QCD thermodynamics in the large-N limit

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Outline

The large-N limit

Results

Conclusions and outlook

Based on:

- M.P., *Thermodynamics of the QCD plasma and the large-N limit*, Phys. Rev. Lett. **103** 232001 (2009), `[arXiv:0907.3719 [hep-lat]]`

Related works: **[Bringoltz and Teper, 2005; Datta and Gupta, 2010]**

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The old perspective: QCD at large N

- Since QCD has no obvious dimensionless expansion parameter (the coupling sets the scale), 't Hooft proposed to use $1/N$ (N being the number of colors) as an expansion parameter **['t Hooft, 1974]**
- Generically, a large-N limit can be interpreted as a 'classical limit'—Identification of coherent states and construction of a classical Hamiltonian **[Yaffe, 1982]**
- The large-N limit of QCD, at fixed 't Hooft coupling $\lambda = g^2 N$ and fixed number of flavors N_f, is a simpler theory ...
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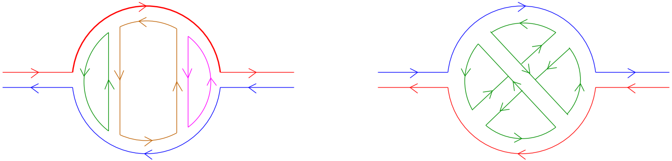

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- Formal connection to string theory: loop expansion in Riemann surfaces for closed string theory with coupling constant $g_{\mathrm{string}} \sim 1/N$ **[Aharony, Gubser, Maldacena, Ooguri and Oz, 1999; Mateos, 2007]**

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The AdS/CFT correspondence

- Maldacena conjectured that the large-N limit of the maximally supersymmetric $\mathcal{N} = 4$ supersymmetric YM (SYM) theory in four dimensions is dual to type IIB string theory in a $AdS_5 \times S^5$ space **[Maldacena, 1997]**

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\mathbf{x}^2\right) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$

- The conjecture arises from the observation that the low-energy dynamics of open strings ending on a stack of N D3 branes in $AdS_5 \times S^5$ can be described in terms of $\mathcal{N} = 4$ SYM
- Geometric interpretation: There exists a correspondence of symmetries in the two theories
- A highly non-trivial correspondence, linking the strongly coupled regime of field theory to the weak-coupling limit of a gravity model
- Identification of the generating functional of connected Green's functions in the gauge theory with the minimum of the supergravity action with given boundary conditions: correlation functions of gauge theory operators from perturbative calculations in the gravity theory **[Gubser, Klebanov and Polyakov, 1998]**
- A stringy realization of the holographic principle: the description of dynamics within a volume of space is "encoded on the boundary" **['t Hooft, 1993; Susskind, 1995]**—see also **[Bousso, 2002]**
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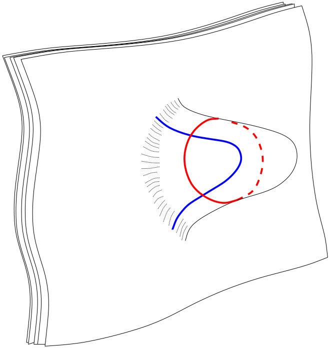

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  - $\mathcal{R}$-symmetry in the gauge theory is $SU(4) \sim SO(6)$ symmetry of S^5
  - The conformal invariance group in the gauge theory is isomorphic to $SO(2, 4)$, the symmetry group of AdS_5
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$$\begin{aligned} g^2 &= 4\pi g_s \\ g^2 N &= \frac{R^4}{l_s^4} \end{aligned}$$

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Non-perturbative predictions for QCD-like theories from holographic models

- ‘Top-down’ approach: break some symmetries of the $\mathcal{N}=4$ theory explicitly, add fundamental matter fields to the gauge theory by including new branes in the string theory **[Bertolini, Di Vecchia, Frau, Lerda, and Marotta, 2001; Graña and Polchinski, 2001; Karch and Katz, 2002]** to get a non-trivial hadron sector with ‘mesons’ and χSB **[Erdmenger, Evans, Kirsch and Threlfall, 2007]**
- Description of hydrodynamic and thermodynamic properties for a strongly interacting system, like the QCD plasma, from gauge/gravity duality—see **[Son and Starinets, 2007; Mateos, 2007; Gubser and Karch, 2009]** and references therein
- ‘Bottom-up’ approach: construct a 5D gravitational background reproducing the main features of QCD **[Polchinski and Strassler, 2001; Erlich, Katz, Son and Stephanov, 2005; Da Rold and Pomarol, 2005; Karch, Katz, Son and Stephanov, 2006]**
- Hard-wall *versus* soft-wall AdS/QCD, and related thermodynamic features **[Herzog, 2007]**

