The Physics of Glueballs

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- V. Mathieu, N. Kochelev, V. Vento
  "The Physics of Glueballs"
  invited review for Int. J. Mod. Phys. E
  arXiv:0810.4453 [hep-ph]
- V. Mathieu, F. Buisseret and C. Semay
  "Gluons in Glueballs: Spin or Helicity ? "
  Phys. Rev. D **77**, 114022 (2008), [arXiv:0802.0088 [hep-ph]]
- V. Mathieu, C. Semay and B. Silvestre-Brac
  "Semirelativistic Potential Model for Three-gluon Glueballs."
  Phys. Rev. D**77** 094009 (2008), [arXiv:0803.0815 [hep-ph]]
- N. Boulanger, F. Buisseret,V. Mathieu, C. Semay
  "Constituent Gluon Interpretation of Glueballs and Gluelumps."
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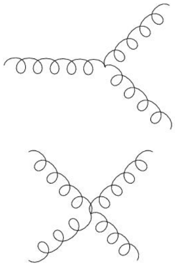
QCD = gauge theory with the color group $\mathcal{SU}(3)$



$$\begin{aligned}\mathcal{L}_{QCD} &= -\frac{1}{4}\mathrm{Tr}\, G_{\mu\nu}G^{\nu\mu} + \sum \bar{q}(\gamma^\mu D_\mu - m)q \\ G_{\mu\nu} &= \partial_\mu A_\mu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]\end{aligned}$$

Quark = fundamental representation **3**
Gluon = Adjoint representation **8**
Observable particles = color singlet **1**

Mesons: $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$

Baryons: $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$

Glueballs: $\mathbf{8} \otimes \mathbf{8} = (\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27}) \oplus (\mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}})$

$\mathbf{8} \otimes \cdots \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8} \oplus \ldots$

Colored gluons $\longrightarrow$ color singlet with only gluons

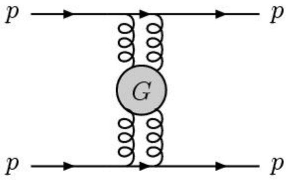
Prediction of the QCD
Production in gluon rich processes (OZI forbidden,...)
Closely linked to the Pomeron:

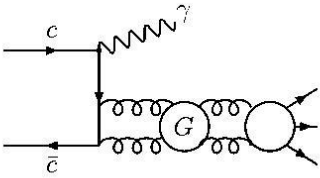
$$J = 0.25M^2 + 1.08$$

Mixing between glueball 0^{++} and light mesons

Candidates: $f_0(1370)$ $f_0(1500)$ $f_0(1710)$

One scalar glueball between those states [Klempt, Phys. Rep. **454**].





Physical States

Pure states: $|gg\rangle$, $|n\bar{n}\rangle$, $|s\bar{s}\rangle$

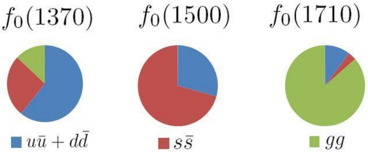
$$|G\rangle = |gg\rangle + \frac{\langle n\bar{n}|gg\rangle}{M_{gg} - M_{n\bar{n}}}\,|n\bar{n}\rangle + \frac{\langle s\bar{s}|gg\rangle}{M_{gg} - M_{s\bar{s}}}\,|s\bar{s}\rangle$$

Analysis of

Production: $J/\psi \to \gamma f_0, \omega f_0, \phi f_0$ Decay: $f_0 \to \pi\pi, K\bar{K}, \eta\eta$

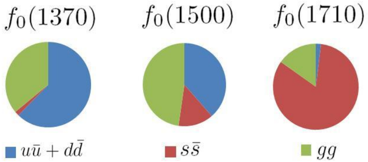
Two mixing schemes

Cheng *et al* [PRD**74**, 094005 (2006)]



$$M_{n\bar{n}} < M_{s\bar{s}} < M_{gg}$$

Close and Kirk [PLB**483**, 345 (2000)]

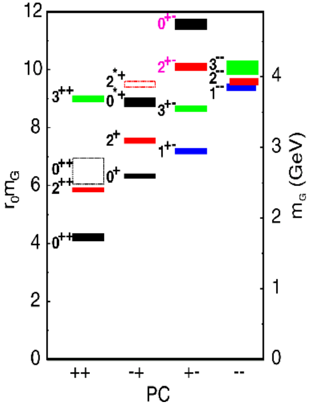


$$M_{n\bar{n}} < M_{gg} < M_{s\bar{s}}$$

Investigation of the glueball spectrum (pure gluonic operators) on a lattice by Morningstar and Peardon [Phys. Rev. D**60**, 034509 (1999)]
Identification of 15 glueballs below 4 GeV

$$\begin{aligned} M(0^{++}) &= 1.730 \pm 0.130 \text{ GeV} \\ M(0^{-+}) &= 2.590 \pm 0.170 \text{ GeV} \\ M(2^{++}) &= 2.400 \pm 0.145 \text{ GeV} \end{aligned}$$

