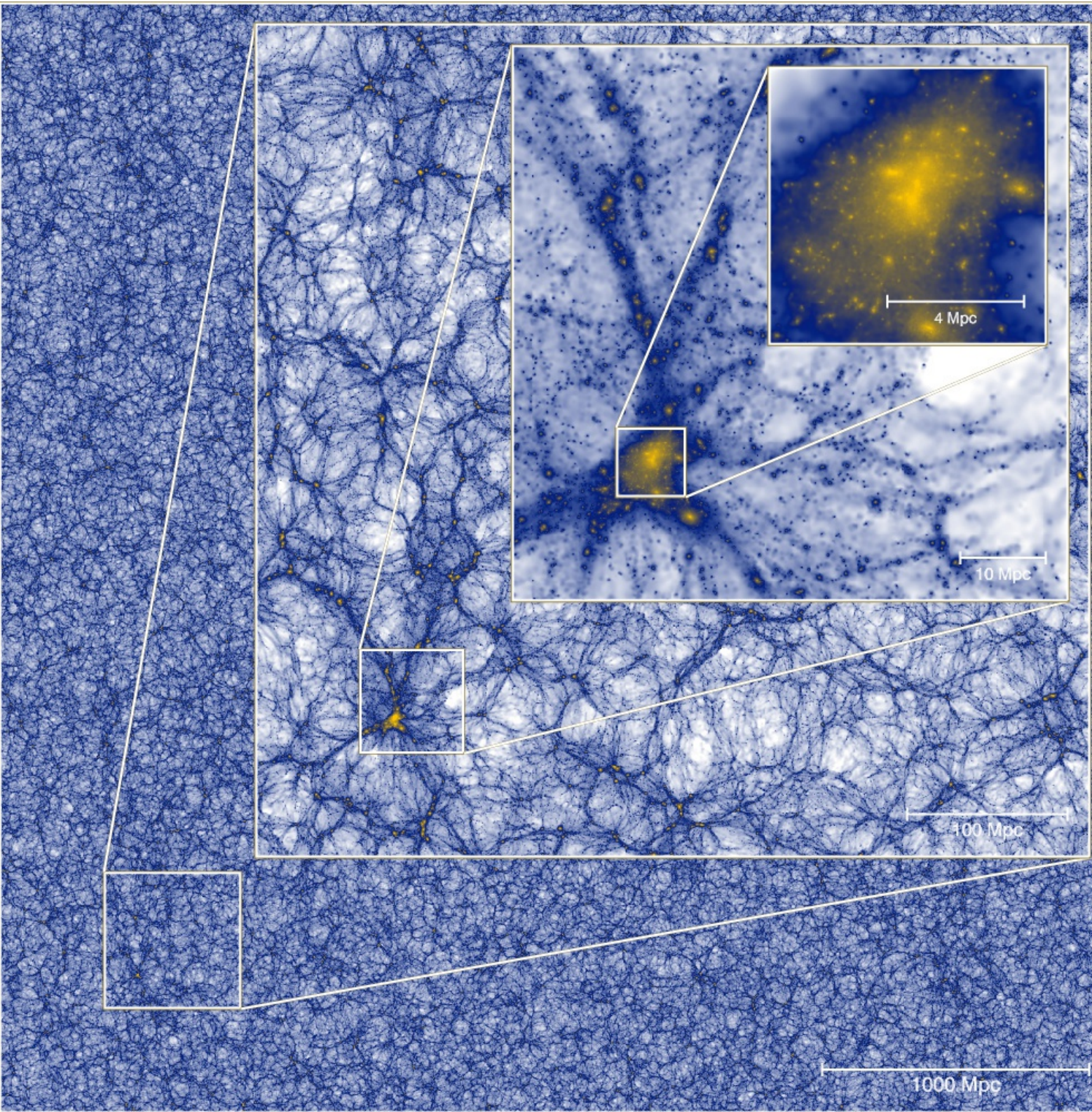
~25 billion (sub)halos  
in mergers trees

$m_p = 6.1 \times 10^9 M_\odot/h$

12288 cores,  
30 TB RAM on  
Supercomputer  
JuRoPa in Juelich

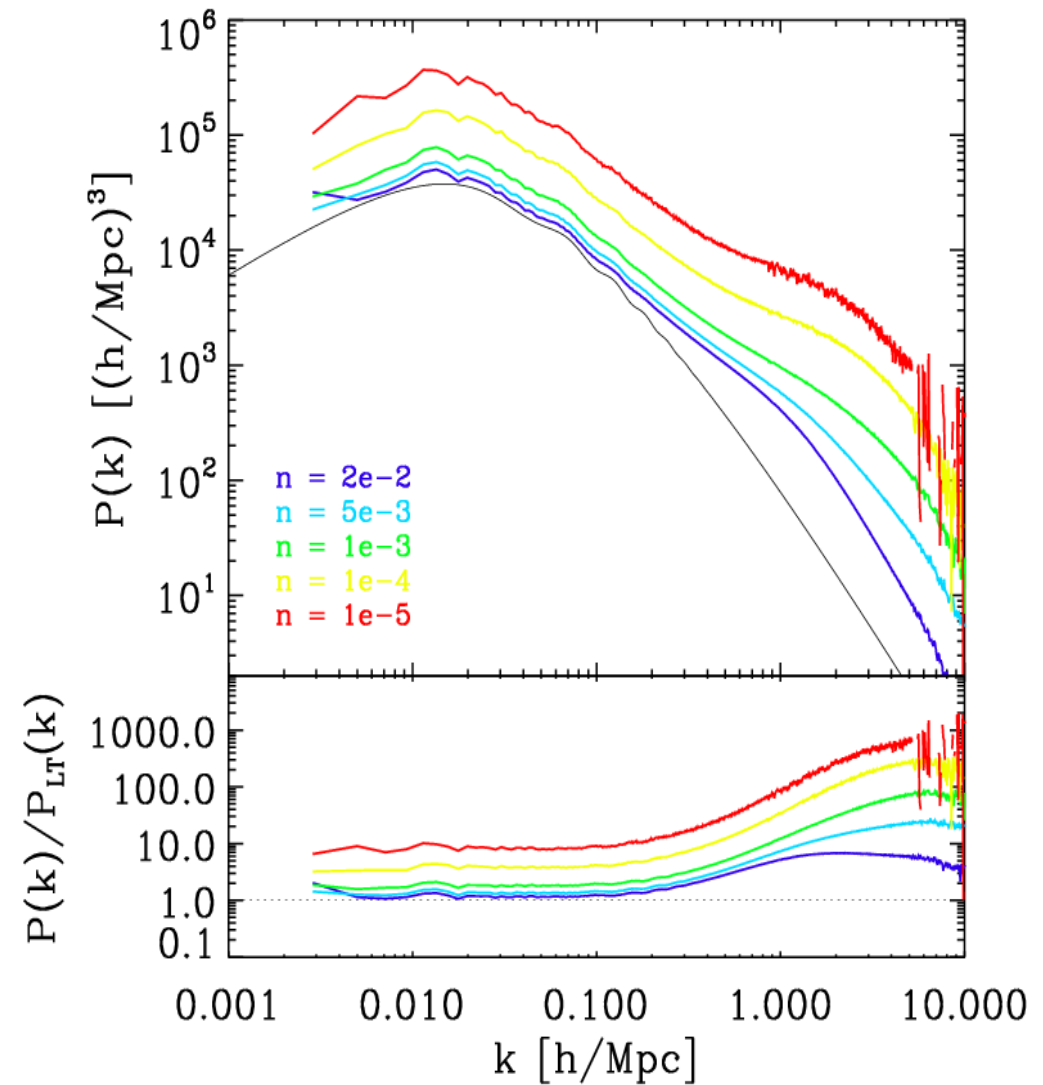
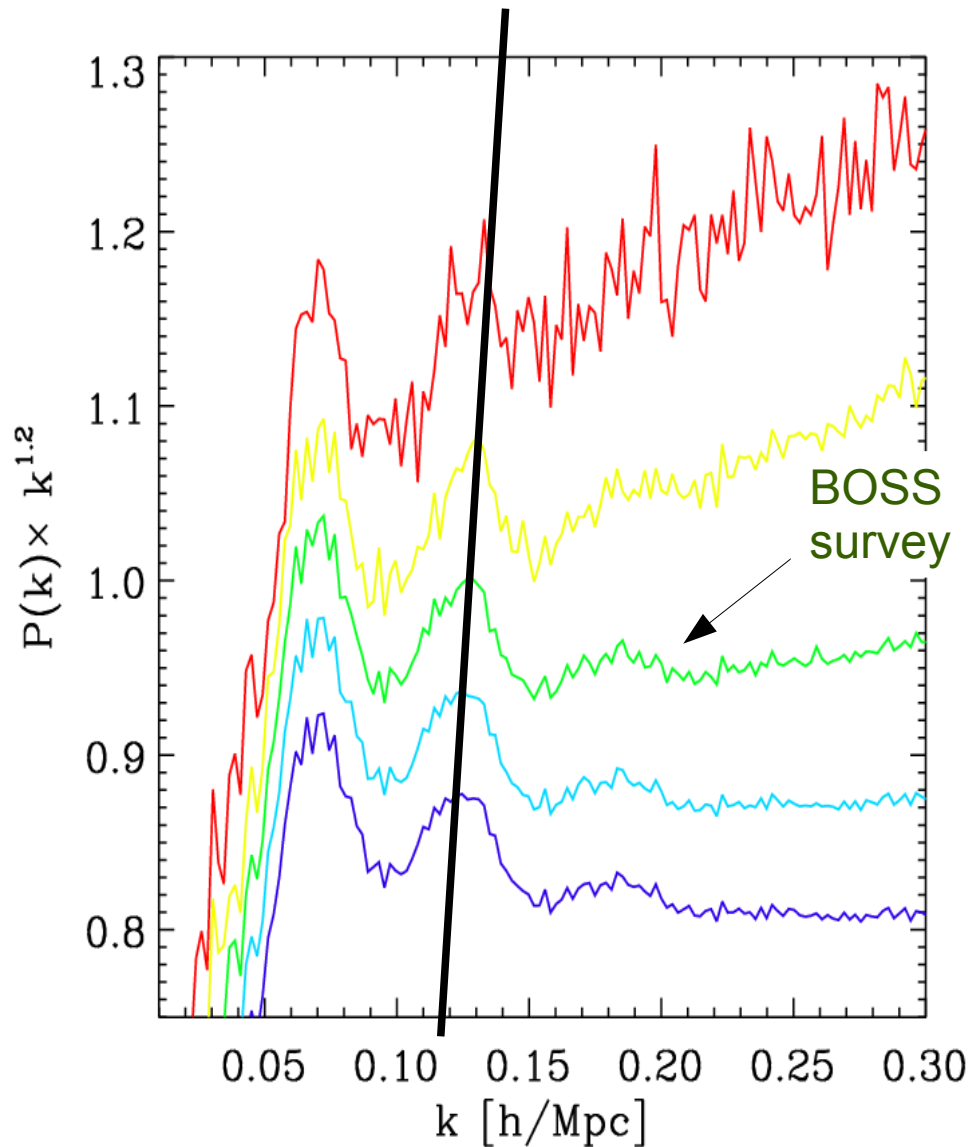
2.7 million CPU-hours

Angulo et al. (2011)



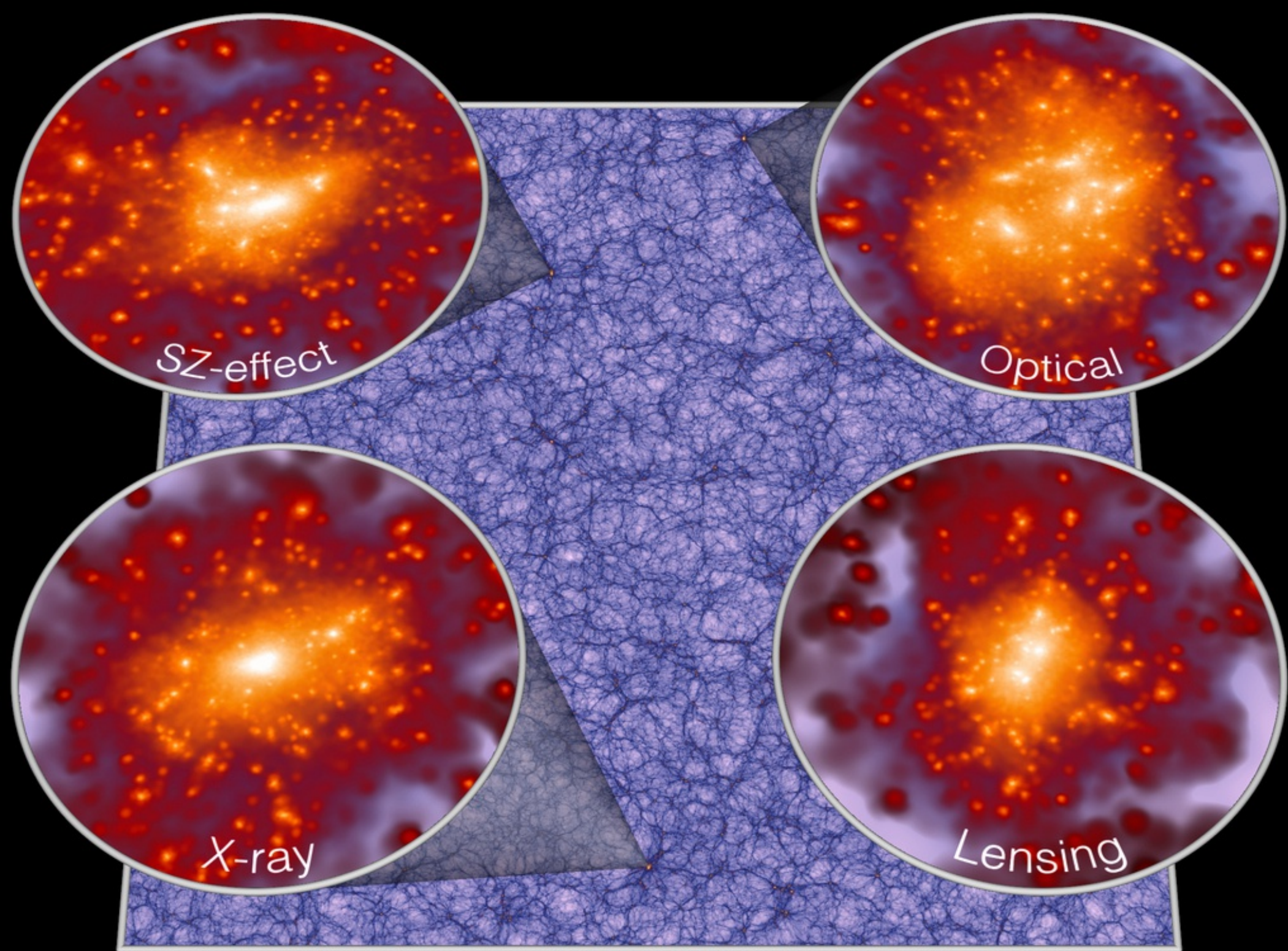
# Different galaxy catalogues in the MXXL simulation trace the BAO features with a scale-dependent bias

POWER SPECTRA OF THE GALAXY DISTRIBUTION AT Z=0 FOR DIFFERENT SPACE DENSITIES



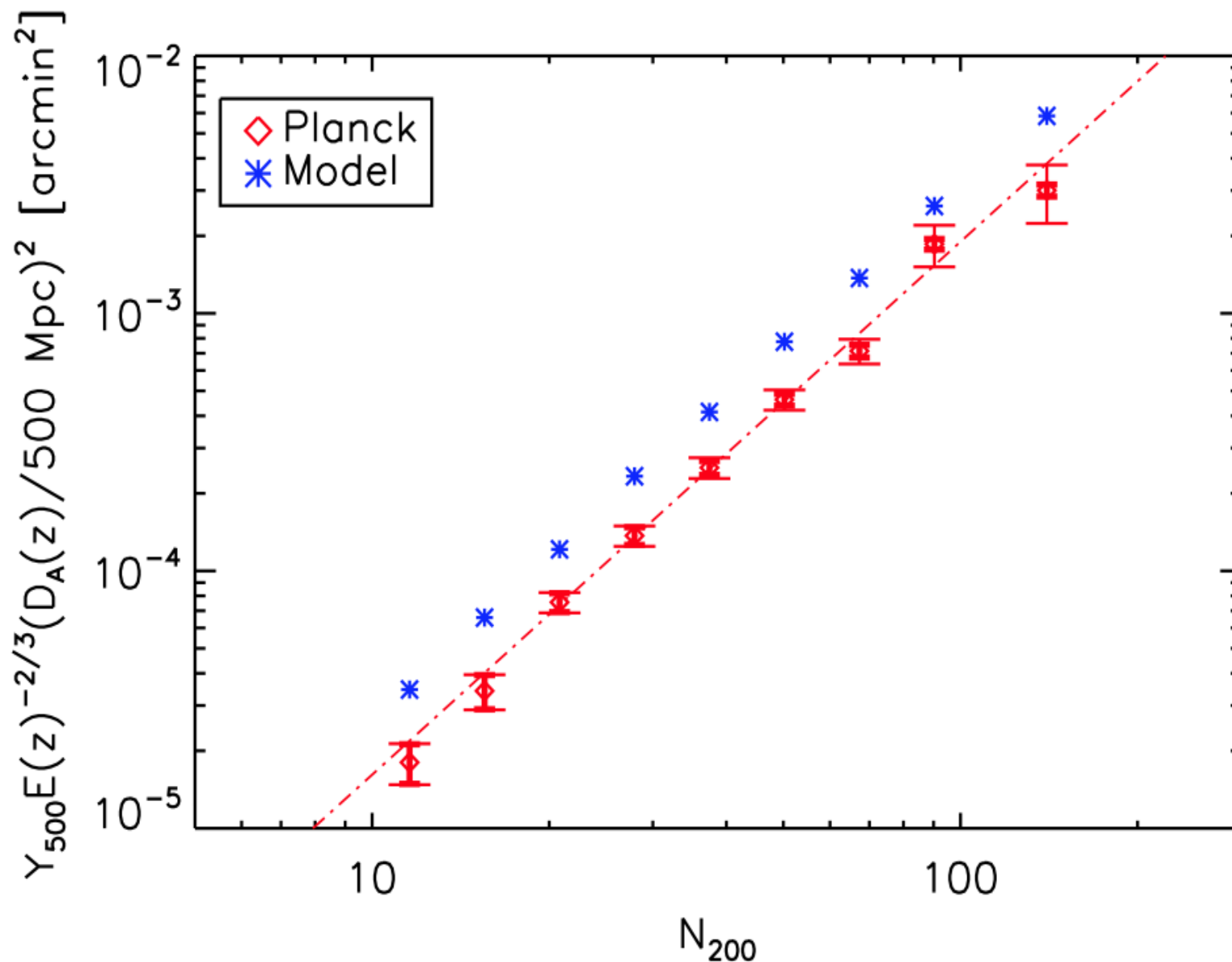
Angulo et al. (2012)





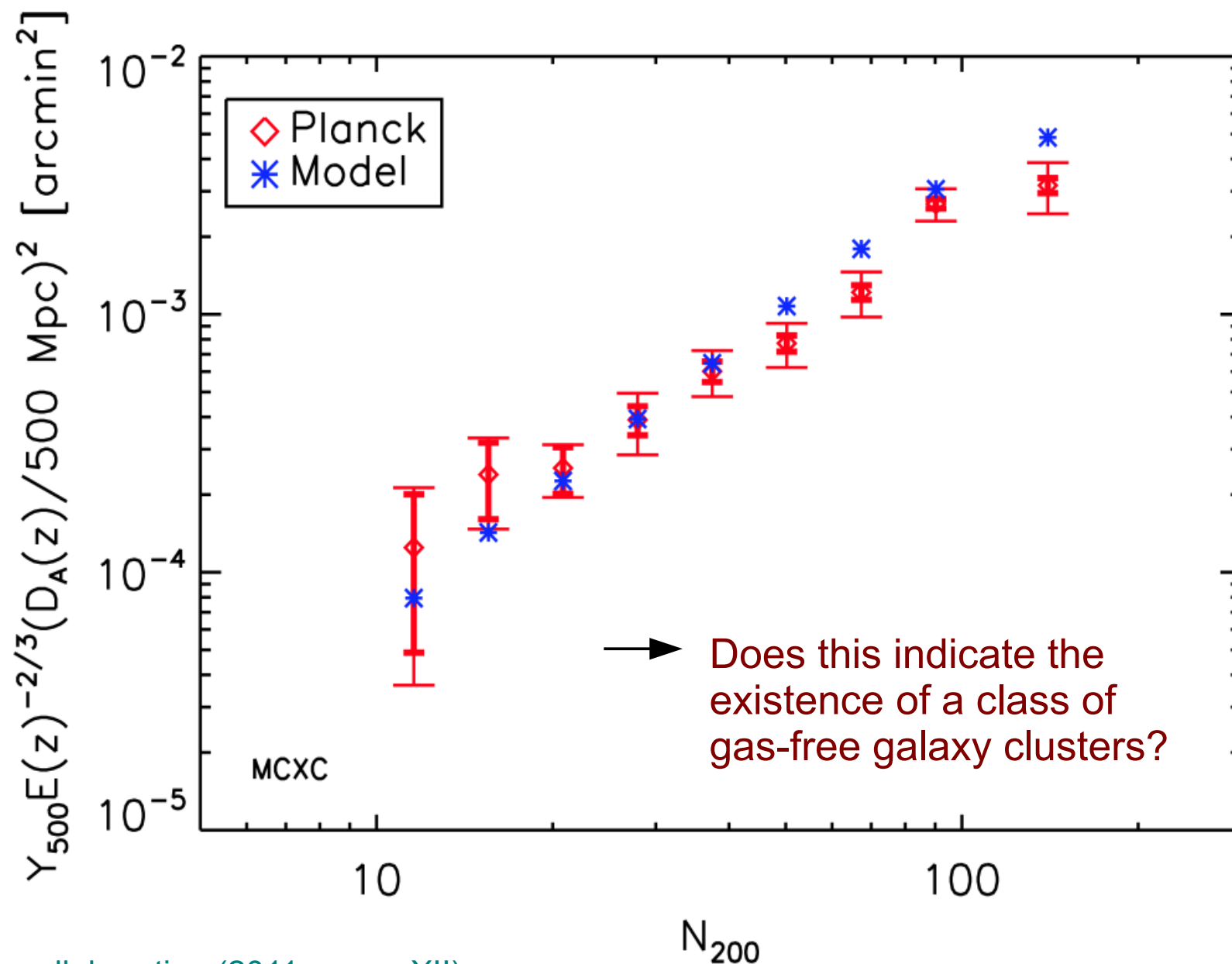
The mean SZ-signal in PLANCK-data for clusters of given optical richness (from the MaxBCG catalogue) is lower by a factor  $\sim 2$  than expected