Improved holographic QCD model

- Kiritsis and collaborators **[Gürsoy, Kiritsis, Mazzanti and Nitti, 2008]** proposed an AdS/QCD model based on a 5D Einstein-dilaton gravity theory, with the fifth direction dual to the energy scale of the $\mathrm{SU}(N)$ gauge theory

$$S_{IHQCD} = -M_P^3 N^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} (\partial \Phi)^2 + V(\lambda) \right] + 2M_P^3 N^2 \int_{\partial M} d^4x \sqrt{h}\, K$$

- Field content on the gravity side: metric (dual to the $\mathrm{SU}(N)$ energy-momentum tensor), dilaton (dual to the trace of F^2) and axion (dual to the trace of $F\tilde{F}$)
- Dilaton potential defined by requiring asymptotic freedom with a logarithmically running coupling in the UV and linear confinement in the IR of the gauge theory
- First-order transition from a thermal-graviton- to a black-hole-dominated regime in the 5D gravity theory dual to the $\mathrm{SU}(N)$ deconfinement transition
- The model successfully reproduces the main non-perturbative spectral and thermodynamical features of the $\mathrm{SU}(3)$ YM theory
- Can also be used to derive predictions for observables such as the plasma bulk viscosity, drag force and jet quenching parameter **[Gürsoy, Kiritsis, Michalogiorgakis and Nitti, 2009]**
- *Caveat*: The effective five-dimensional Newton constant $G_5 = 1/\left(16\pi M_P^3 N^2\right)$ becomes small only in the large-N limit; at finite N, string interactions can be non-negligible above a scale $M_P N^{2/3} \simeq 2.5$ GeV in $\mathrm{SU}(3)$

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$$V(\lambda) = \frac{12}{\ell^2}\left[1 + V_0\lambda + V_1\lambda^{4/3}\sqrt{\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)}\right]$$

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- Kiritsis and collaborators **[Gürsoy, Kiritsis, Mazzanti and Nitti, 2008]** proposed an AdS/QCD model based on a 5D Einstein-dilaton gravity theory, with the fifth direction dual to the energy scale of the $\mathrm{SU}(N)$ gauge theory
- Field content on the gravity side: metric (dual to the $\mathrm{SU}(N)$ energy-momentum tensor), dilaton (dual to the trace of F^2) and axion (dual to the trace of $F\tilde{F}$)
- Dilaton potential defined by requiring asymptotic freedom with a logarithmically running coupling in the UV and linear confinement in the IR of the gauge theory
- First-order transition from a thermal-graviton- to a black-hole-dominated regime in the 5D gravity theory dual to the $\mathrm{SU}(N)$ deconfinement transition
- The model successfully reproduces the main non-perturbative spectral and thermodynamical features of the $\mathrm{SU}(3)$ YM theory
- Can also be used to derive predictions for observables such as the plasma bulk viscosity, drag force and jet quenching parameter **[Gürsoy, Kiritsis, Michalogiorgakis and Nitti, 2009]**
- *Caveat*: The effective five-dimensional Newton constant $G_5 = 1/\left(16\pi M_P^3 N^2\right)$ becomes small only in the large-N limit; at finite N, string interactions can be non-negligible above a scale $M_P N^{2/3} \simeq 2.5$ GeV in $\mathrm{SU}(3)$