Quenched approximation (gluodynamics)
→ mixing with quarks is neglected

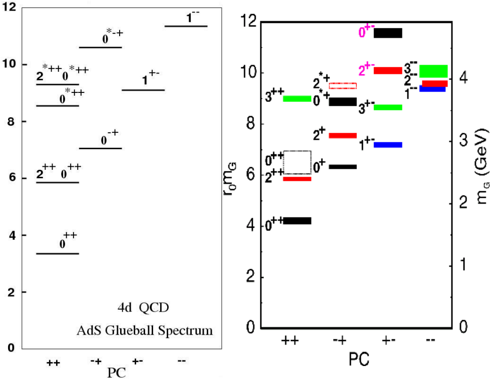


Lattice studies with $n_f = 2$ exist. The lightest scalar would be sensitive to the inclusion of sea quarks but no definitive conclusion.

AdS/QCD

AdS/CFT correspondance:
Correspondance between conformal theories and string theories in AdS spacetime
QCD not conformal → breaking conformal invariance somehow

- Introduction of a black hole in AdS to break conformal invariance
- Parameter adjusted on 2^{++}
- Same hierarchy but some states are missing (spin 3,...)



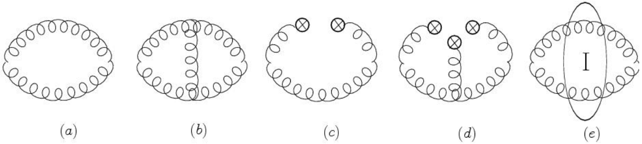
[R. C. Brower *et al.*, Nucl. Phys. B**587**, 249 (2000)]

Gluonic currents: $J_S(x) = \alpha_s G^a_{\mu\nu}(x) G^a_{\mu\nu}(x)$ $\quad J_P(x) = \alpha_s G^a_{\mu\nu}(x) \widetilde{G}^a_{\mu\nu}(x)$

$$\Pi(Q^2) = i \int d^4x \; e^{iq\cdot x} \langle 0|T J_G(x) J_G(0)|0\rangle = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi(s)}{s+Q^2} ds$$

Theoretical side (OPE):

$$J_G(x) J_G(0) = C_{(a)+(b)+(e)} \mathbf{1} + C_{(c)} G^a_{\mu\nu} G^{\mu\nu}_a + C_{(d)} f_{abc} G^a_{\alpha\beta} G^b_{\beta\gamma} G^b_{\gamma\alpha} + \cdots$$



Confinement parameterized with condensates $\langle 0|\alpha_s G^a_{\mu\nu} G^{\mu\nu}_a|0\rangle, \ldots$

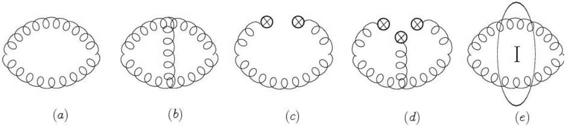
Phenomenological side:

$$\mathrm{Im}\Pi(s) = \sum_i \pi f^2_{G_i} m^4_{G_i} \delta(s - m^2_{G_i}) + \pi\theta(s - s_0) \mathrm{Im}\Pi(s)^{\mathrm{Cont}}$$

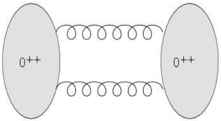
Sum rules calculation by Forkel [Phys. Rev. D**71**, 054008 (2005)]

$$M(0^{++}) = 1.25 \pm 0.20 \text{ GeV} \quad M(0^{-+}) = 2.20 \pm 0.20 \text{ GeV} \tag{1}$$

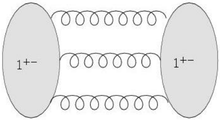
Two gluons in interaction



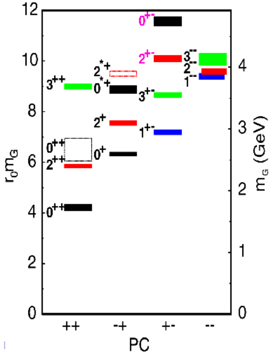
Two gluons → $C = +$

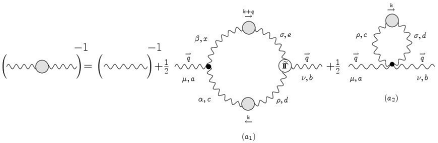


Three gluons → $C = -$



Works on constituent models by Bicudo, Llanes-Estrada, Szczepaniak, Simonov,...