**SZ vs. OPTICAL RICHNESS MEASURED BY PLANCK COMPARED TO MODEL EXPECTATIONS**



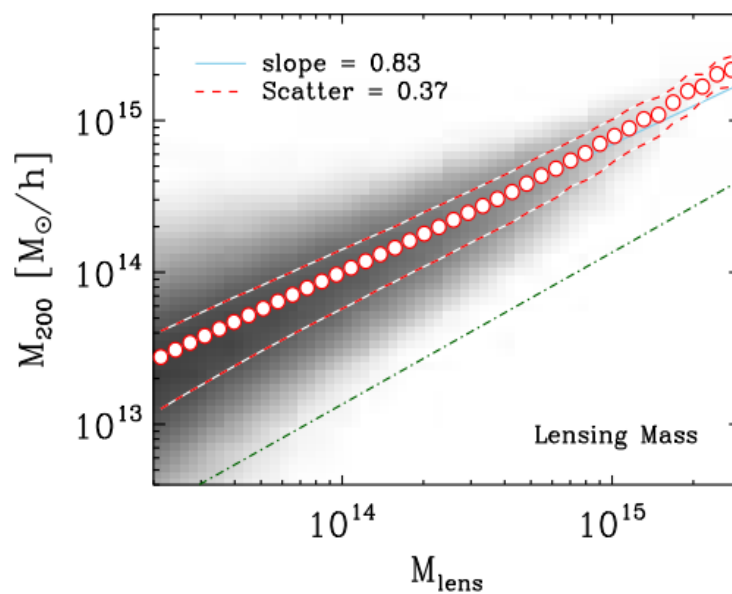
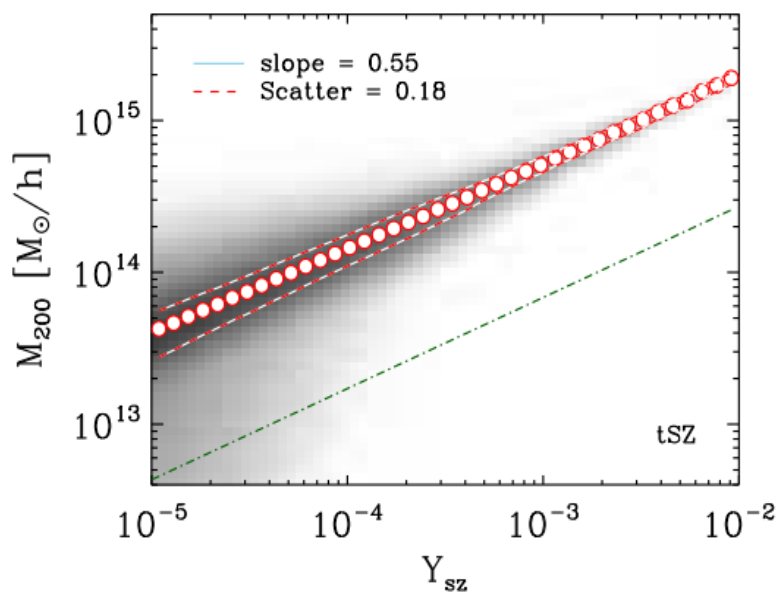
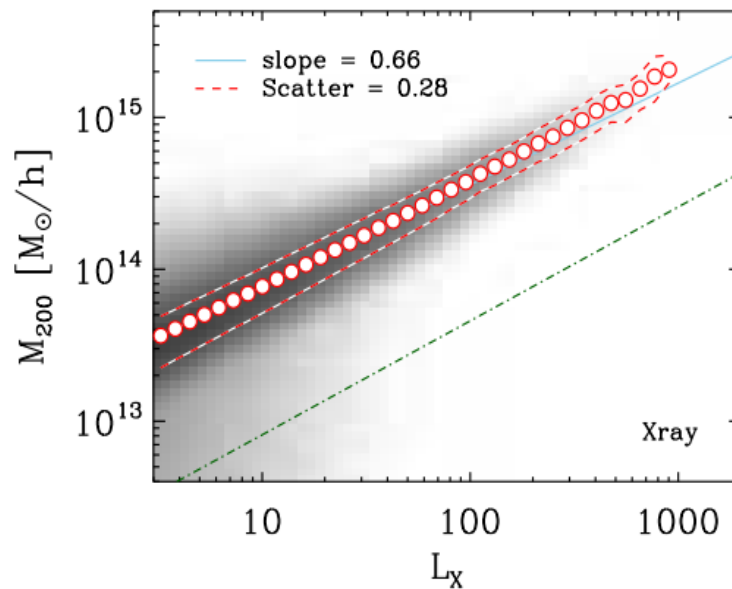
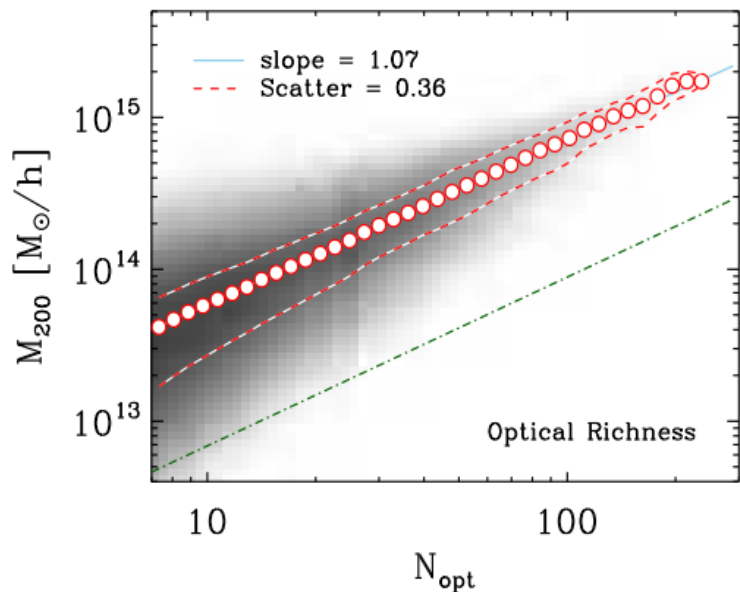
For the subsample of clusters with individual X-ray measurements, the PLANCK-data for the SZ matches the model expectations

### SZ vs. OPTICAL REACHNESS FOR X-RAY DETECTED CLUSTER SUBSAMPLE



Mock catalogues from the MXXL allow a study of the expected cluster scaling relations, and their modification due to systematic effects

## DIFFERENT CLUSTER SCALING RELATIONS AND THEIR SCATTER

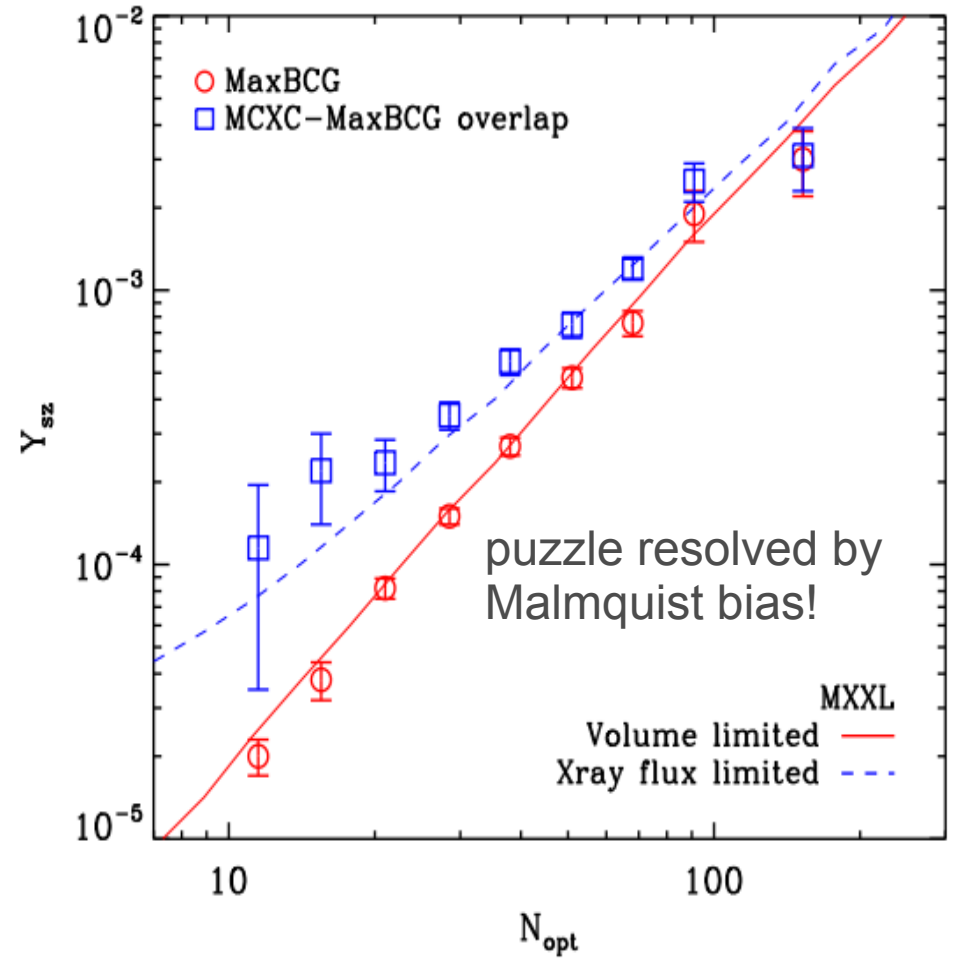
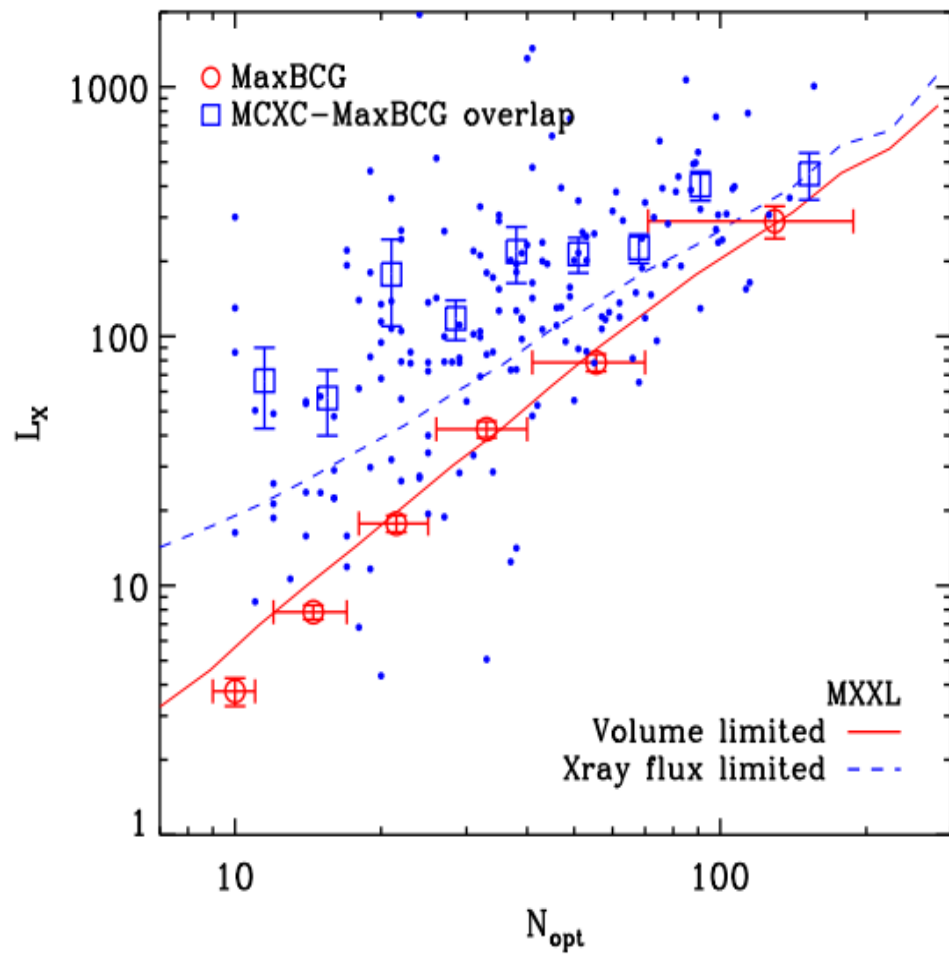


**Systematic effects that influence the scaling relations:**

- Sample selection
- Spurious cluster identification
- Miscentering
- Contamination due to line-of-sight foreground structures

# The biases introduced in the measured relations can quantitatively account for the difference detected in the PLANCK analysis

## $L_x/SZ$ vs. OPTICAL REACHNESS FOR DIFFERENT SAMPLES



Angulo, Springel, White, Frenk, Jenkins & Baugh (2012)

# Dynamics of structure formation in baryonic matter

## BASIC EQUATIONS

Astrophysical plasmas are extremely thin, with (usually) negligible viscosity

### Euler equations of inviscid ideal gas dynamics

$$\frac{\partial \rho_c}{\partial t} + \frac{1}{a} \nabla_c (\rho_c \mathbf{v}) = 0$$

$$\frac{\partial (\rho_c \mathbf{v})}{\partial t} + \frac{1}{a} \nabla_c [(\rho_c \mathbf{v} \mathbf{v}^T + P_c) \mathbf{v}] = -H(a) \rho_c \mathbf{v} - \frac{\rho_c}{a^2} \nabla_c \Phi_c$$

$$\frac{\partial (\rho_c e)}{\partial t} + \frac{1}{a} \nabla_c [(\rho_c e + P_c) \mathbf{v}] = -2H(a) \rho_c e - \frac{\rho_c \mathbf{v}}{a^2} \nabla_c \Phi_c$$

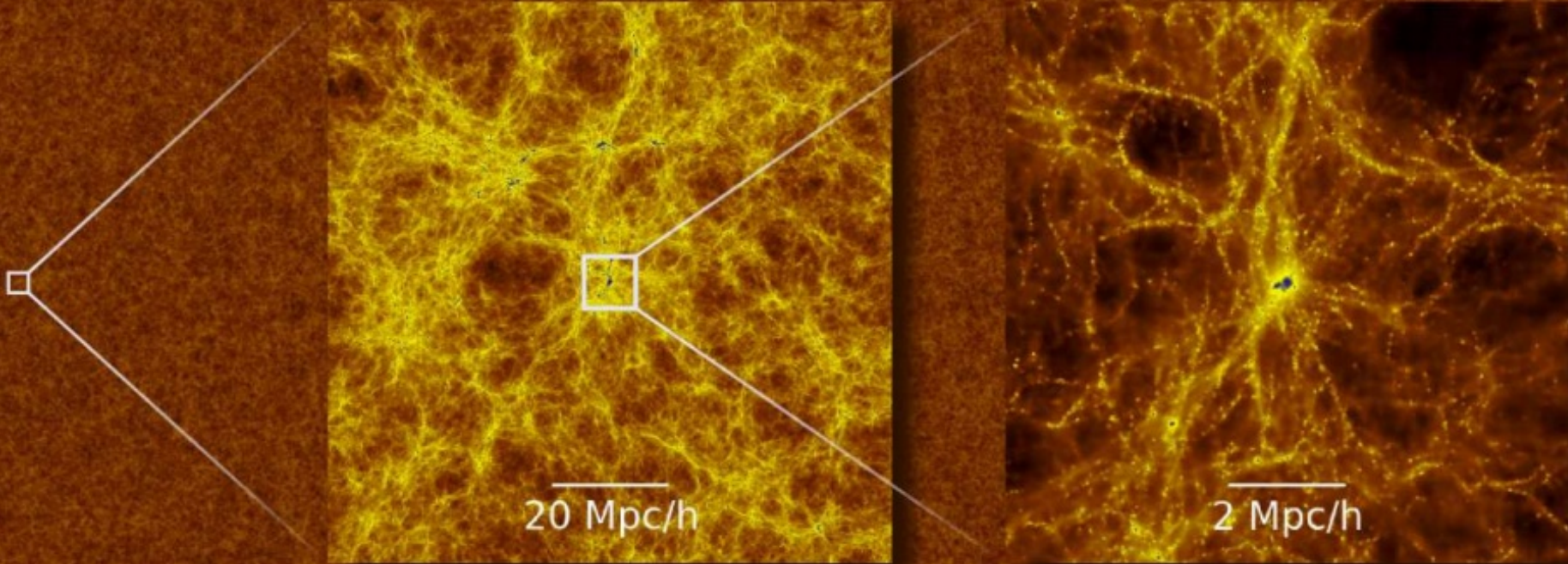
$$\nabla_c^2 \Phi_c = 4\pi G [\rho_c(\mathbf{x}) - \bar{\rho}_c]$$

### Important hydrodynamical processes

- Shock waves
- Turbulence
- Radiative transfer
- Magnetic fields
- Star formation
- Supernova explosions
- Black holes, etc...

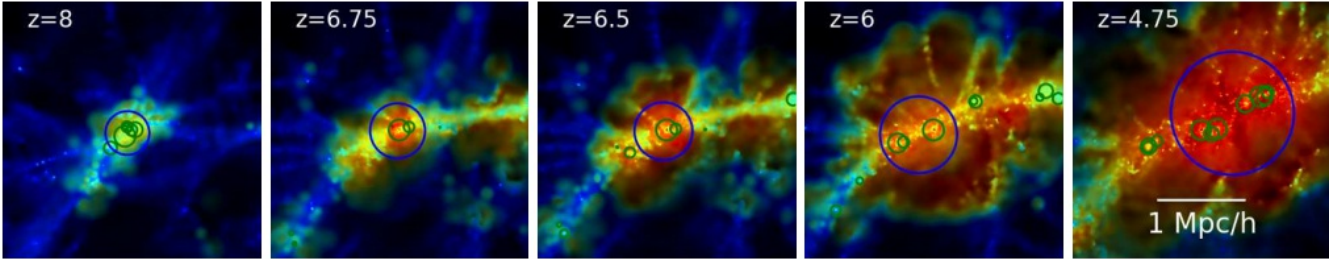
The *MassiveBlack* simulation is the largest astrophysical SPH simulation to date

TRACKING THE FORMATION OF THE FIRST QUSARS ON A PETAFLOP MACHINE



Di Matteo, Springel, et al. (2011)

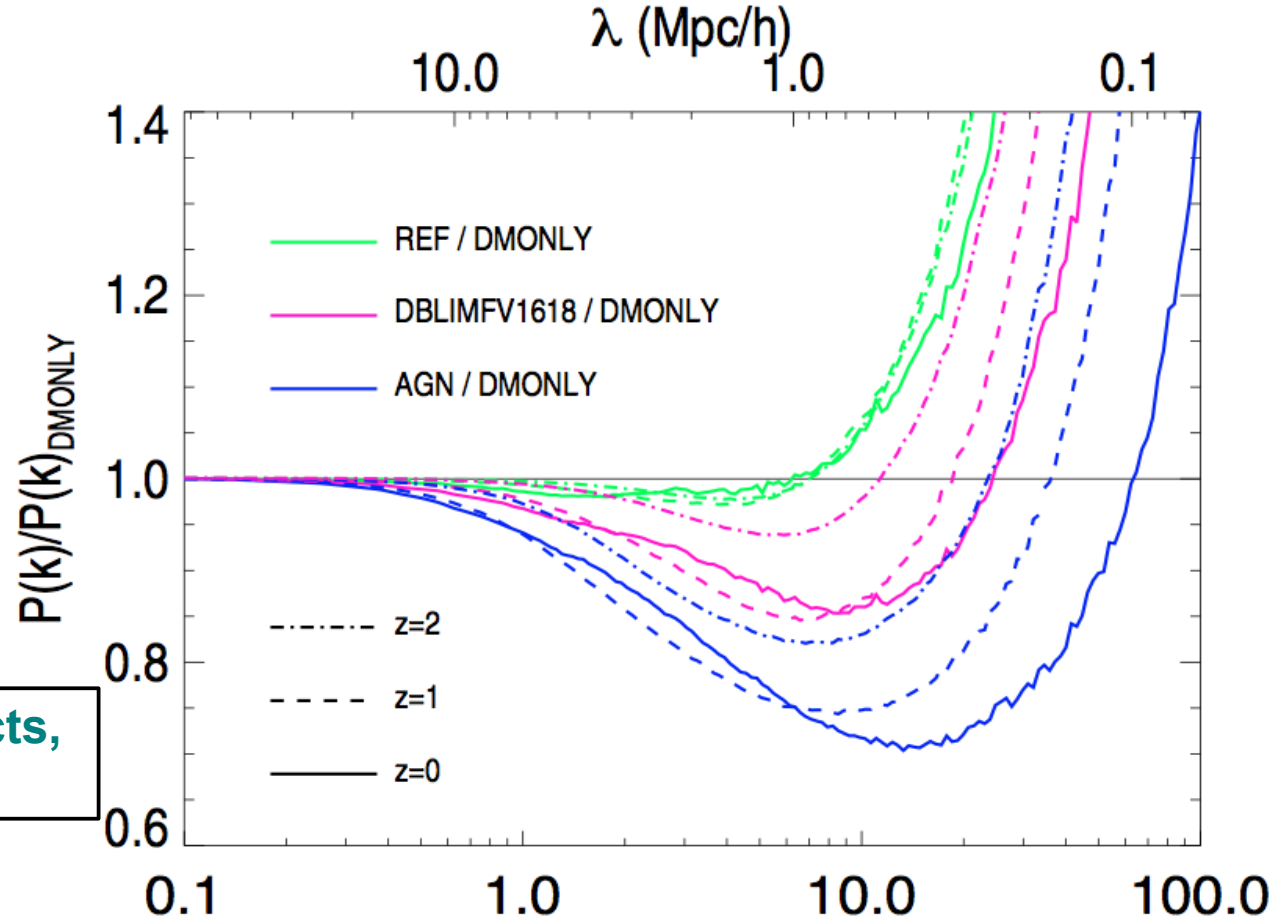
- ▶  $2 \times 3200^3 \sim 65.5$  billion particles
- ▶ 533 Mpc/h box
- ▶  $10^5$  cores on Kraken (Cray XT-5)
- ▶ Multi-threaded P-GADGET3 code



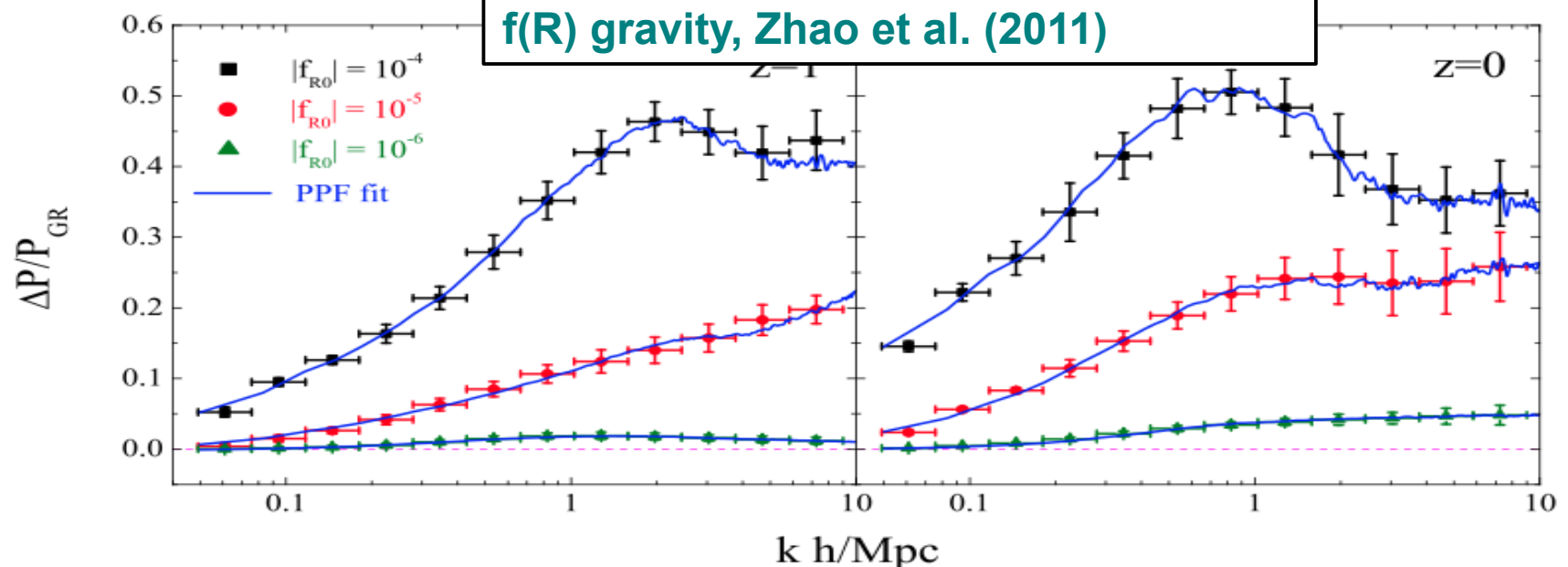
Baryonic physics and modified gravity effects on the matter power spectrum are of similar magnitude