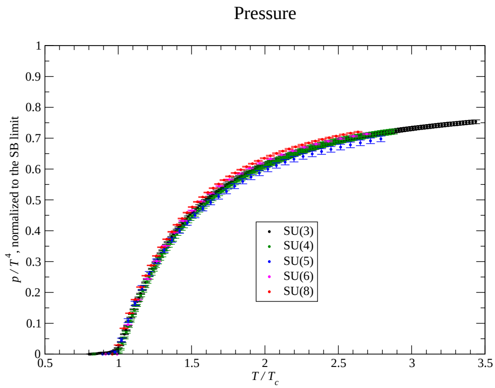
Outline

The large-N limit

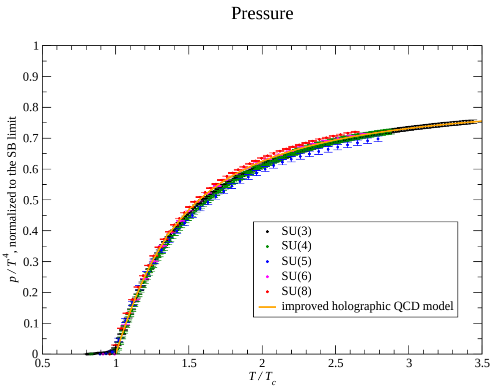
Results

Conclusions and outlook

Lattice results *vs.* Improved holographic QCD model



Lattice results *vs.* Improved holographic QCD model



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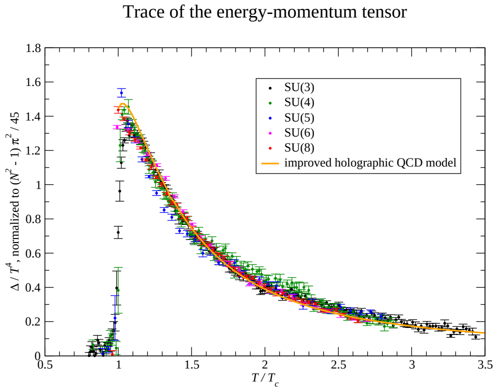

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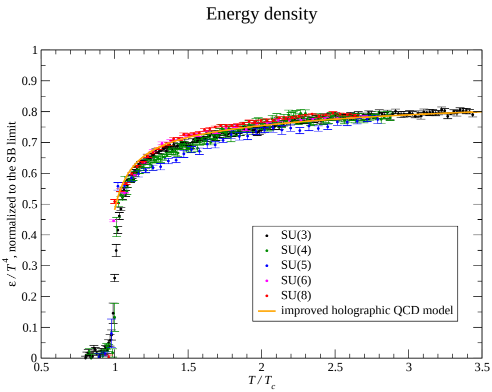

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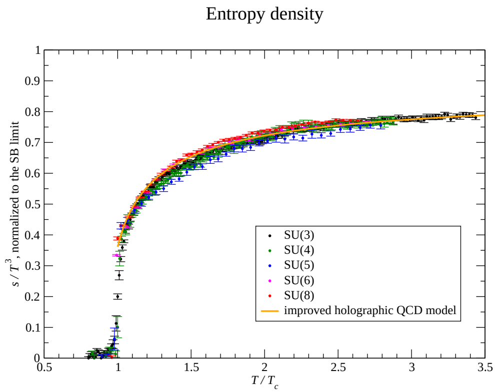


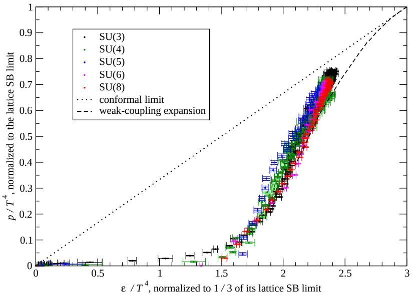
Lattice results for $\mathrm{SU}(N)$ *vs.* AdS/CFT?

The $\mathrm{SU}(N)$ plasma tends to become exactly conformally invariant only in the $T \rightarrow \infty$ limit, where it is no longer strongly coupled

Lattice results for $\mathrm{SU}(N)$ *vs.* AdS/CFT?

In the temperature range investigated in this work, the lattice results approach approximate scale-invariance only for $T \simeq 3T_c$, where the plasma is still (relatively) strongly coupled . . .

$p(\varepsilon)$ equation of state and approach to conformality

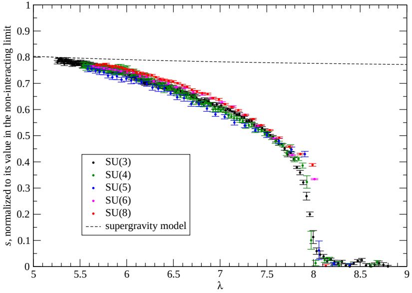


Lattice results for $\mathrm{SU}(N)$ *vs.* AdS/CFT?

...and the entropy density is comparable with the supergravity prediction for $\mathcal{N} = 4$ SYM **[Gubser, Klebanov and Tseytlin, 1998]**

$$\frac{s}{s_0} = \frac{3}{4} + \frac{45}{32}\zeta(3)(2\lambda)^{-3/2} + \dots$$

Entropy density *vs.* 't Hooft coupling



Lattice results for $\mathrm{SU}(N)$ *vs.* AdS/CFT?

. . . in which the entropy density is comparable with the supergravity prediction for $\mathcal{N} = 4$ SYM **[Gubser, Klebanov and Tseytlin, 1998]**

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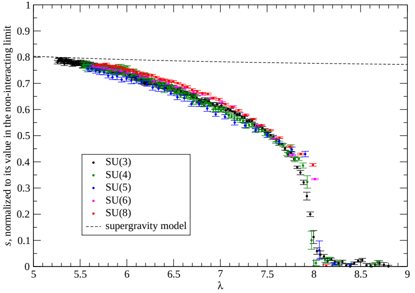
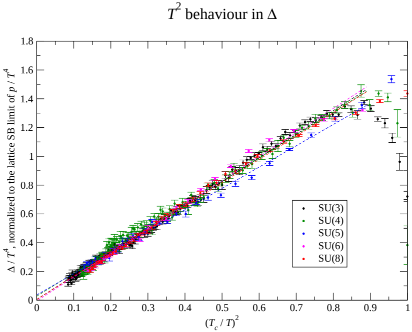
Entropy density *vs.* 't Hooft coupling



Note that a comparison of $\mathcal{N} = 4$ SYM and full-QCD lattice results for the drag force on heavy quarks also yields $\lambda \simeq 5.5$ **[Gubser, 2006]**

T^2 contributions to the trace anomaly?

The trace anomaly reveals a characteristic T^2-behavior, possibly of non-perturbative origin **[Megías, Ruiz Arriola and Salcedo, 2003; Pisarski, 2006; Andreev, 2007; Buisseret, 2009]**

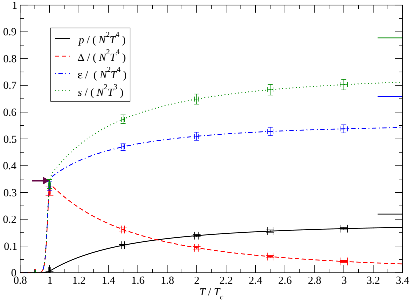


Extrapolation to $N \to \infty$

Based on the parametrization **[Bazavov *et al.*, 2009]**:

$$\frac{\Delta}{T^4} = \frac{\pi^2}{45}(N^2 - 1) \cdot \left(1 - \left\{1 + \exp\left[\frac{(T/T_c) - f_1}{f_2}\right]\right\}^{-2}\right)\left(f_3 \frac{T_c^2}{T^2} + f_4 \frac{T_c^4}{T^4}\right)$$

Extrapolation to the large-N limit



Outline

Conclusions

- Equilibrium thermodynamic observables in $\mathrm{SU}(N)$ YM theories at temperatures $0.8T_c \leq T \leq 3.4T_c$ show a mild dependence on N
- Successful comparison with holographic predictions
- Δ seems to have a T^2 dependence also at large N

Projects for the future

Projects for the future - I

(in case 'plan A' fails . . .)

- $\mathrm{SU}(N)$ screening masses and spatial string tensions, comparisons with AdS/CFT **[Bak, Karch and Yaffe, 2007]** and with IHQCD **[Alanen, Kajantie and Suur-Uski, 2009]**
- Observables related to thermodynamic fluctuations: specific heat, speed of sound *et c.* **[Gavai, Gupta and Mukherjee, 2005]**
- Renormalized Polyakov loops **[Dumitru *et al.*, 2004; Gupta, Hübner and Kaczmarek, 2008; Gavai, 2010]**
- Transport coefficients **[Meyer, 2007]**

Projects for the future - II

(in case 'plan A' fails . . .)

- High-precision thermodynamics for $\mathrm{SU}(N)$ theories in 3D (work in progress with Caselle, Castagnini, Feo and Gliozzi; see also **[Bialas, Daniel, Morel and Petersson, 2008]**)