**POWER SPECTRA IN DIFFERENT MODELS**

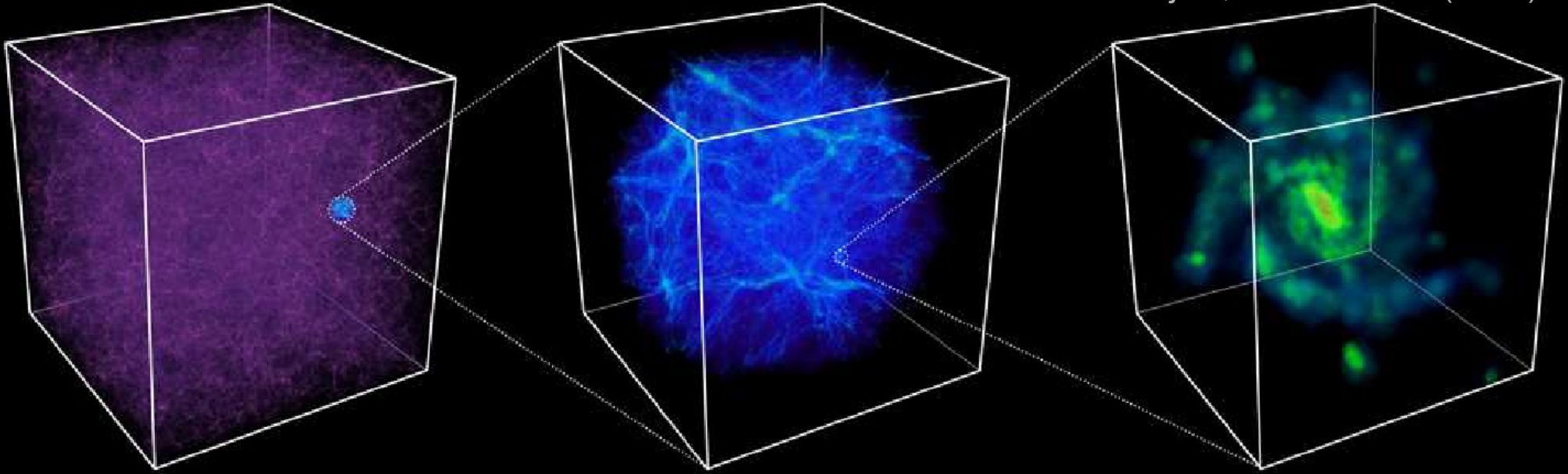
baryonic feedback effects, Semboloni et al. (2011)



f(R) gravity, Zhao et al. (2011)





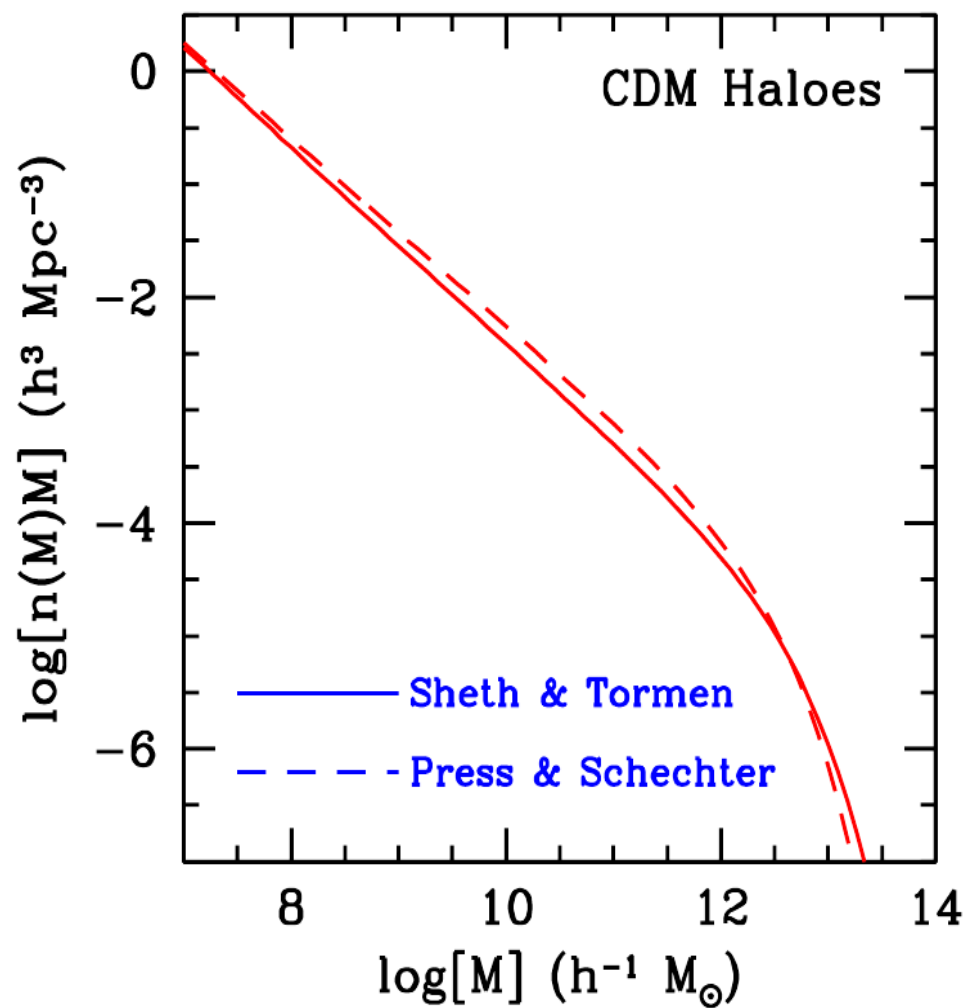
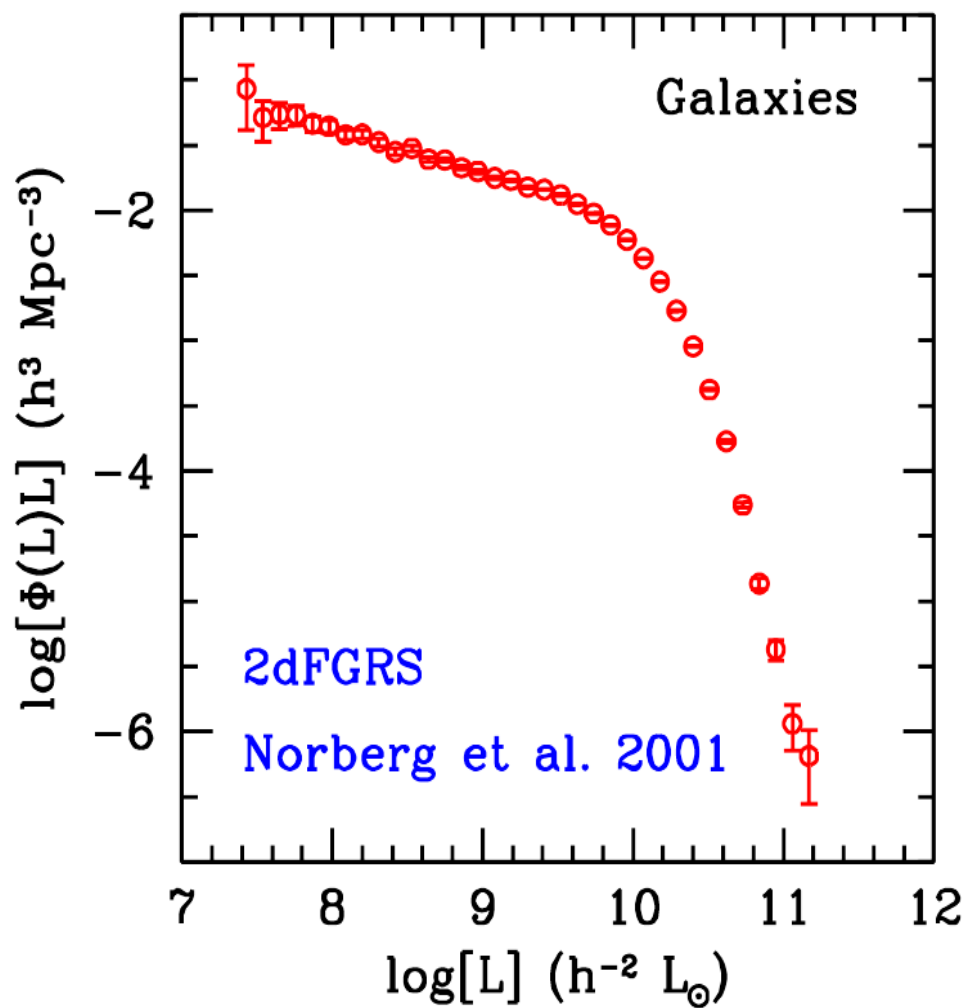


## Hydrodynamical simulations aim to predict:

- Morphology of galaxies
- Fate of the diffuse gas, WHIM, metal enrichment
- X-ray atmospheres in halos
- Turbulence in halos and accretion shocks
- Large-scale regulation of star formation in galaxies through feedback processes from stars and black holes
- Transport processes (e.g. conduction)
- Radiative transfer
- Dynamical transformations (e.g. ram-pressure stripping)
- Magnetic fields

A long standing issue in galaxy formation theory: The shapes of the CDM halo mass function and the galaxy luminosity function are very different

THE OBSERVED LF COMPARED TO THE SHAPE OF THE CDM HALO MASS FUNCTION



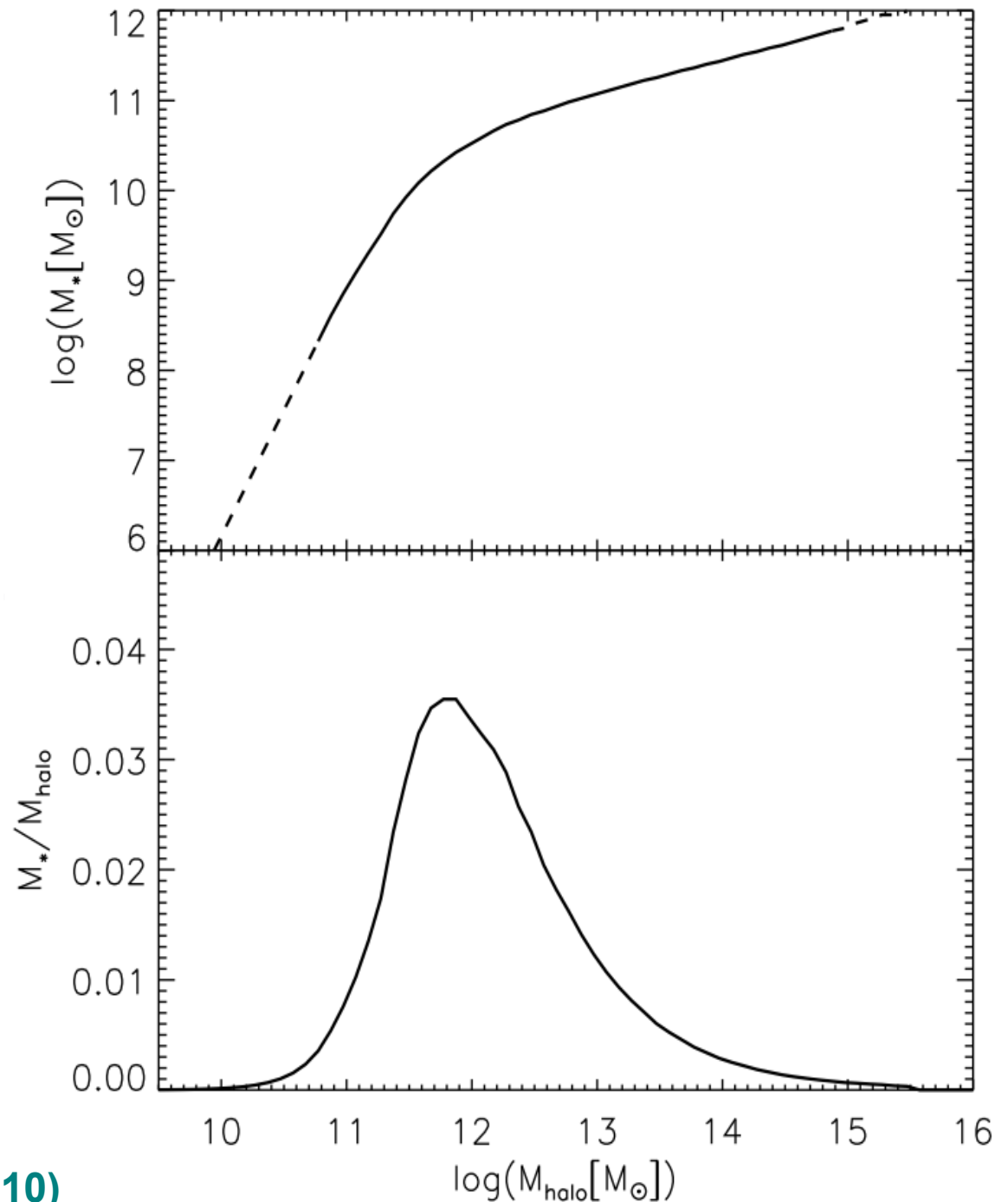
van den Bosch et al. (2004)

Abundance matching  
gives the expected halo  
mass – stellar mass  
relation in  $\Lambda$ CDM

STELLAR MASSES FROM  
SDSS/DR7 MATCHED TO  $\Lambda$ CDM  
SIMULATION EXPECTATIONS

**Assumption:**

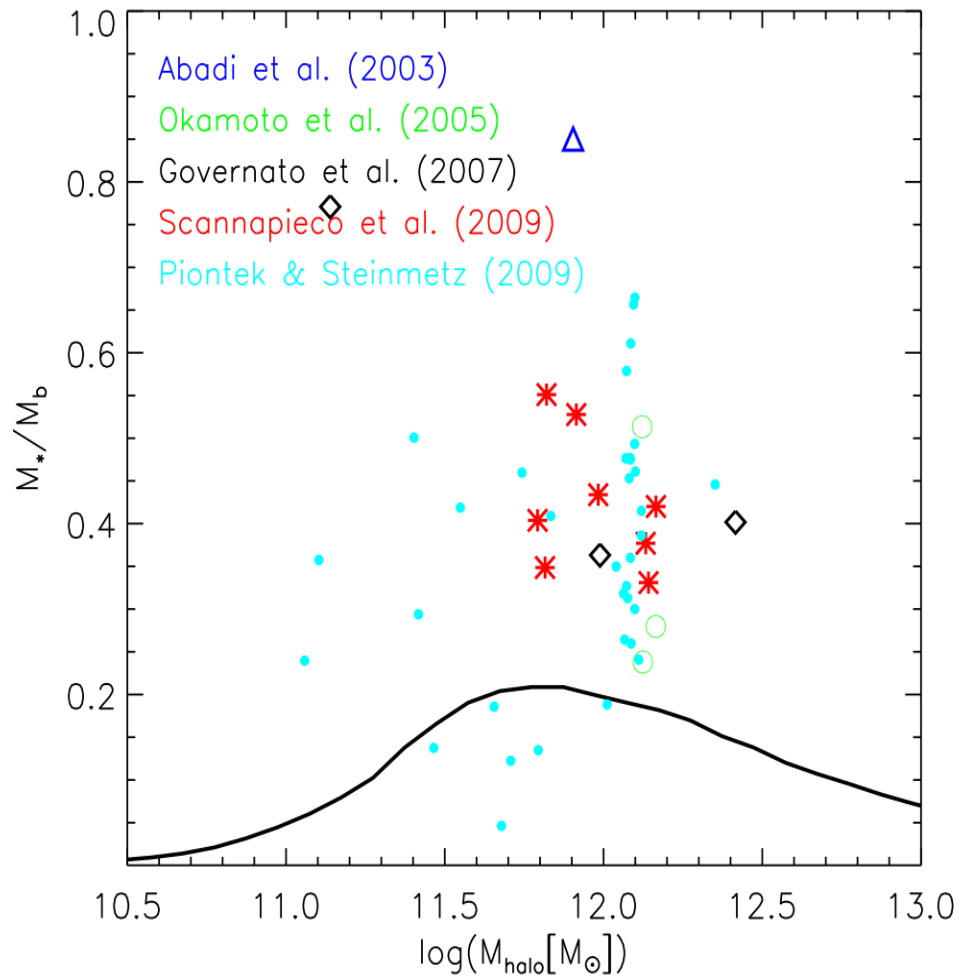
Stellar mass is monotonically  
increasing with halo mass



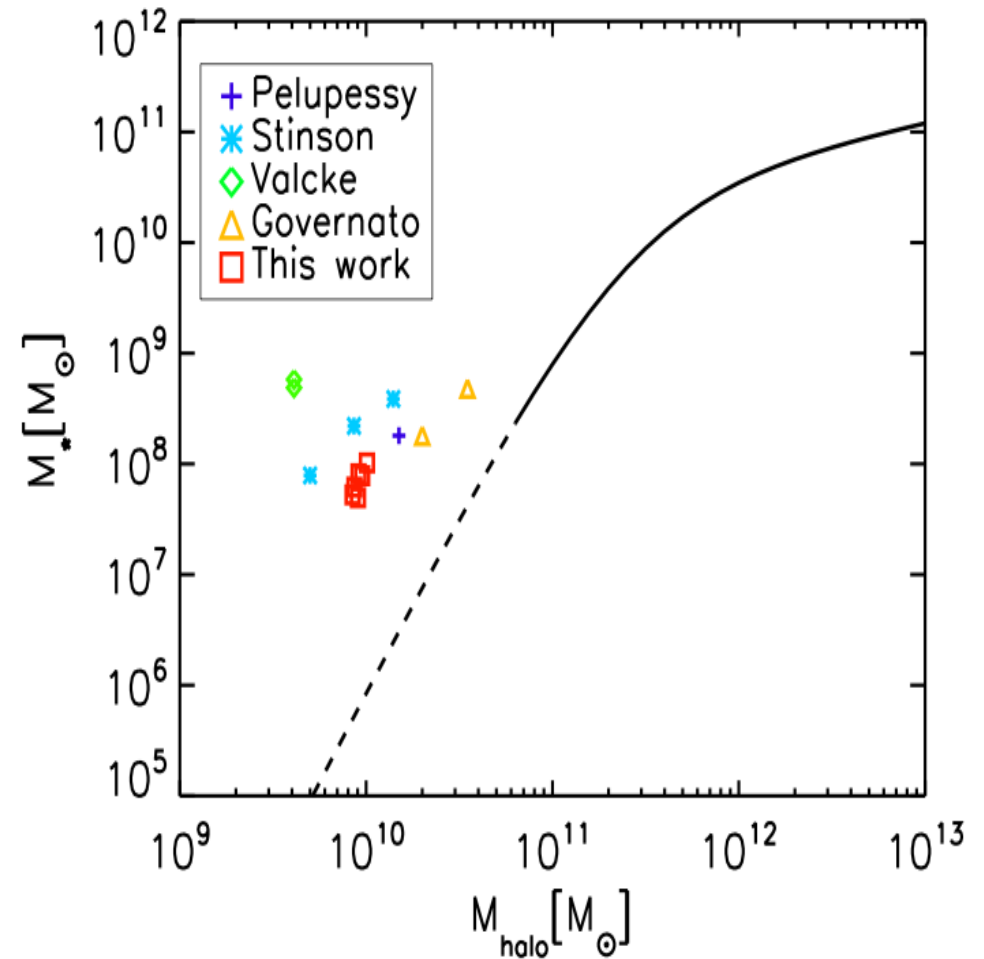
Guo, White & Boylan-Kolchin (2010)

# Current cosmological hydrodynamic simulations have trouble to explain such a low galaxy formation efficiency

## GALAXY FORMATION EFFICIENCY AS A FUNCTION OF HALO MASS



Guo, White & Boylan-Kolchin (2010)



Sawala & White (2010)

Future progress with cosmological simulations requires....

- **Better resolution (more computing power...)**
- **Higher accuracy of numerical codes**
- **More complete and realistic physics models**

# Trouble ahead in the Exaflop regime ?

10<sup>18</sup>

in

2018

How long would the Millennium-XXL take on a Exaflop Supercomputer at peak performance?

15 min

One of the main problems:

***Power Consumption***

Petaflop Computer: **6 MW**

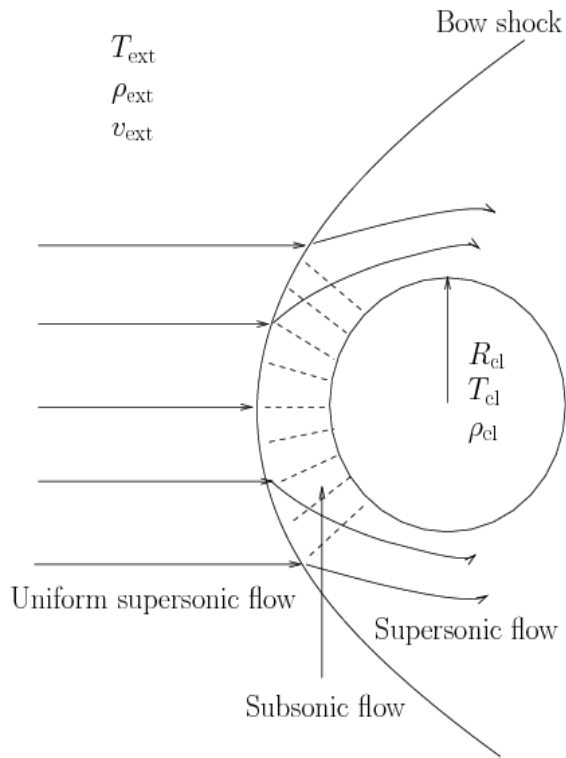
Exaflop Computer: ~ **GW ?**

Need to get this down to **20-40 MW**

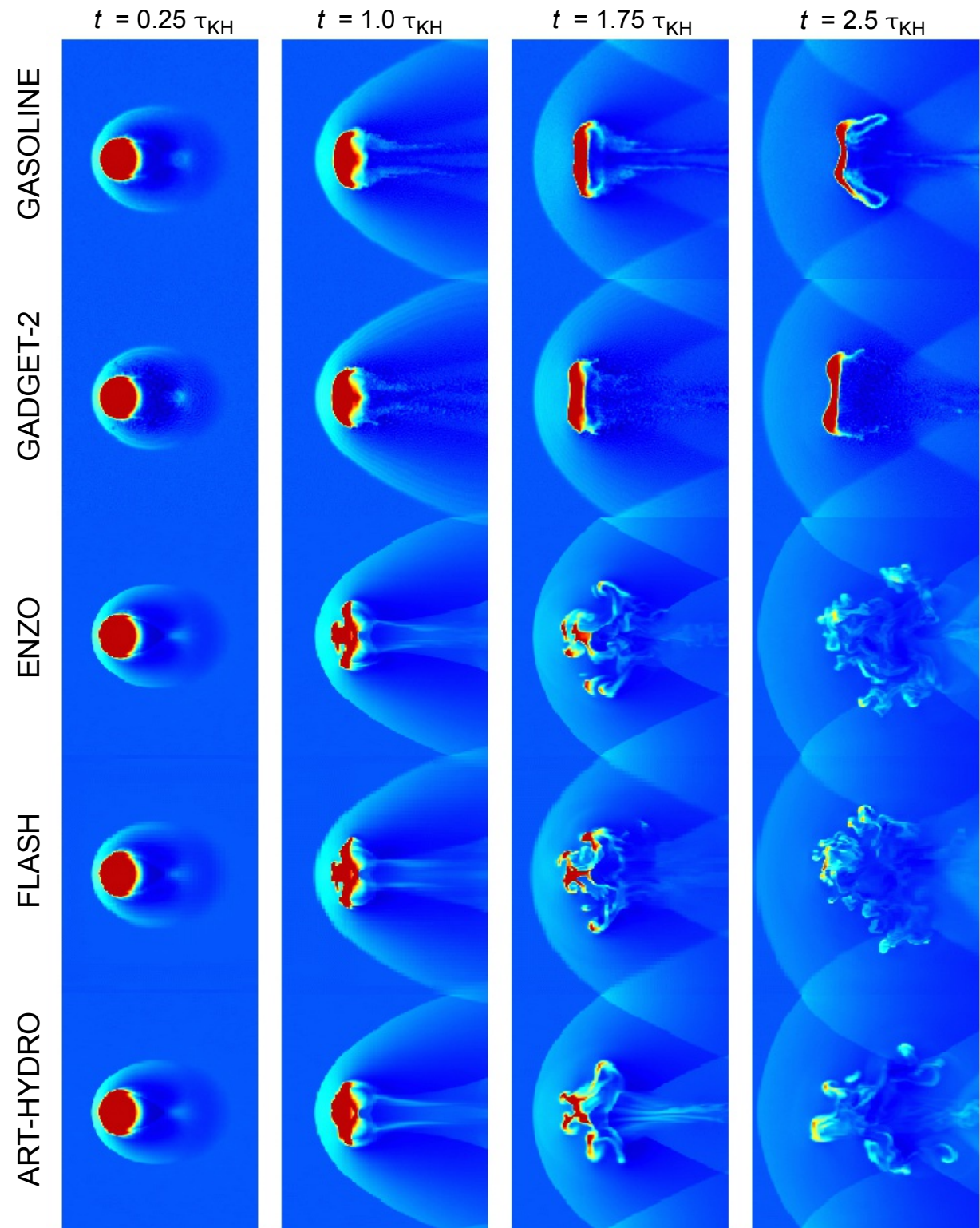


A cloud moving through ambient gas shows markedly different long-term behavior in SPH and Eulerian mesh codes

### DISRUPTION OF A CLOUD BY KELVIN-HELMHOLTZ INSTABILITIES



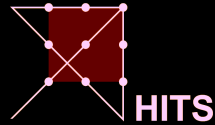
Agertz et al. (2007)



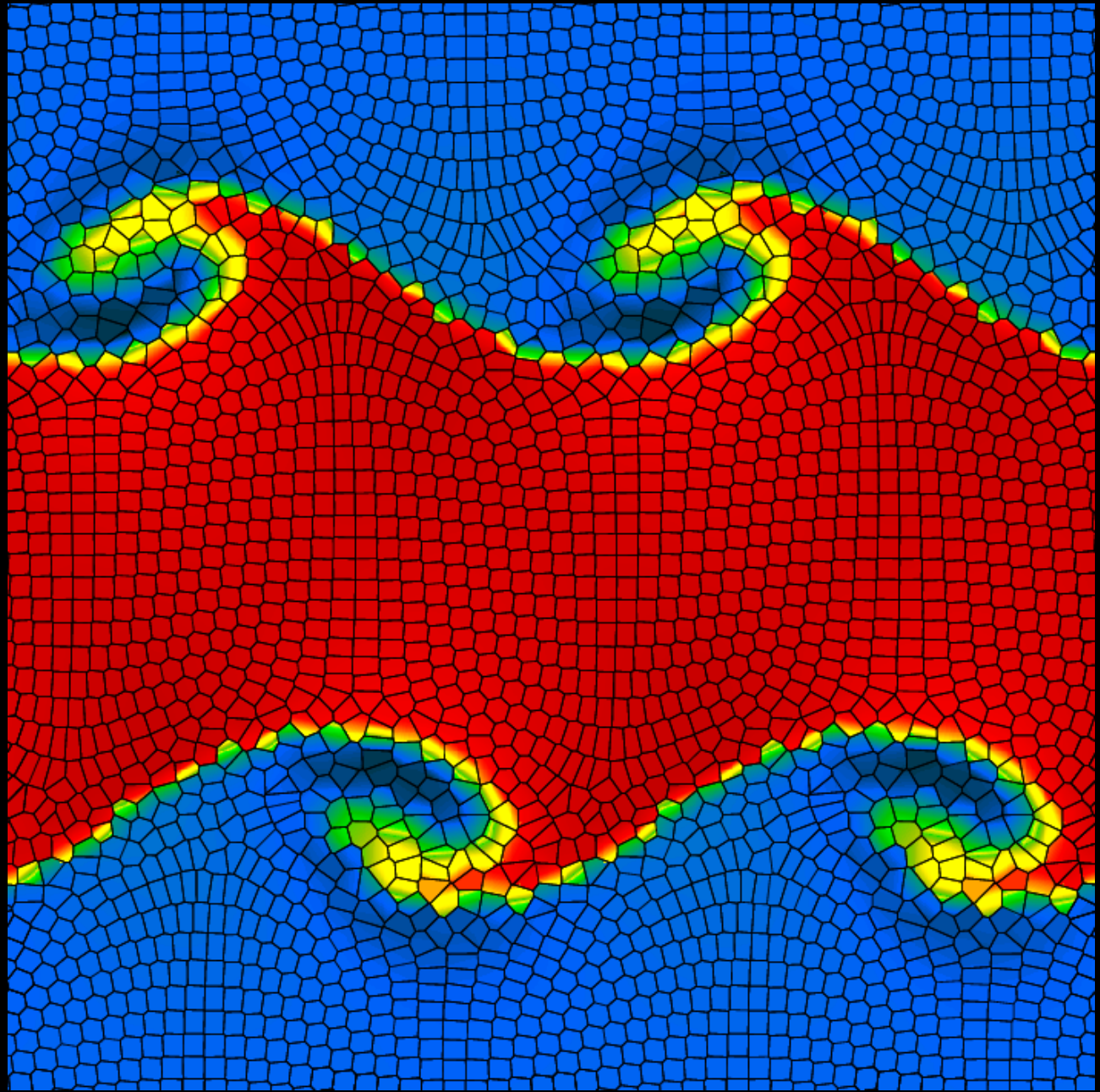
# Moving-mesh hydrodynamics with AREPO

Volker Springel

Heidelberg Institute for Theoretical Studies



UNIVERSITÄT HEIDELBERG

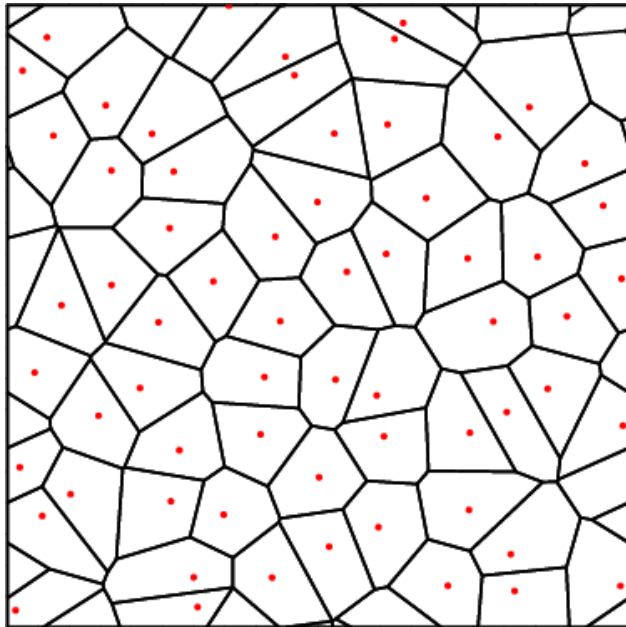




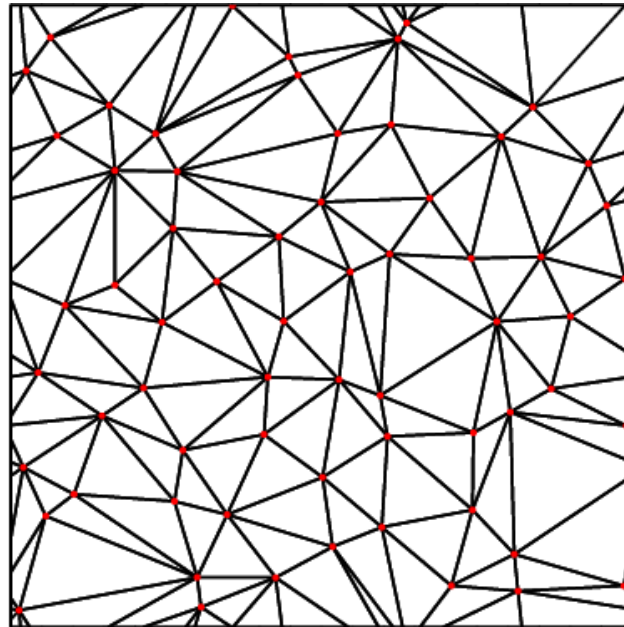
# Voronoi and Delaunay tessellations provide unique partitions of space based on a given sample of mesh-generating points

## BASIC PROPERTIES OF VORONOI AND DELAUNAY MESHES

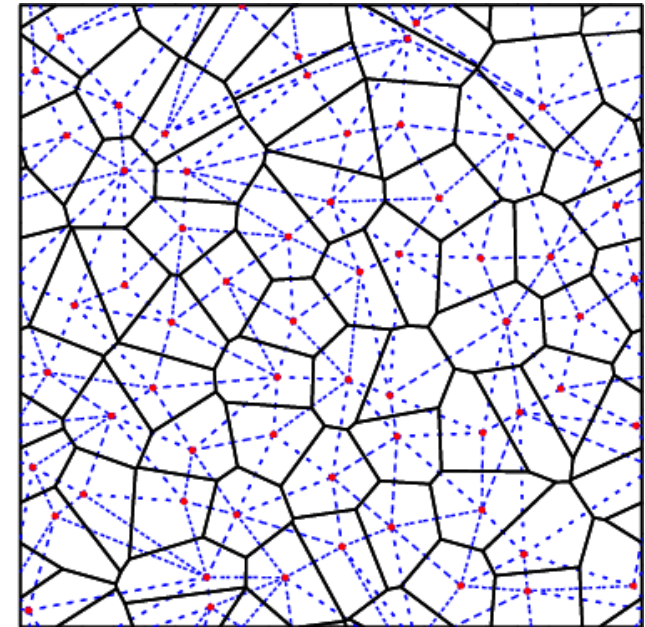
Voronoi mesh



Delaunay triangulation



both shown together



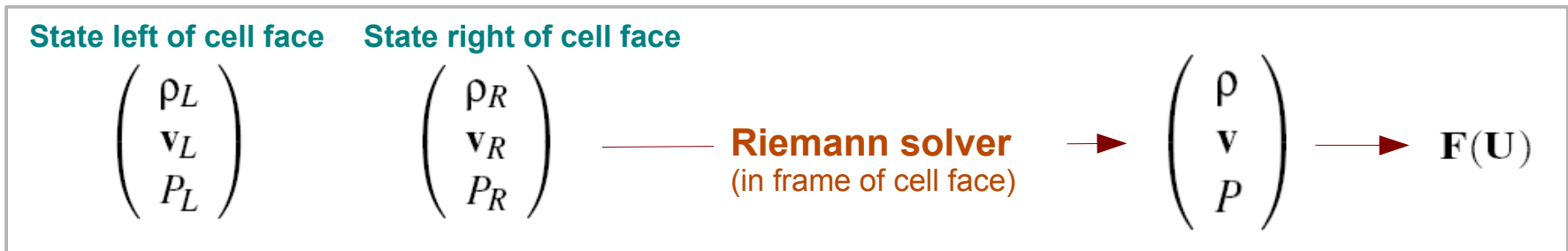
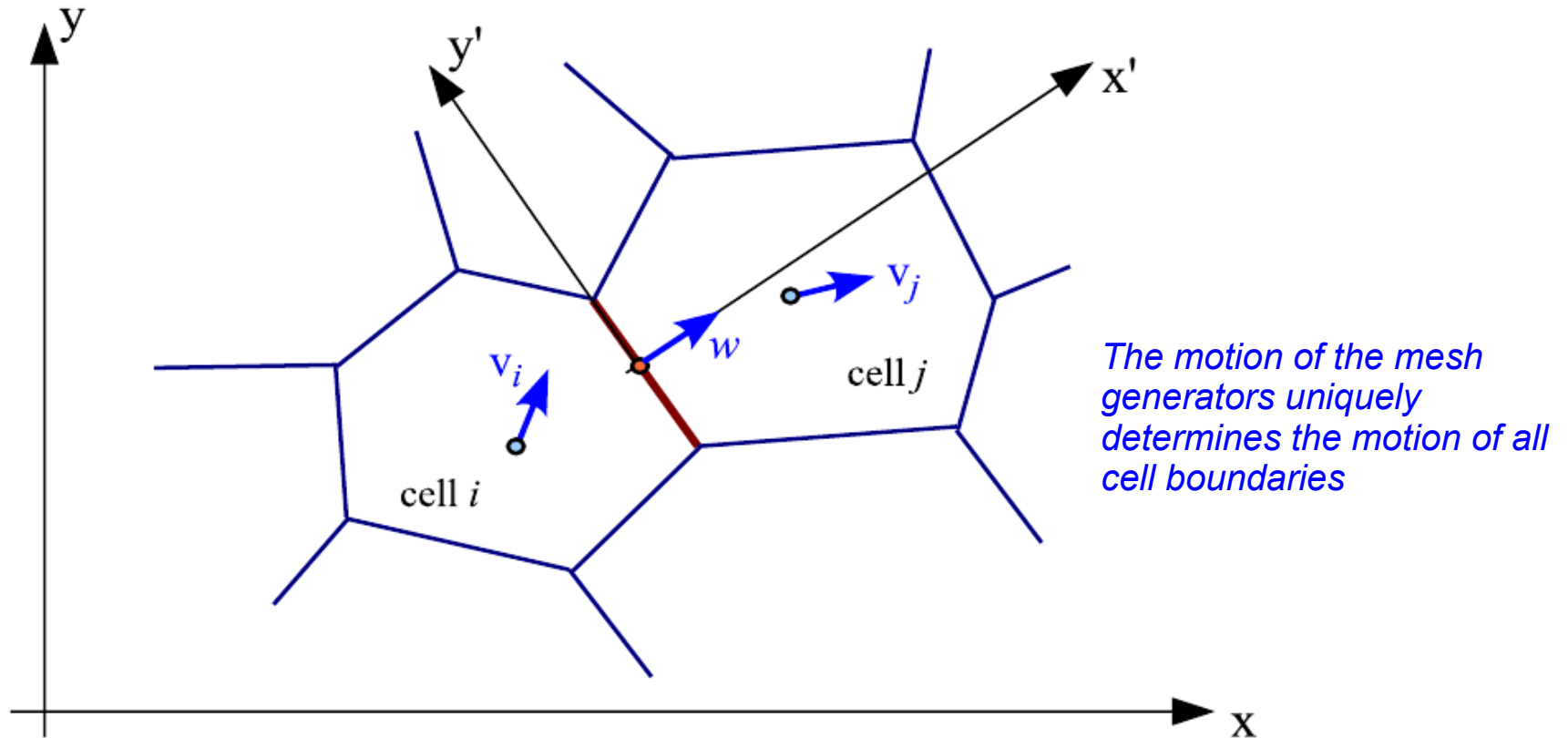
Each Voronoi cell contains the **space closest** to its generating point

The Delaunay triangulation contains only triangles with an **empty circumcircle**. The Delaunay triangulation maximizes the minimum angle occurring among all triangles.

The centres of the circumcircles of the Delaunay triangles are the vertices of the Voronoi mesh. In fact, the two tessellations are the topological **dual graph** to each other.

The fluxes are calculated with an exact Riemann solver in the frame of the moving cell boundary

SKETCH OF THE FLUX CALCULATION

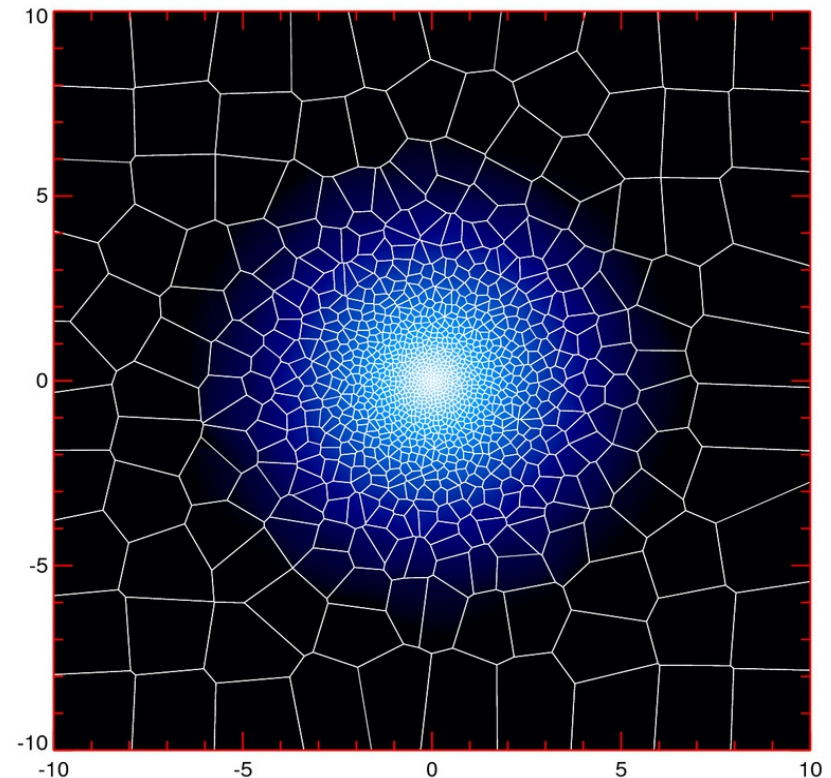
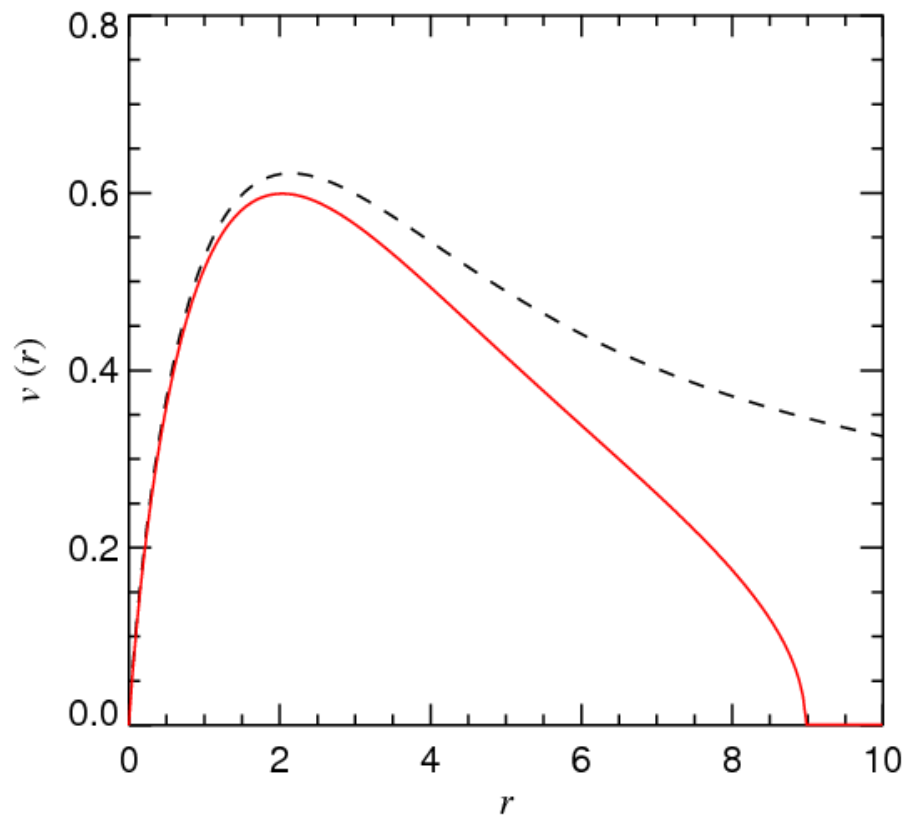


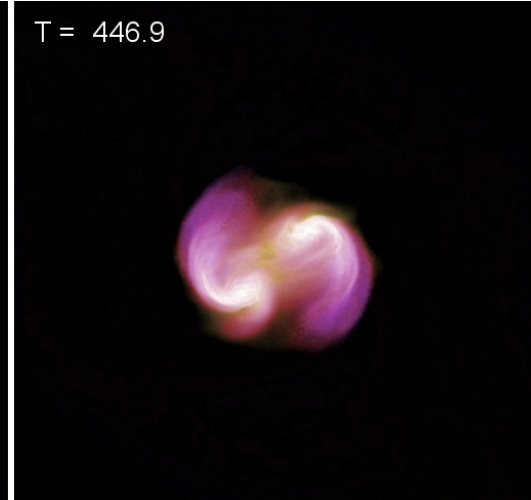
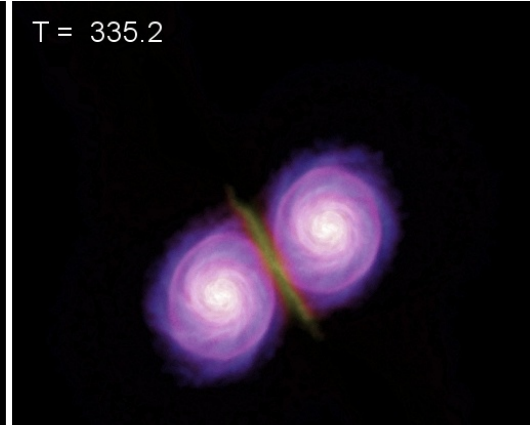
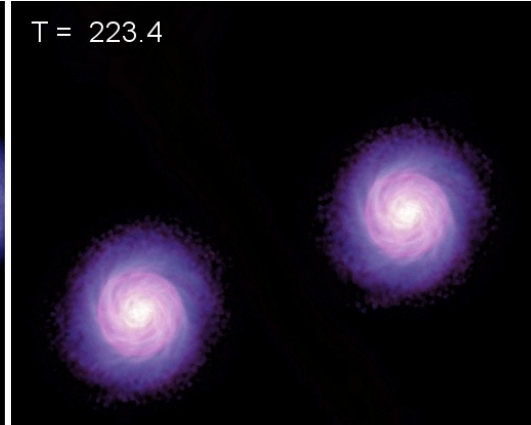
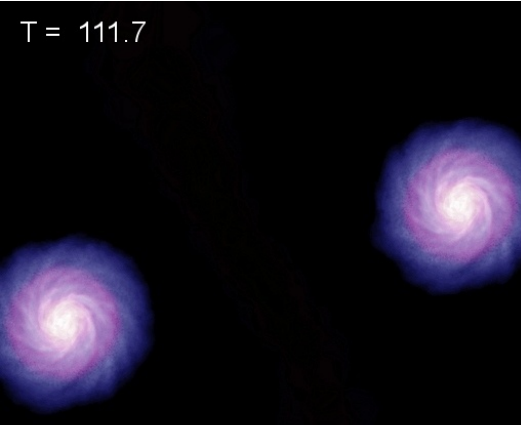
A differentially rotating gaseous disk with strong shear can be simulated well with the moving mesh code

## MODEL FOR A CENTRIFUGALLY SUPPORTED, THIN DISK

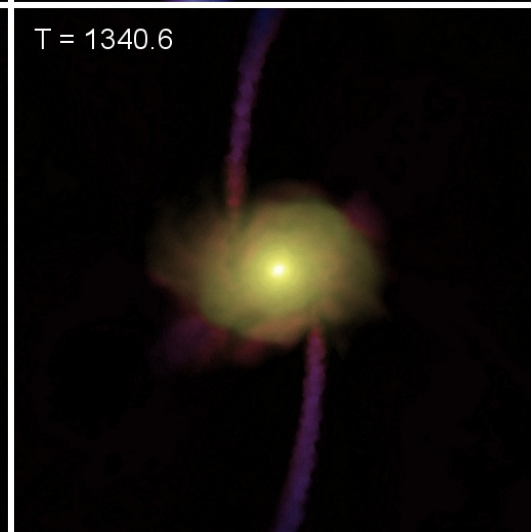
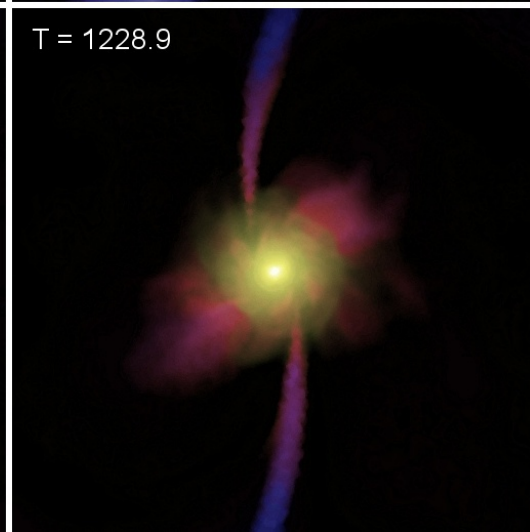
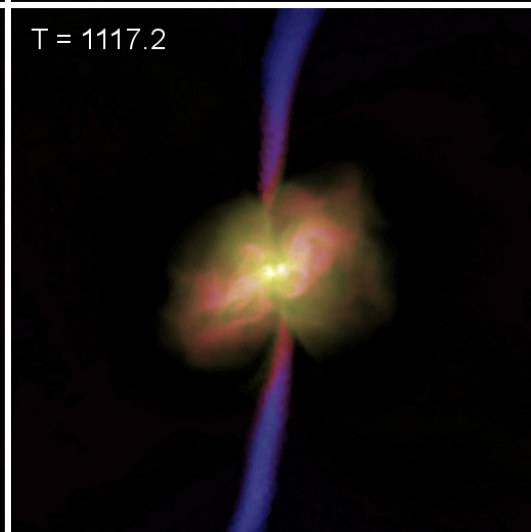
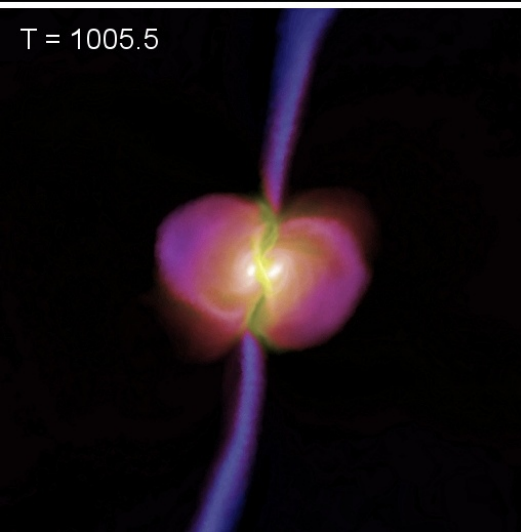
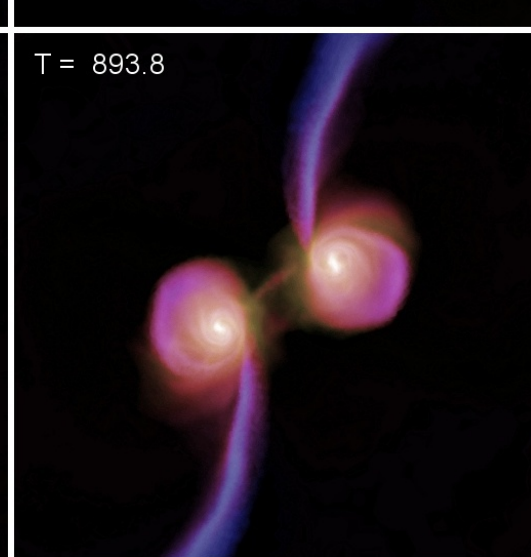
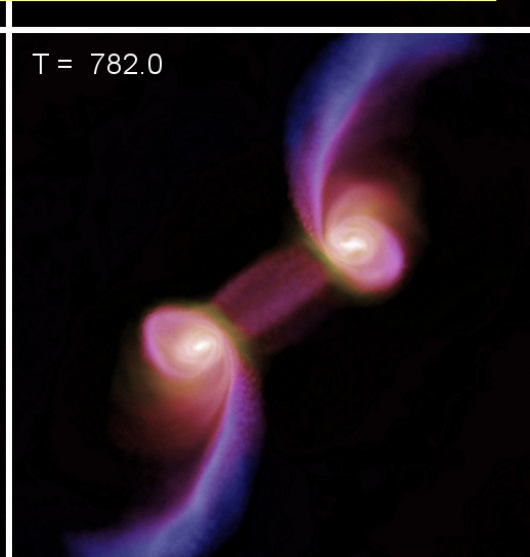
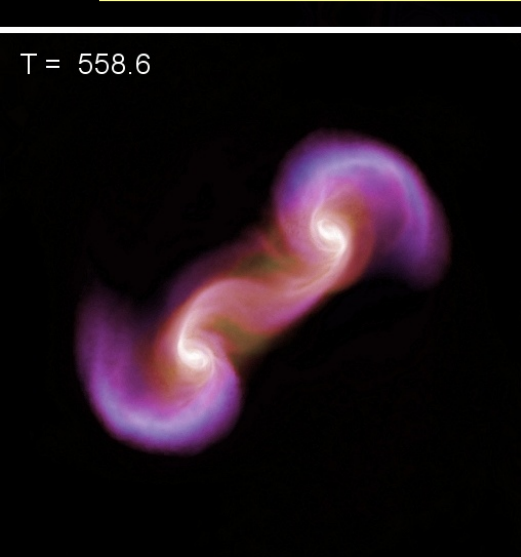
$$\Sigma(r) = \Sigma_0 \exp(-r/h)$$

$$v_c^2(r) \equiv r \frac{\partial \Phi}{\partial r} = 2 \frac{Gm}{h} y^2 [I_0(y)K_0(y) - I_1(y)K_1(y)]$$



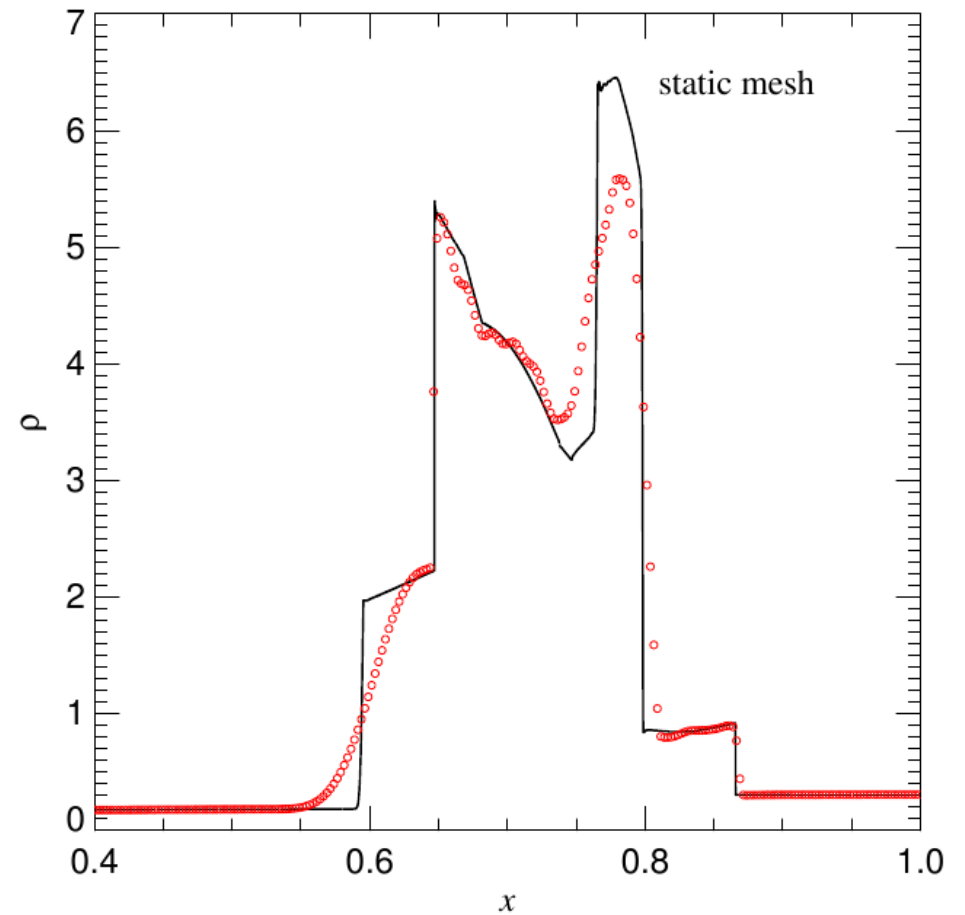
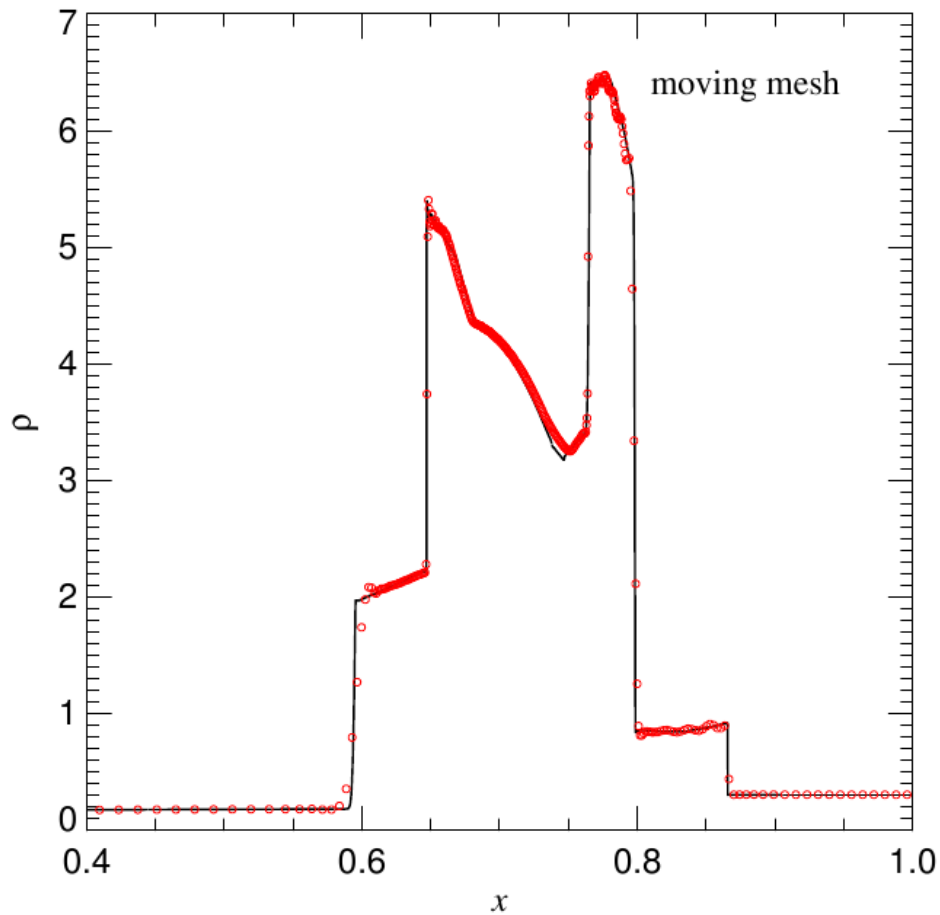


Galaxy collision simulation with the moving mesh code



The moving-mesh code deals well with problems that involve complicated shock interactions

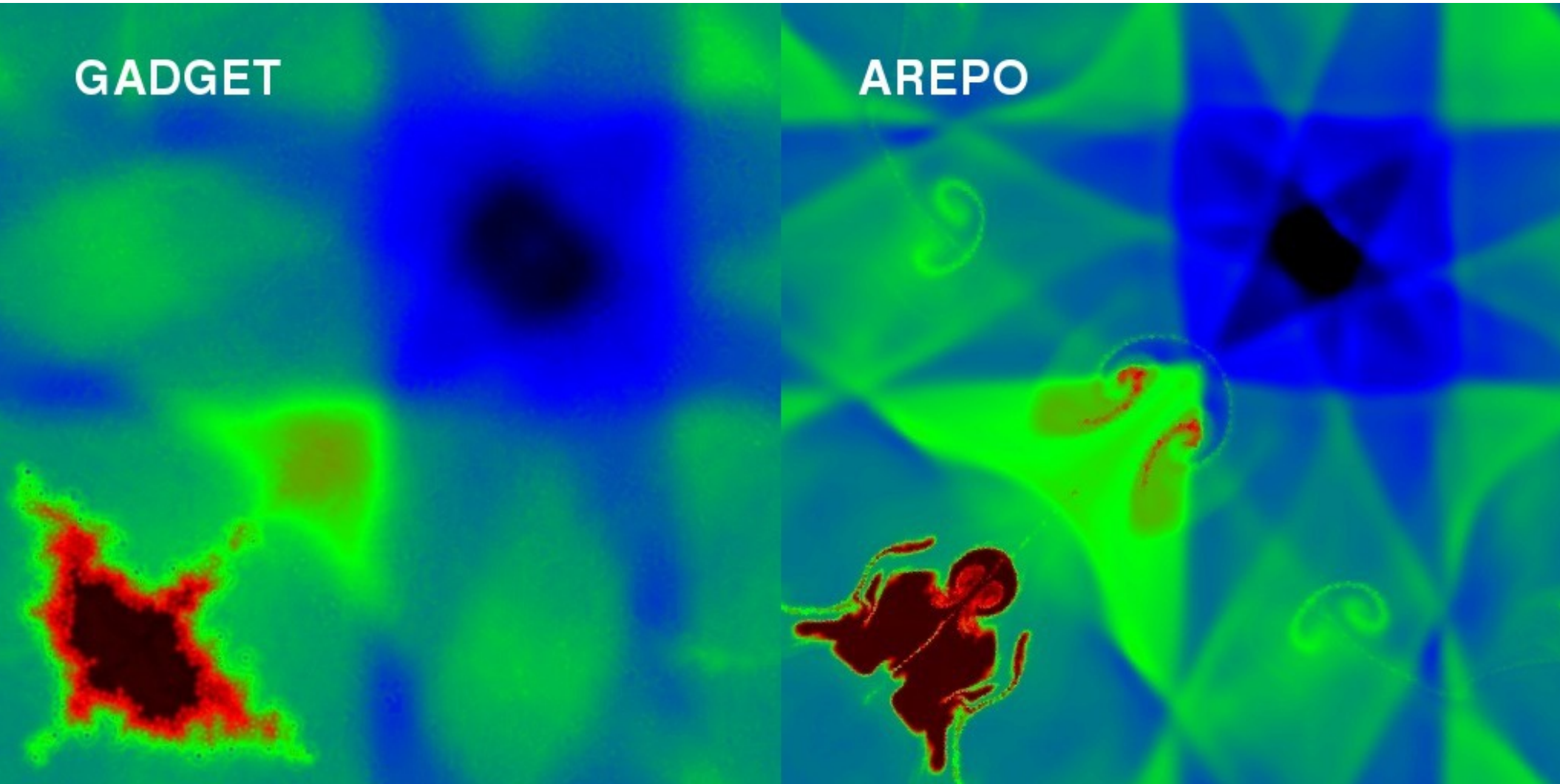
**WOODWARD & COLELLA'S INTERACTING DOUBLE BLAST PROBLEM**



# Interacting shock waves reveal significant differences in vorticity production

TWO-DIMENSIONAL IMPLOSION PROBLEM

Sijacki et al. (2011)



But in the end: **Does it matter  
for cosmological simulations?**

# Moving-mesh cosmology: First applications of AREPO

Mark Vogelsberger

Debora Sijacki

Dusan Keres

Paul Torrey

Lars Hernquist

Volker Springel

4 new papers, astro-ph (2011)

20 Mpc/h box, WMAP7 cosmology

Resolutions:  $2 \times 128^3$ ,  $2 \times 256^3$ ,  $2 \times 512^3$

AREPO and GADGET runs

**equal physics, equal gravity solver**

Andreas Bauer & VS (2011)

Subsonic turbulence in moving-mesh and SPH

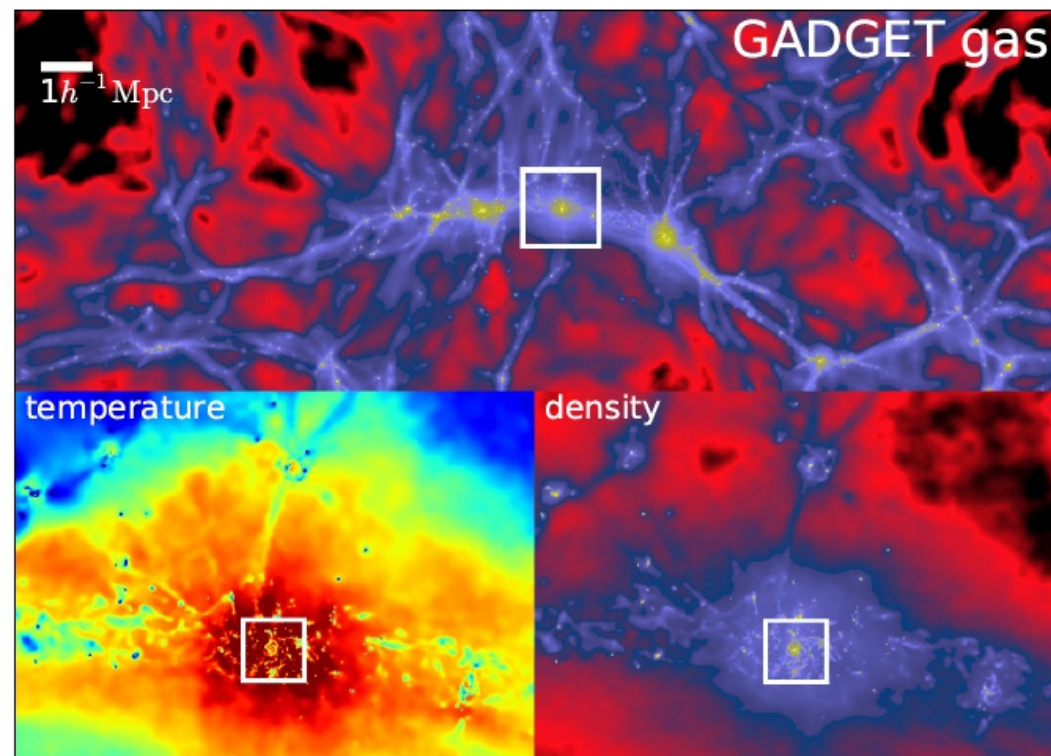
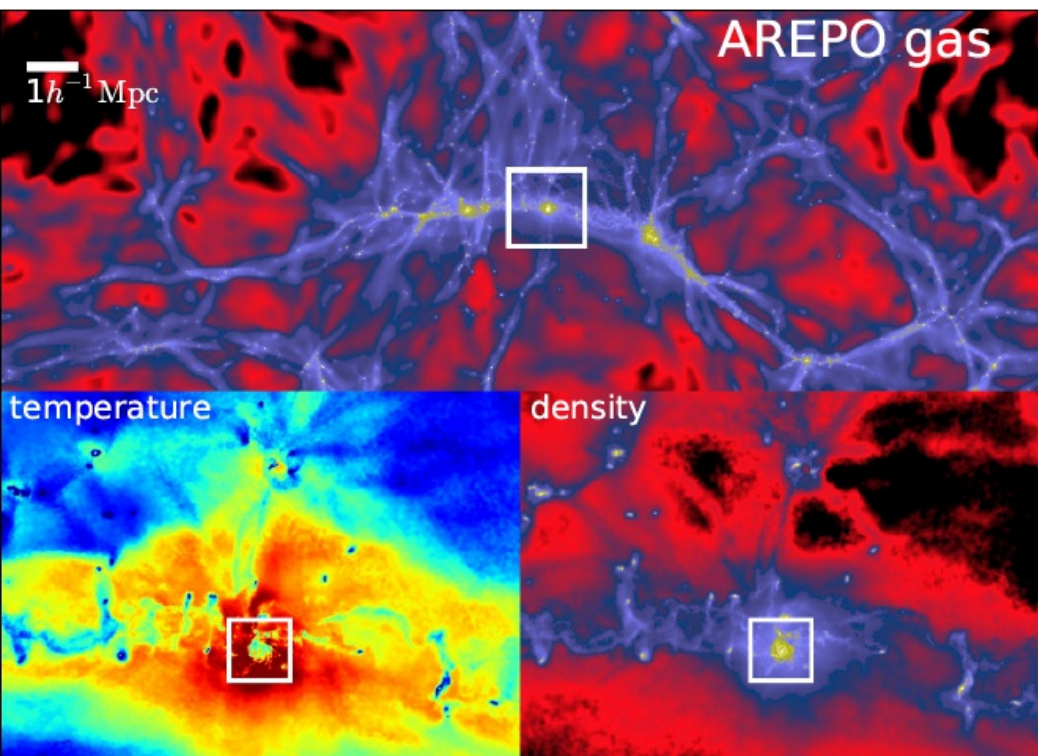
Thomas Greif, VS, et al. (2011)

Population III star formation

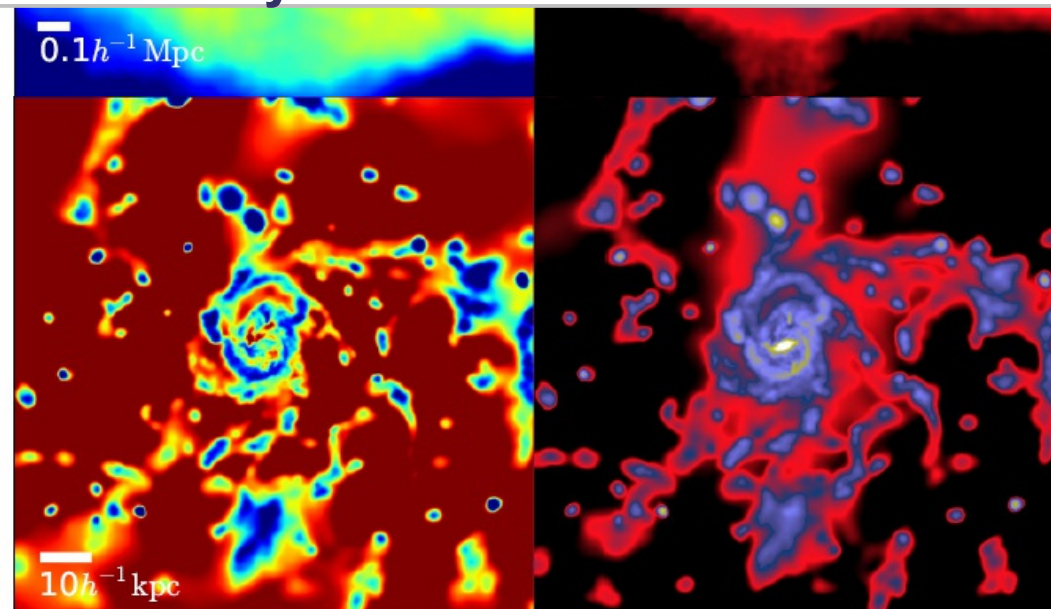
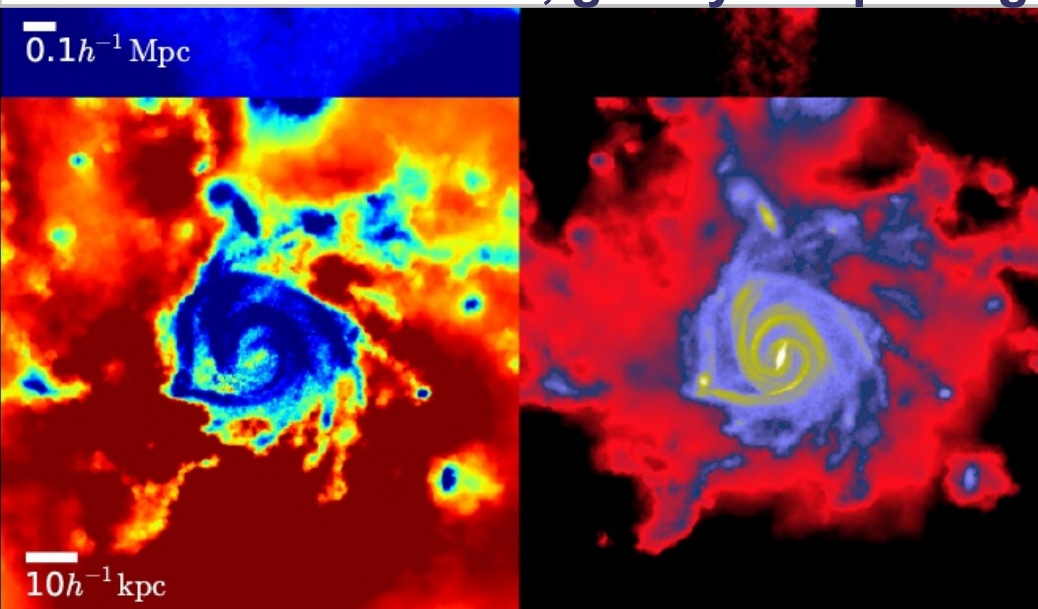


# On large scales, the code produces similar results as standard SPH techniques

## GAS AND TEMPERATURE FIELDS IN A COSMOLOGICAL HYDRODYNAMIC SIMULATION

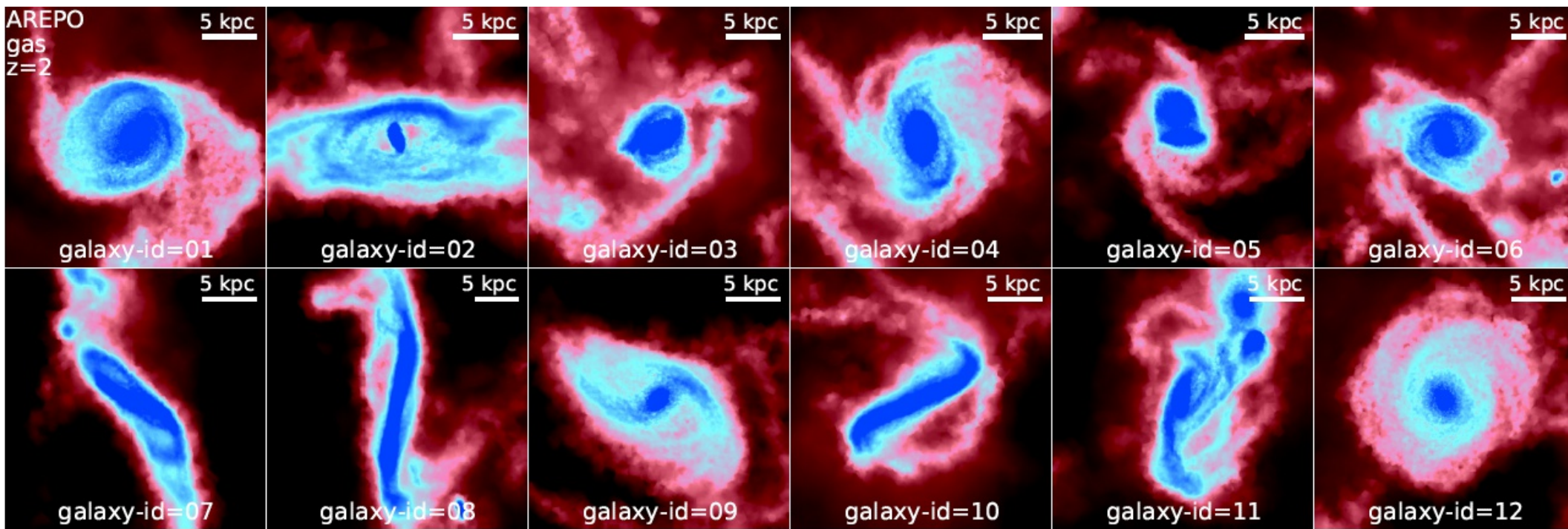


## But on small scales, galaxy morphologies look very different

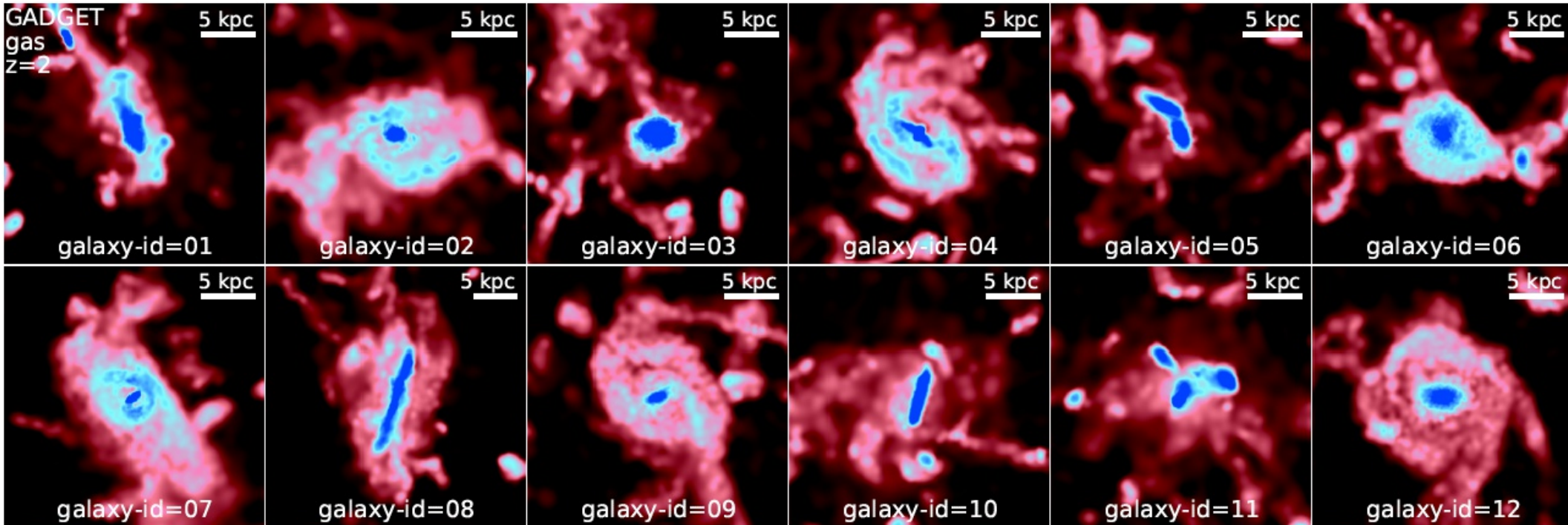


# Projected gas densities in matching AREPO and SPH halos

## AREPO:

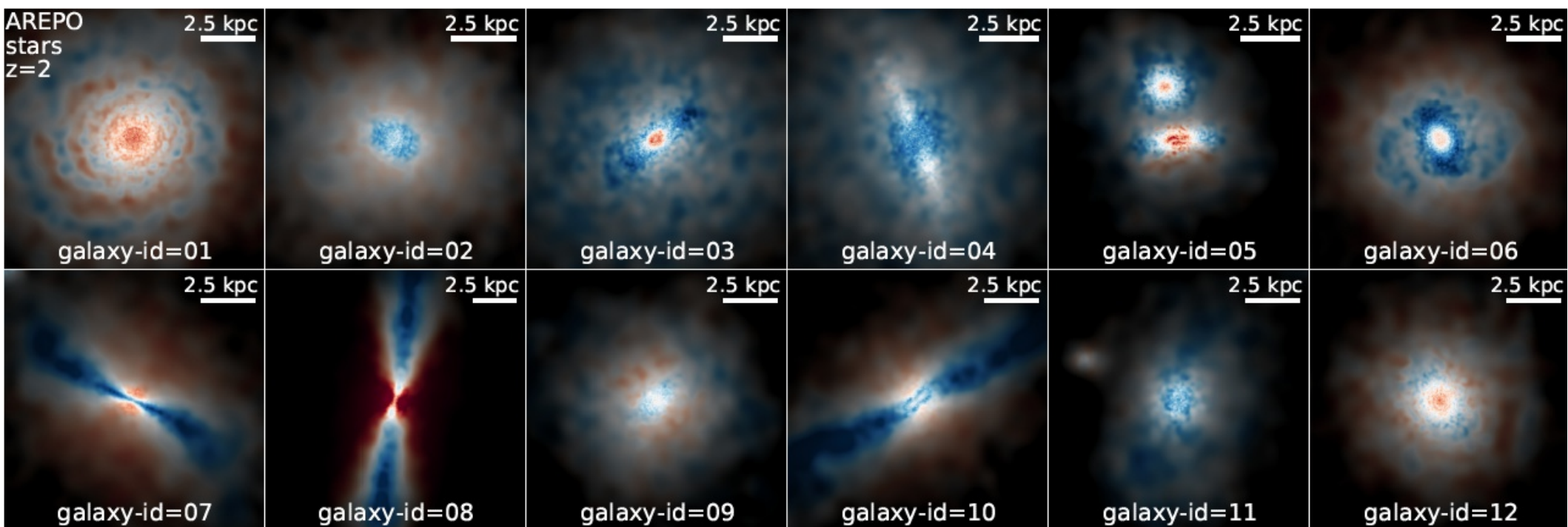


## SPH:

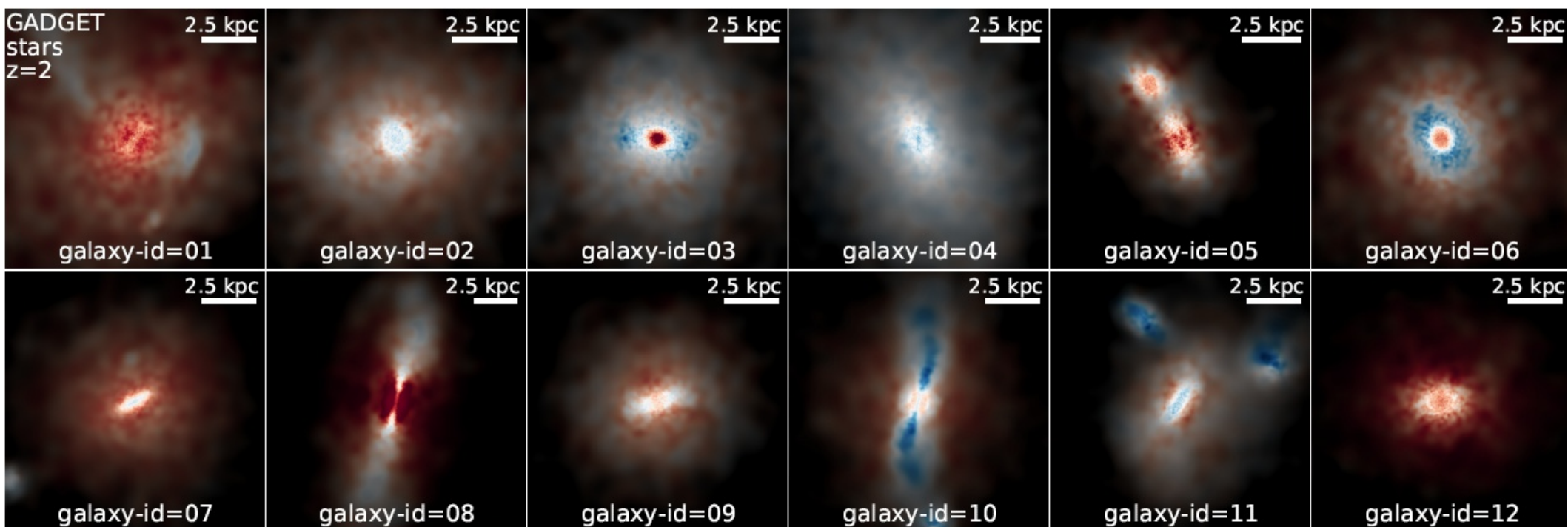


# Projected stellar densities in matching AREPO and SPH halos

## AREPO:



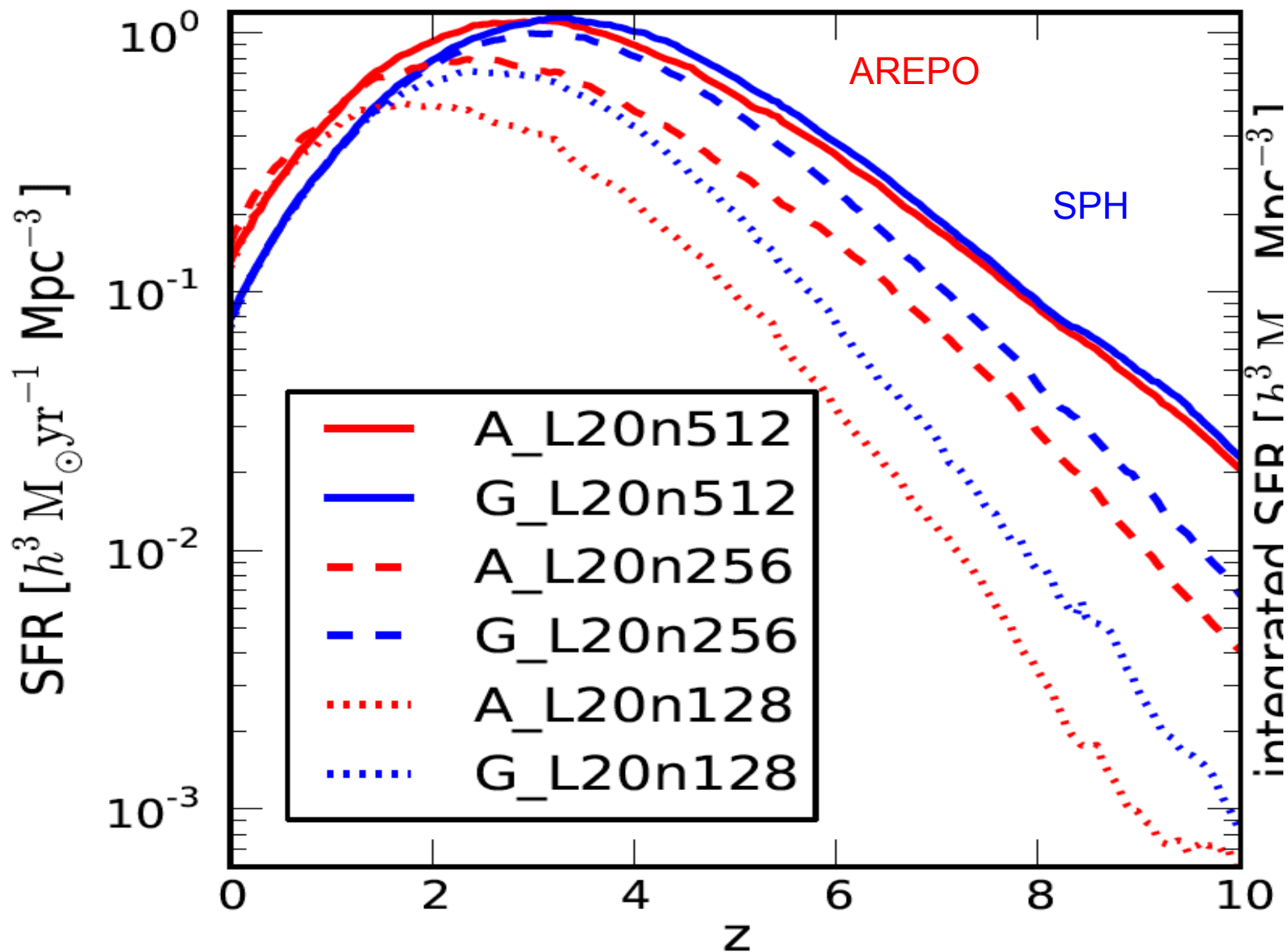
## SPH:



Compared with SPH, the cosmic star formation rate density is higher in AREPO at low redshift

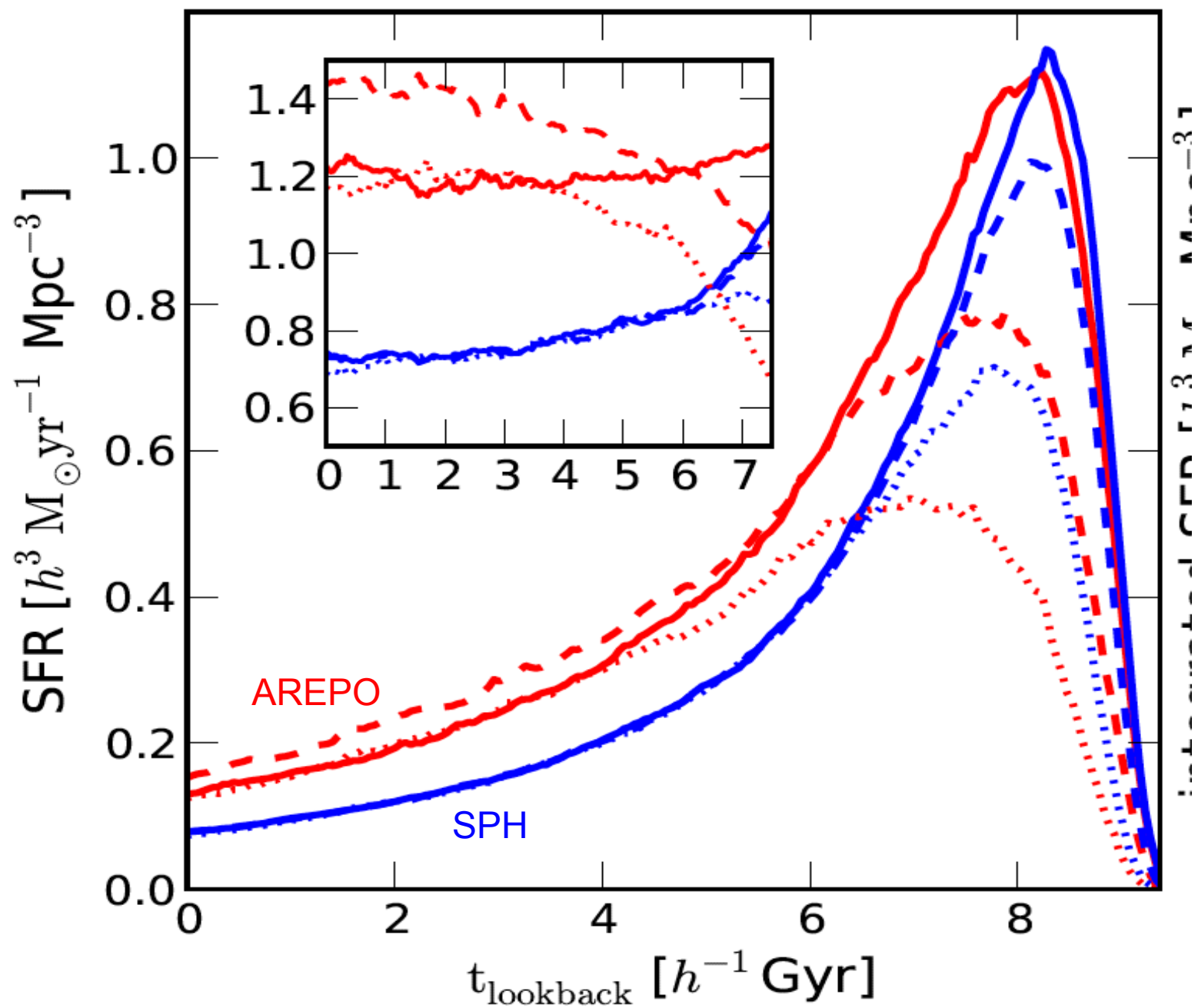
SFR-DENSITY AS A FUNCTION OF REDSHIFT FOR DIFFERENT RESOLUTIONS AND CODES

Vogelsberger et al. (2011)



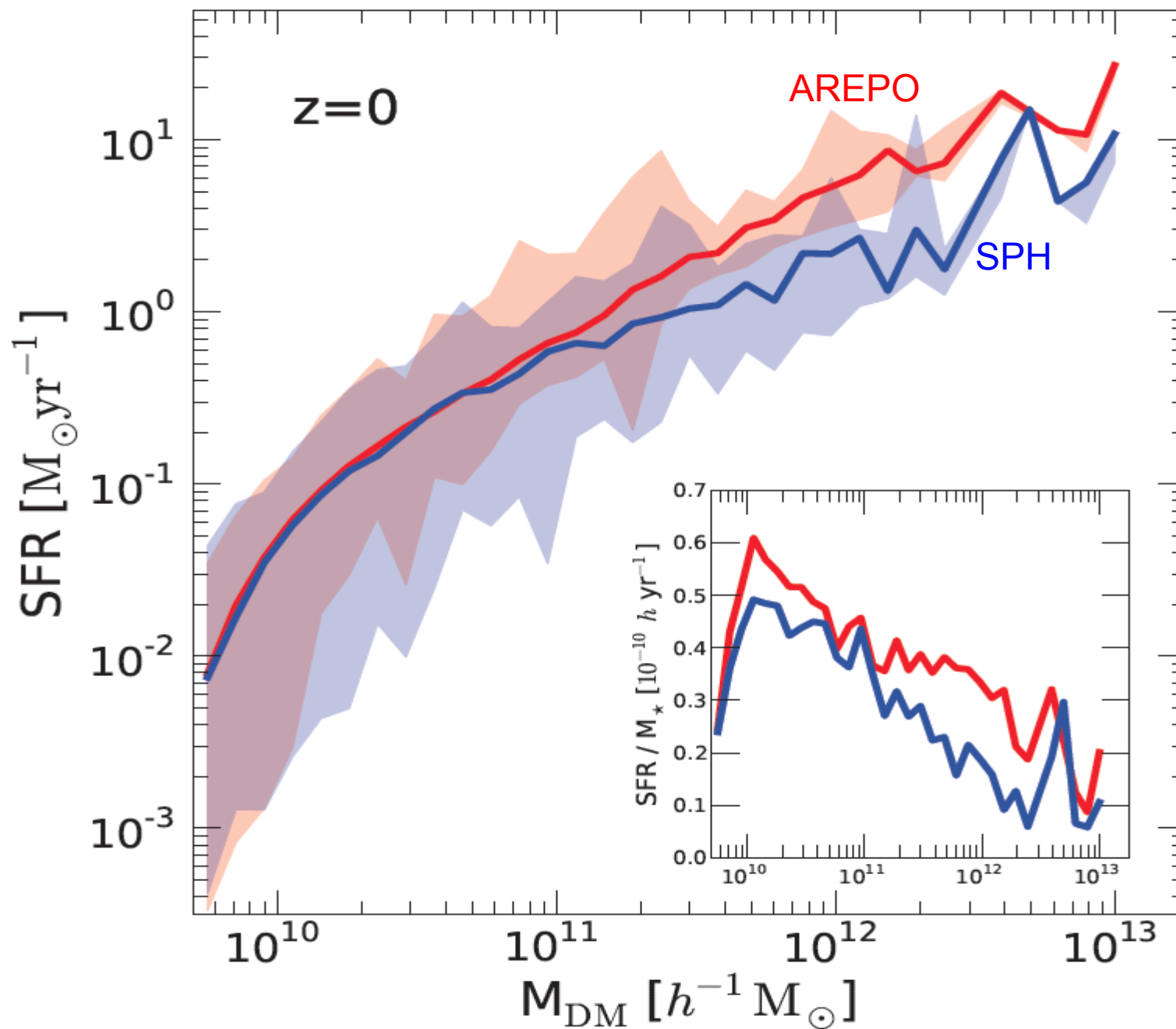
Compared with SPH, the cosmic star formation rate density is higher in AREPO at low redshift

SFR-DENSITY AS A FUNCTION OF TIME FOR DIFFERENT RESOLUTIONS AND CODES



# The difference in star formation originates in massive halos

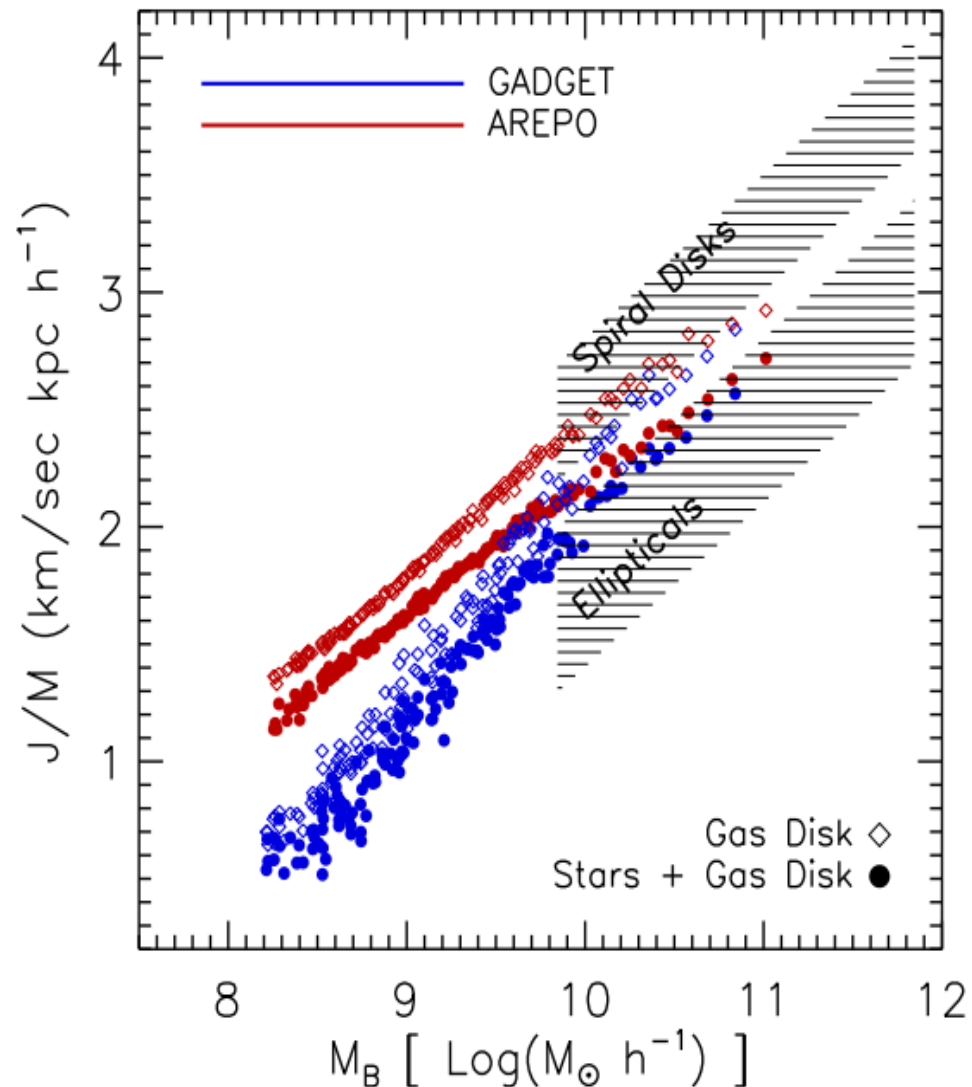
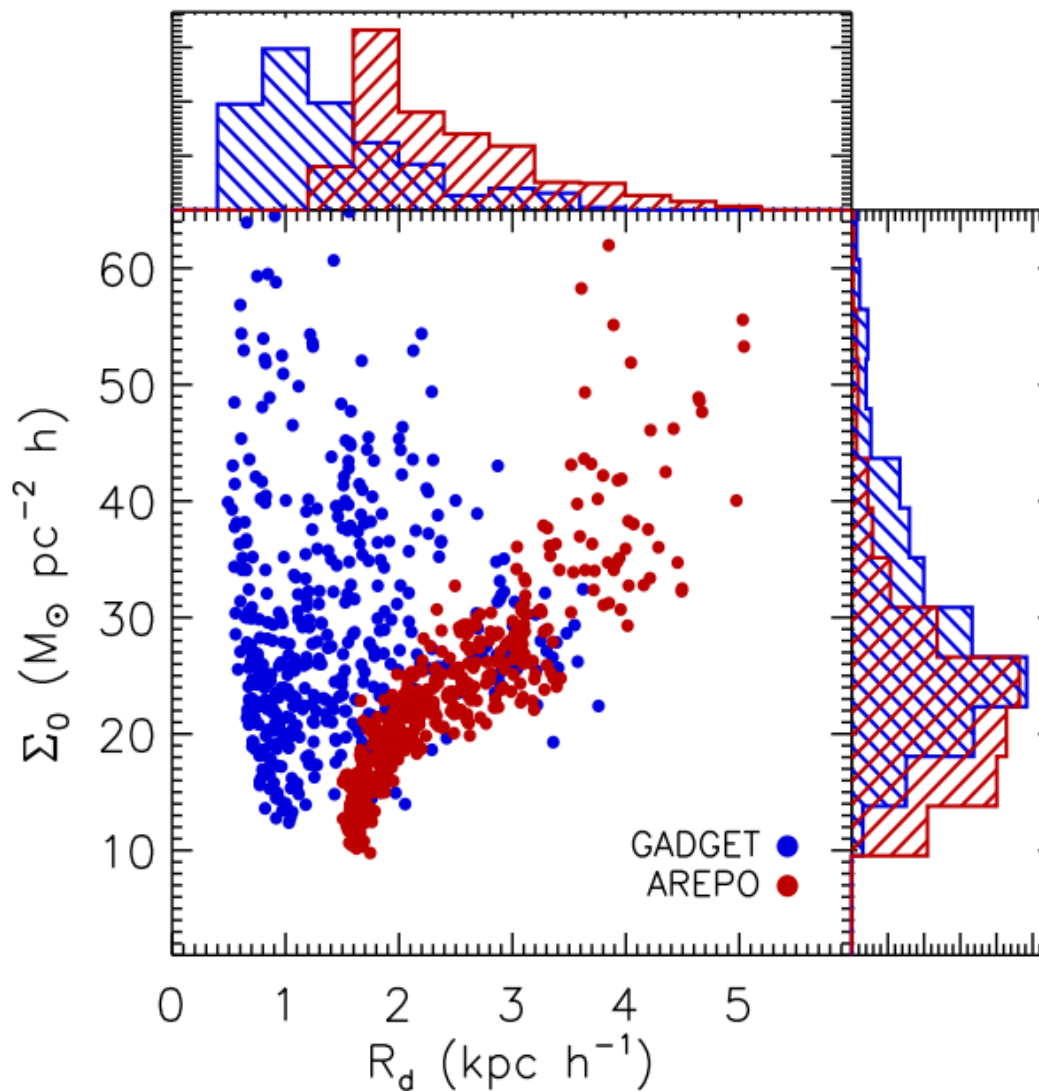
## STAR FORMATION RATE AS A FUNCTION OF HALO MASS



# Gasous disk scale lengths are much larger in the moving-mesh code

## DISK SCALE LENGTHS AND ANGULAR MOMENTUM IN GADGET AND AREPO

Torrey et al. (2011)



# Satellite mass loss and orbital decay is different in SPH and AREPO

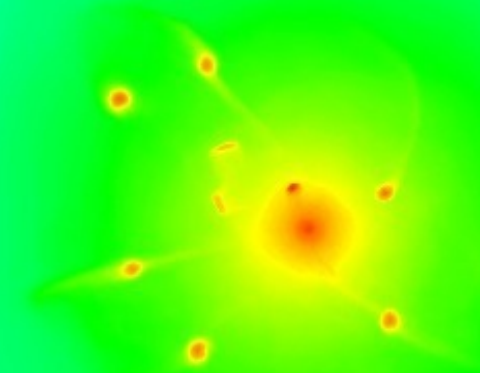
FIDUCIAL GAS BLOBS IN ORBIT IN A CLUSTER

Sijacki et al. (2011)

GADGET



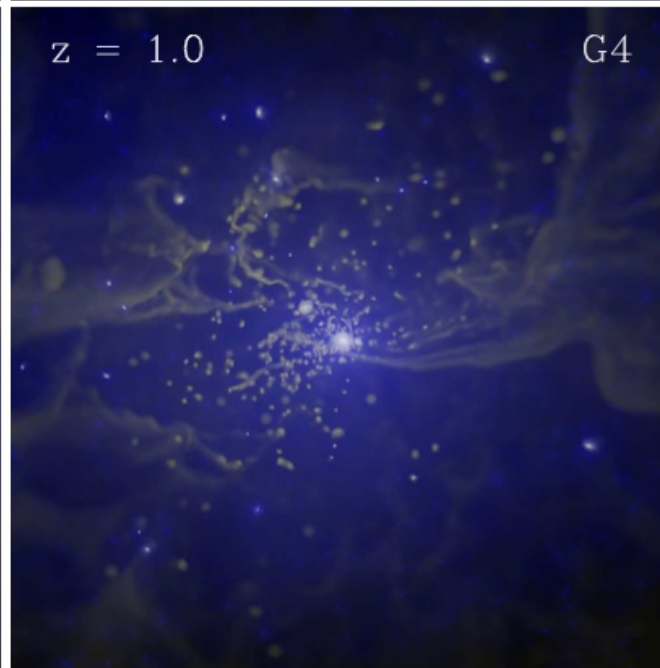
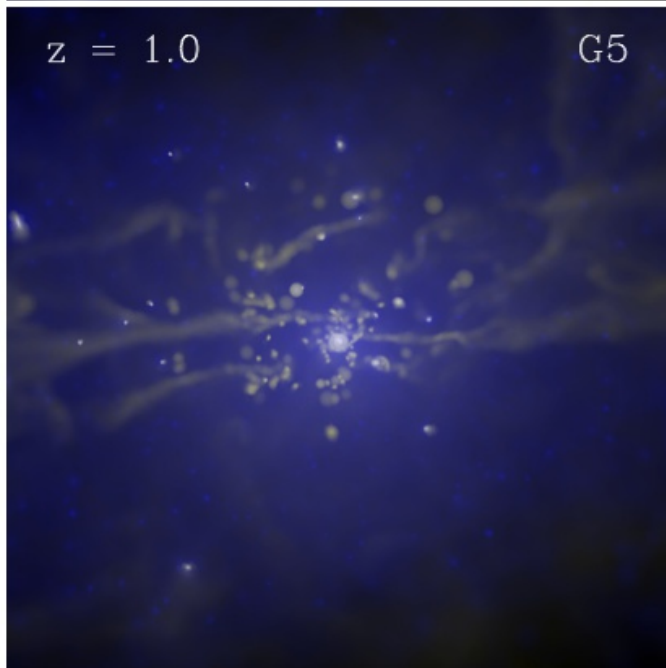
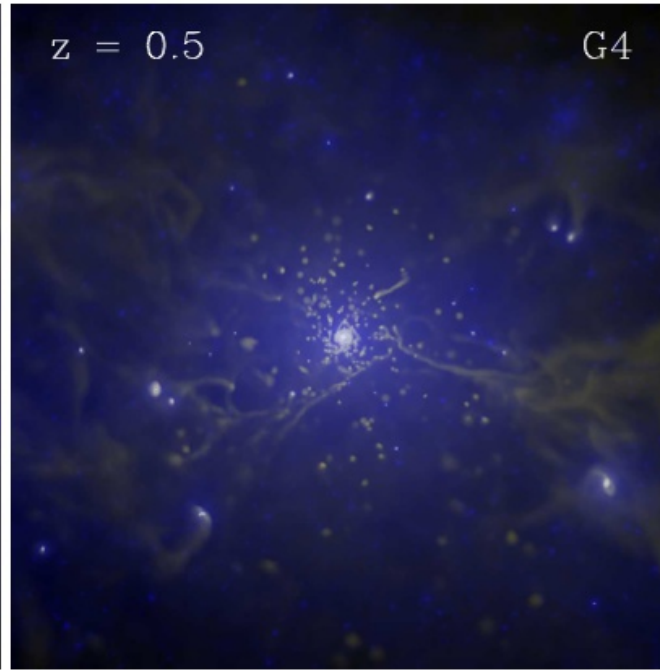
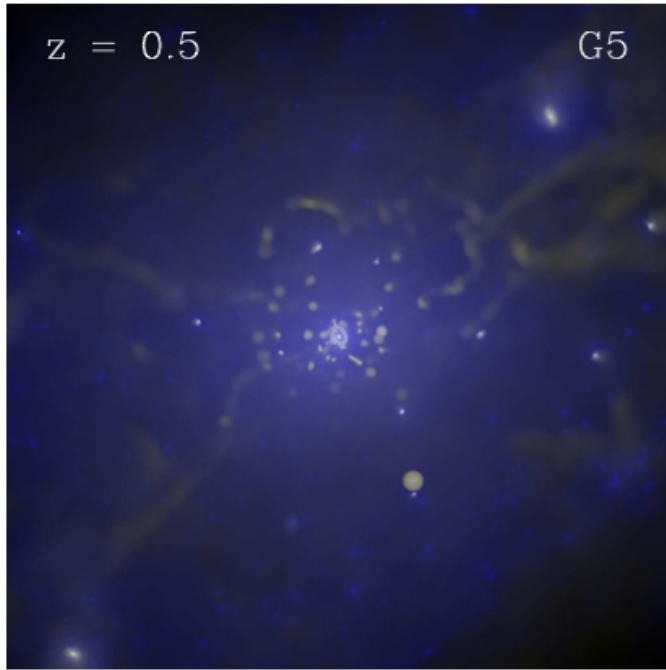
AREPO





# Clumpy gas distribution around Aquila galaxy in GADGET

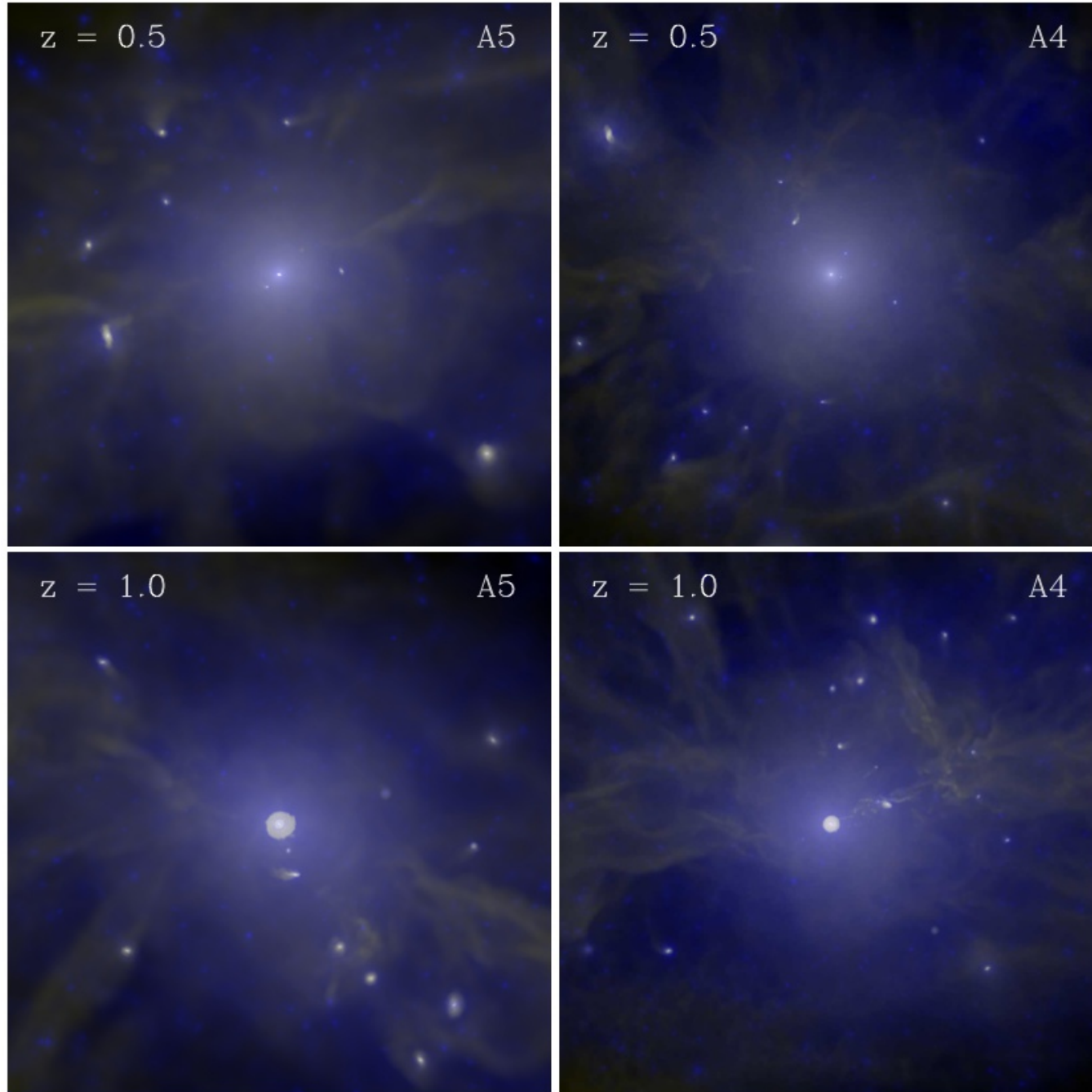
## GAS BLOBS IN ORBIT AROUND AQUILA AT DIFFERENT TIMES AND RESOLUTIONS



Also seen, e.g, in ERIS  
(Guedes et al., 2011)

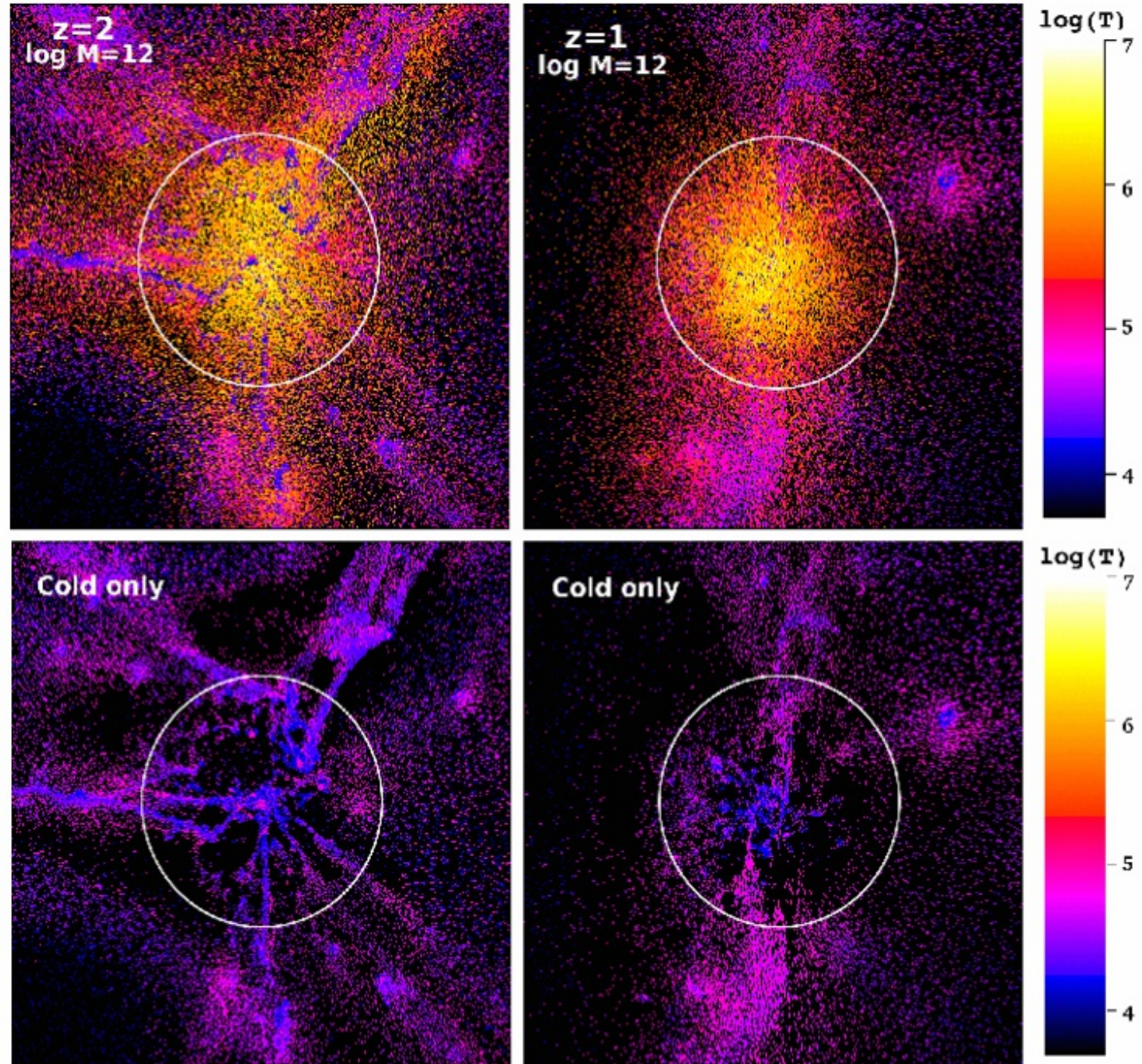
# Smooth gas distribution around Aquila galaxy in AREPO

## GAS IN THE HALO AT DIFFERENT TIMES AND RESOLUTIONS



# How do galaxies get their gas?

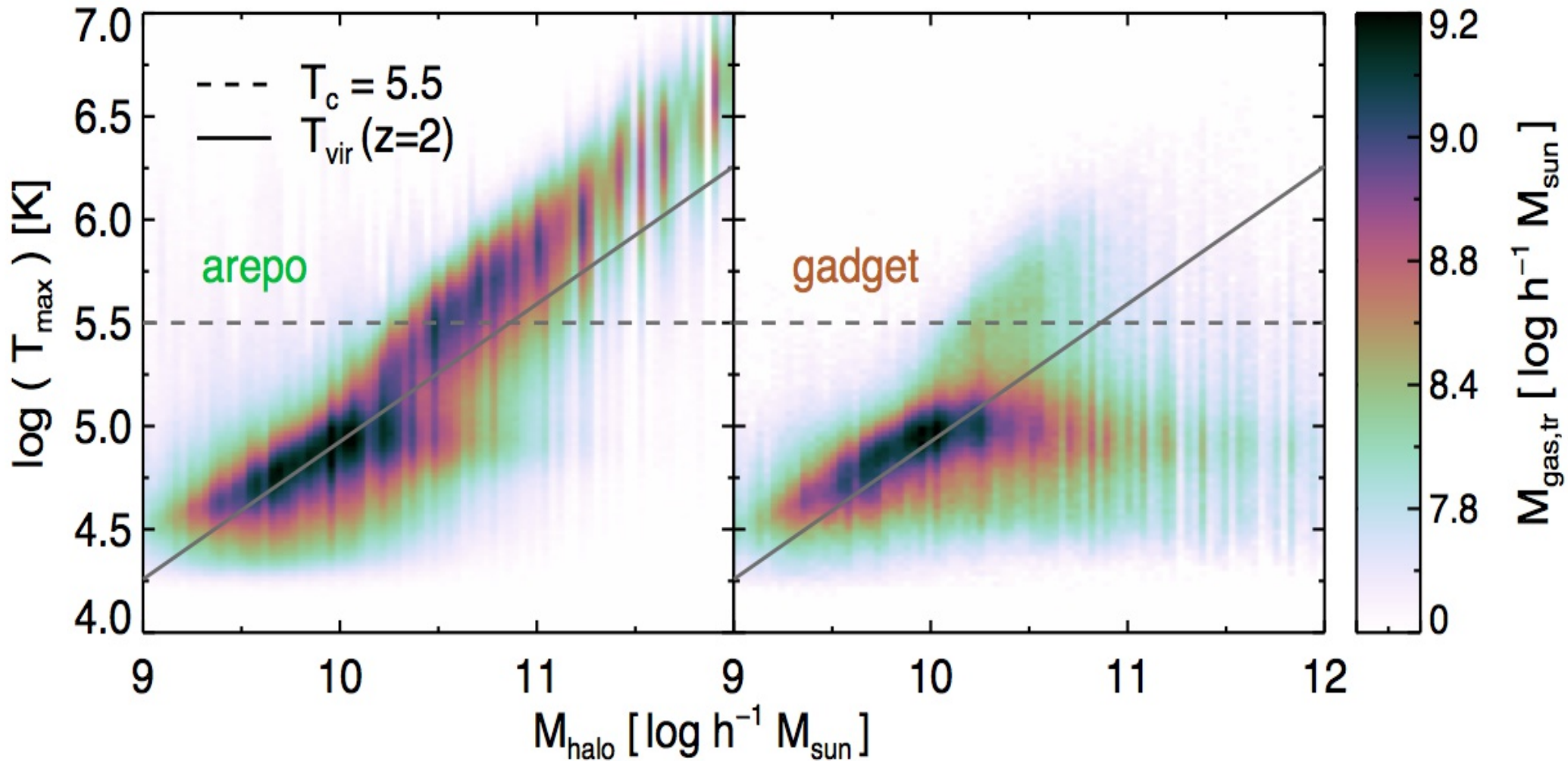
"COLD ACCRETION" HAS BEEN SUGGESTED AS DOMINANT MODE EVEN FOR LARGE HALOS



Keres et al. (2005, 2009)

There are marked differences in cold vs. hot accretion for massive galaxies

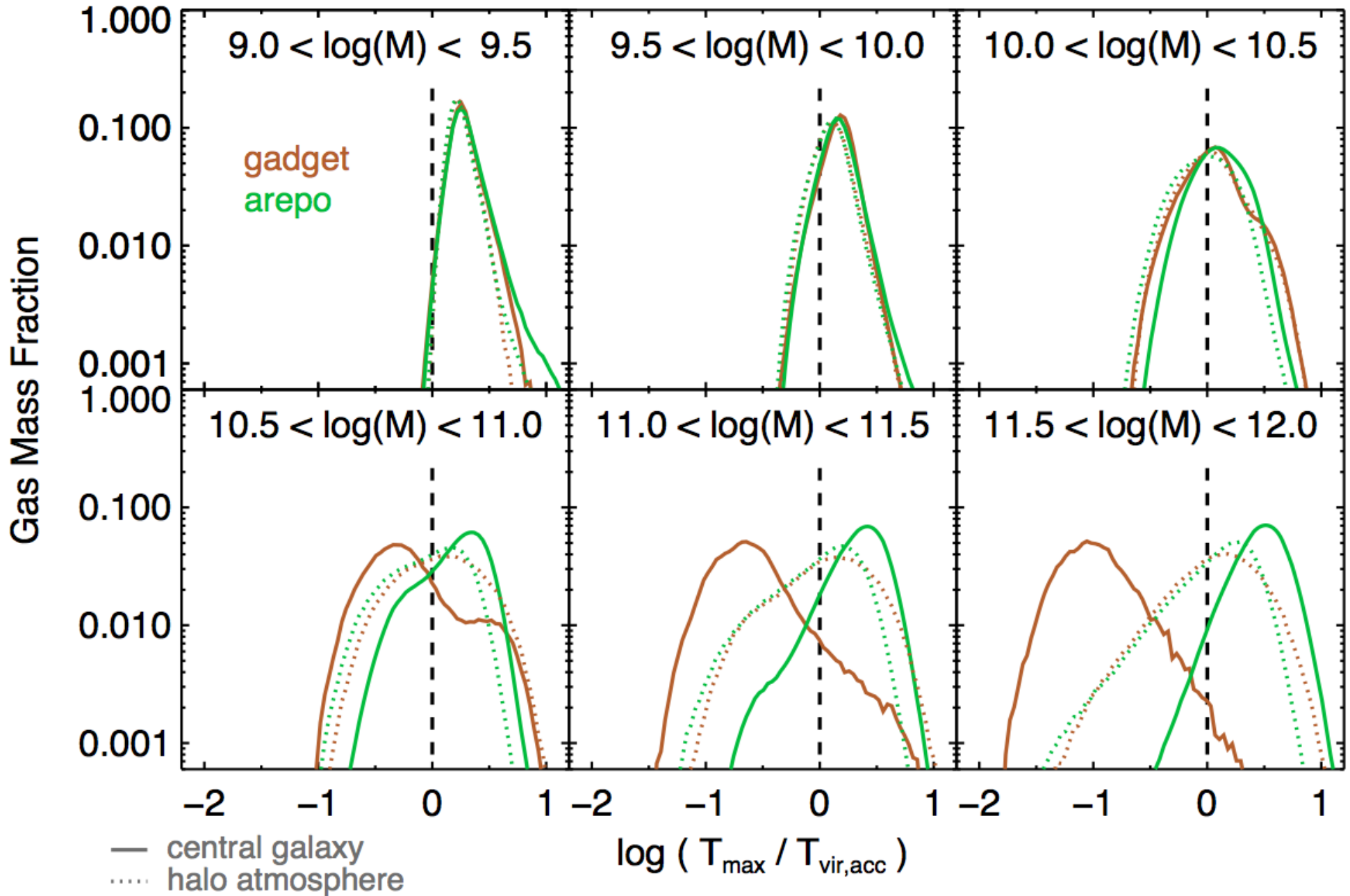
PAST MAXIMUM TEMPERATURE OF GAS ACCRETED ONTO CENTRAL GALAXIES



Nelson et al. (2012)

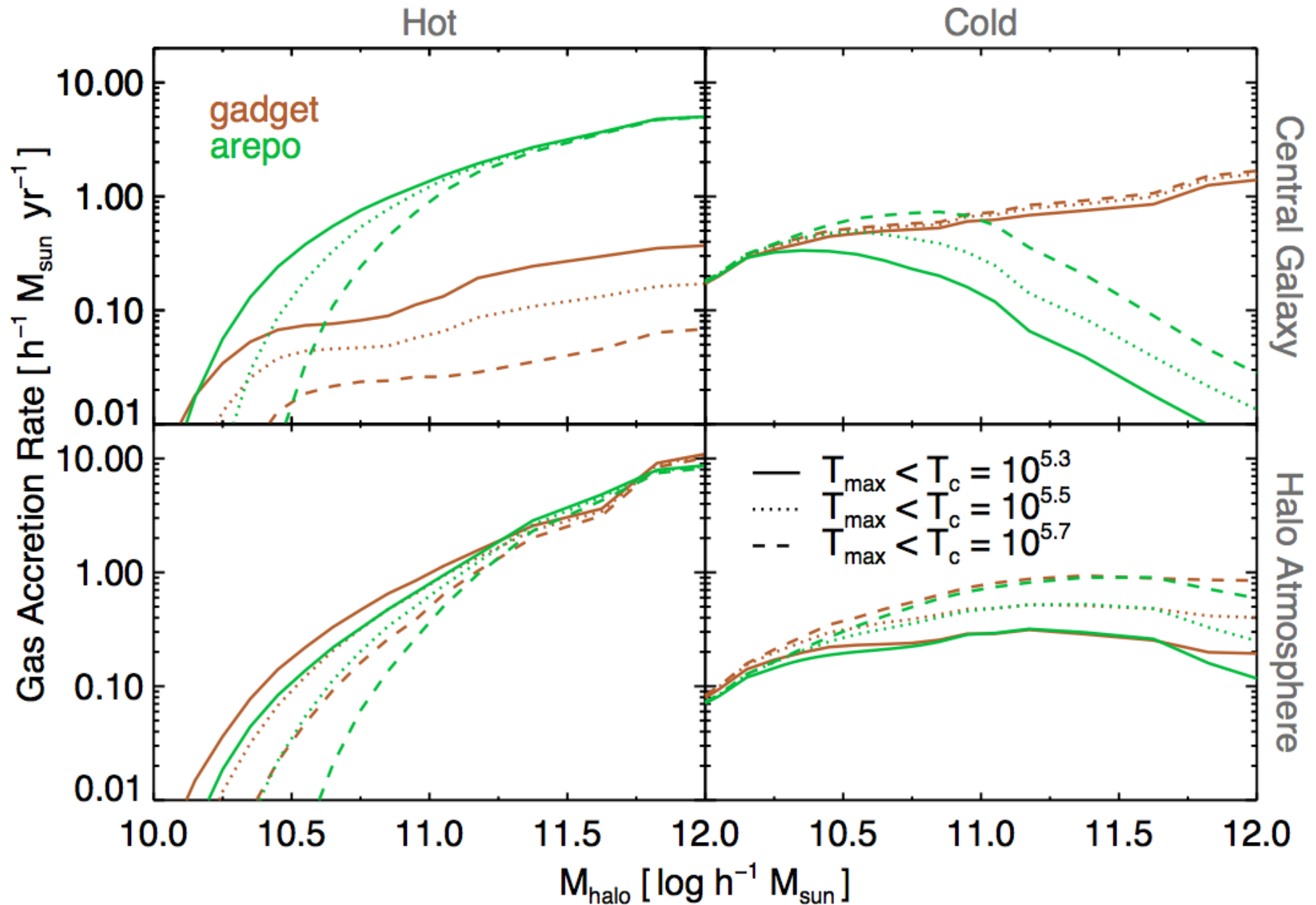
There are marked differences in cold vs. hot accretion for massive galaxies

DISTRIBUTION OF PAST MAXIMUM TEMPERATURE OF ACCRETED GAS AT  $Z = 2$



The relative importance of "hot" and "cold" modes of accretion are different for massive halos

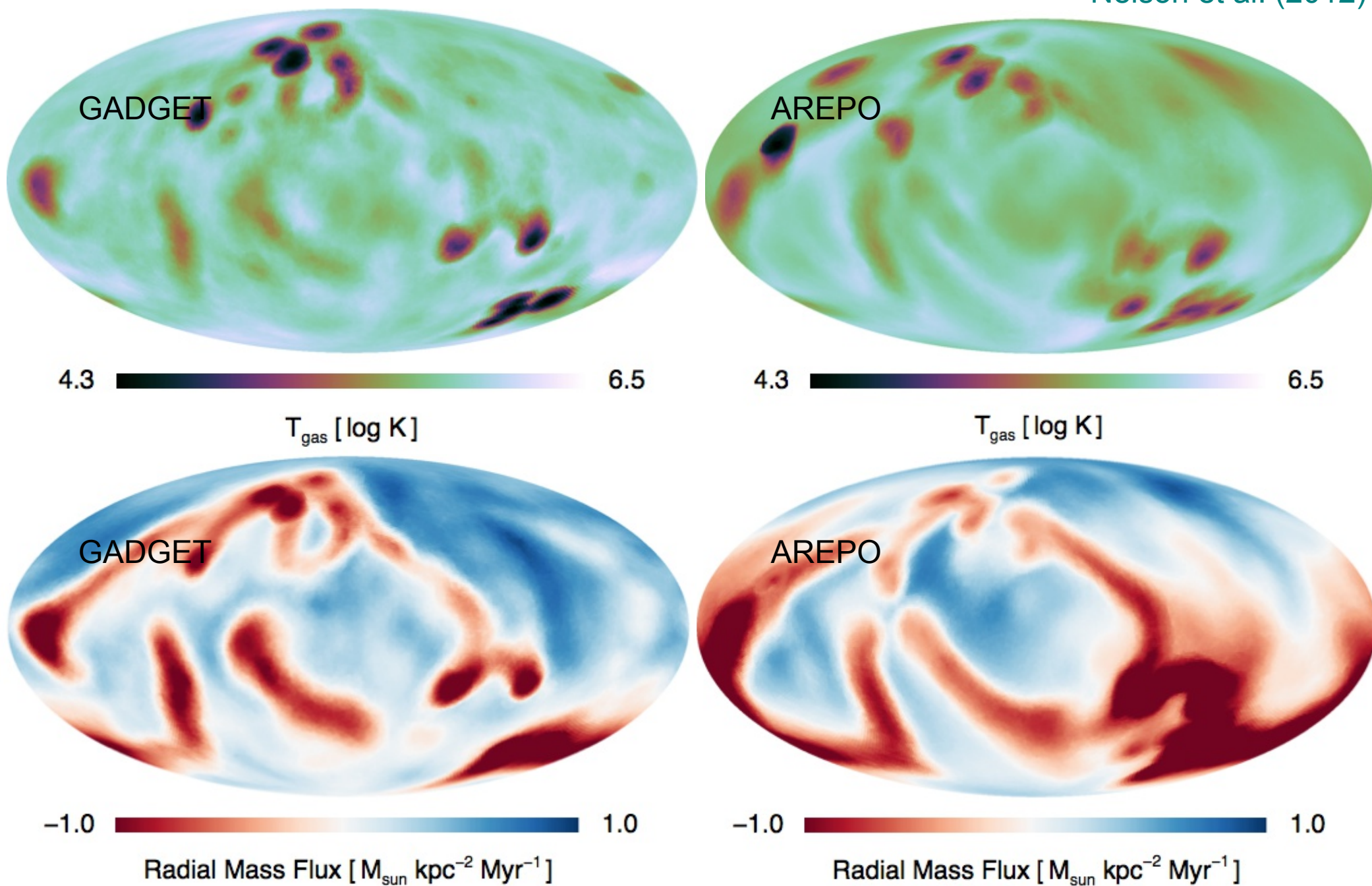
ACCRETION RATES OF HOT AND COLD GAS AS A FUNCTION OF HALO MASS AT  $Z = 2$



At the virial radius, only moderate differences in the gas flow are seen

ALL-SKY MAPS OF GAS PROPERTIES AROUND A TYPICAL  $\log(M)=11.5$  HALO AT  $Z=2$

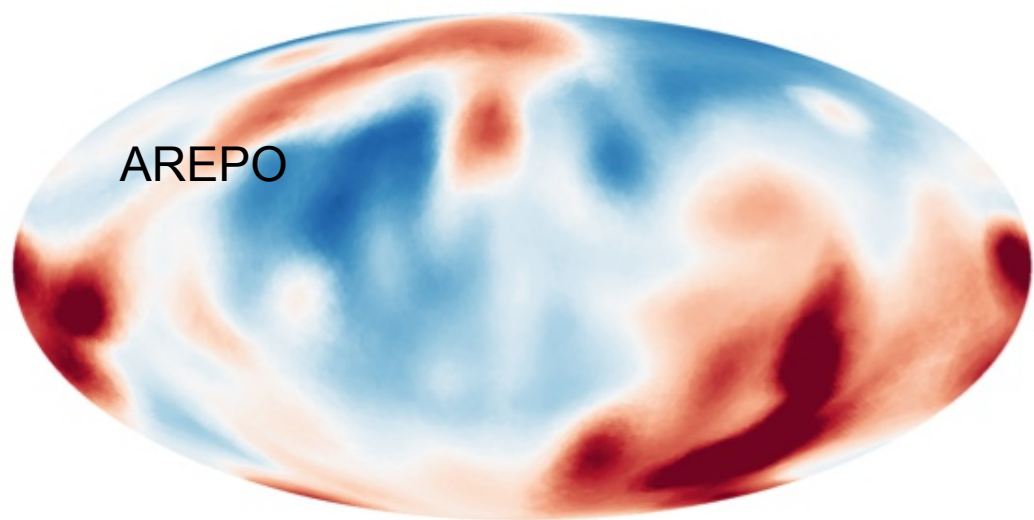
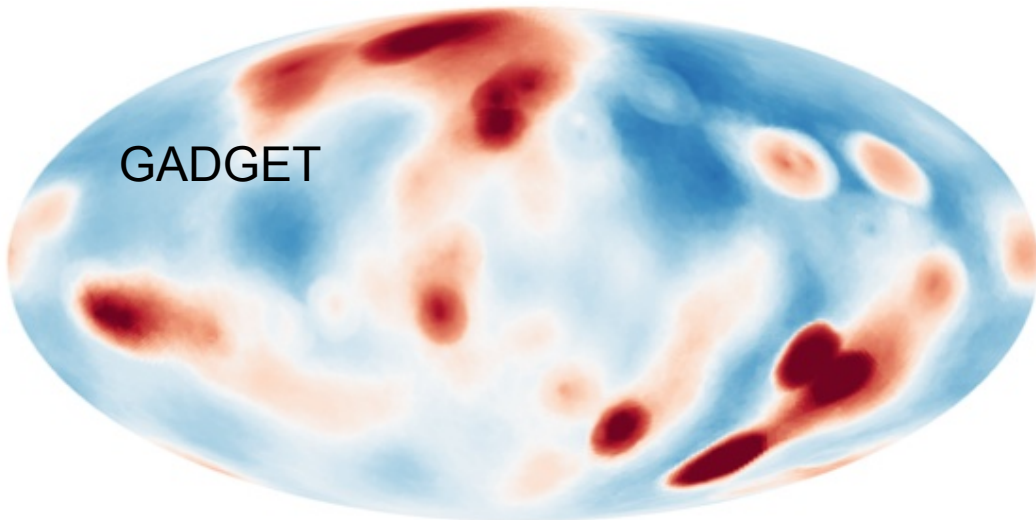
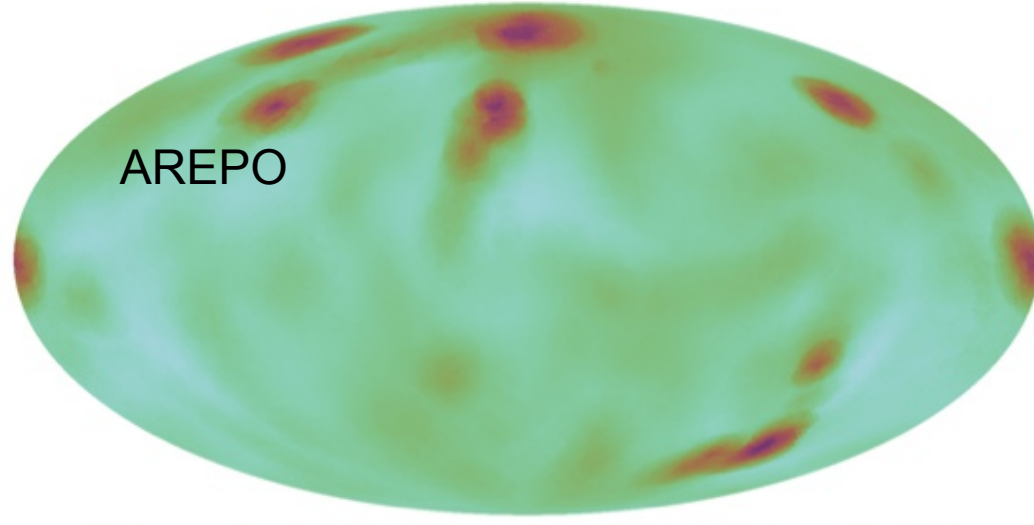
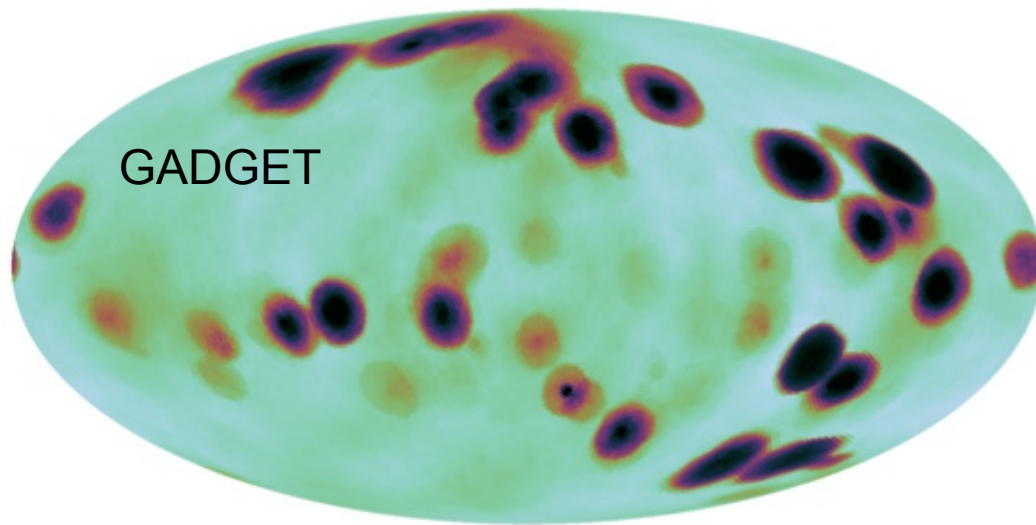
Nelson et al. (2012)



At half the virial radius, pronounced differences in the gas flow are apparent

ALL-SKY MAPS OF GAS PROPERTIES AROUND A TYPICAL HALO

Nelson et al. (2012)





## Summary points

- Direct numerical simulations have become indispensable for studying the **non-linear growth of structures** in  $\Lambda$ CDM and modified gravity cosmologies.
- Current numerical techniques allow high-resolution simulations with an unprecedented dynamic range.  
**One presently reaches  $N > 10^{11}$ , with a dynamic range of  $10^5 - 10^7$  in 3D.**
- **Understanding galaxy formation** physics remains a **serious challenge** in  $\Lambda$ CDM, both at the faint and the bright end.
- **New moving-mesh techniques** provide an accurate alternative technique for structure formation simulations. First results suggest that they are of great help to arrive at more reliable predictions for galaxy formation.