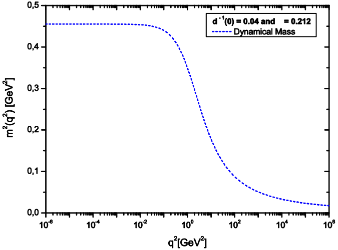




Gluons massless in the Lagrangian
Non-perturbative effects → dynamical mass
Cornwall [PRD**26** 1453 (1982)]

$$m^2(q^2) = m_0^2 \left[\frac{\ln\left(\frac{q^2+\rho m_0^2}{\Lambda^2}\right)}{\ln\left(\frac{\rho m_0^2}{\Lambda^2}\right)} \right]^{\gamma}$$

Gluons ∼ heavy quarks



Gluonium models for the low-lying glueballs with Spinless Salpeter Hamiltonian

$$H_{gg} = 2\sqrt{\boldsymbol{p}^2 + m^2} + V(r)$$

bare mass $m = 0$ and effective mass $\mu = \langle\Psi|\sqrt{\boldsymbol{p}^2}|\Psi\rangle$ state dependent (Simonov 1994)
Gluons spin-1 particles with the usual rules of spin coupling

Brau and Semay [Phys. Rev. D**70**, 014017 (2004)]

Two-gluon glueballs $\rightarrow C = +$ and $P = (-1)^L$

$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ with $S = 0, 1, 2$.

$$H^0 = 2\sqrt{\boldsymbol{p}^2} + \frac{9}{4}\sigma r - 3\frac{\alpha_s}{r}.$$

Cornell potential does not lift the degeneracy between states with different S

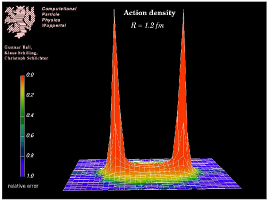
Corrections of order $\mathcal{O}(1/\mu^2)$, with

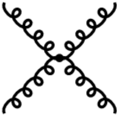
$$\mu = \langle\Psi|\sqrt{\boldsymbol{p}^2}|\Psi\rangle$$

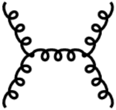
Structures coming from the OGE

$$V_{\text{oge}} = \lambda\Bigg[\left(\frac{1}{4} + \frac{1}{3}\boldsymbol{S}^2\right)U(r) - \frac{\pi}{\mu^2}\delta(\boldsymbol{r})\left(\frac{5}{2}\boldsymbol{S}^2 - 4\right)$$
$$\frac{3}{2\mu^2}\frac{U'(r)}{r}\boldsymbol{L}\cdot\boldsymbol{S} - \frac{1}{6\mu^2}\left(\frac{U'(r)}{r} - U''(r)\right)T\Bigg]$$

with $\lambda = -3\alpha_S$ and $U(r) = \exp(-\mu r)/r$.







Smearing of the attractive $\delta^3(\boldsymbol{r})$ by a gluon size

$$\rho(\boldsymbol{r}, \gamma) = \exp(-r/\gamma)/r$$

Replacement of potentials by convolutions with the size function

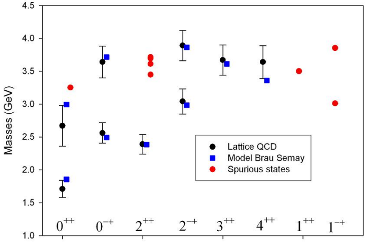
$$V(\boldsymbol{r}) \to \widetilde{V}(\boldsymbol{r}, \gamma) = \int V(\boldsymbol{r} + \boldsymbol{r}^*)\rho(\boldsymbol{r}^*, \gamma) d\boldsymbol{r}^*$$

Parameters

$$\sigma = 0.21 \text{ GeV}^2 \quad \alpha_s = 0.50 \quad \gamma = 0.5 \text{ GeV}^{-1}$$

Good agreement with lattice QCD
Gluons = spin 1 $\to$ spurious $J = 1$ states and indetermination of states
For instance, 0^{++} can be $(L, S) = (0, 0)$ or $(2, 2)$

J^{PC}	(L, S)		
0^{++}	(0,0)	(2,2)	$(0,0)^*$
0^{-+}	(1,1)	$(1,1)^*$	
2^{++}	(0,2)	(2,0)	(2,2)
2^{-+}	(1,1)	$(1,1)^*$	
3^{++}	(2,2)		
4^{++}	(2,2)		
1^{++}	(2,2)		
1^{-+}	(1,1)	$(1,1)^*$	

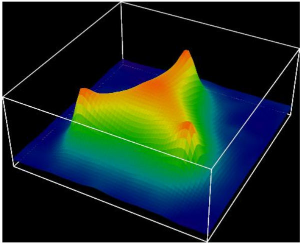


The Model

Extension of the model to three-gluon systems

$$H = \sum_{i=1}^{3} \sqrt{\boldsymbol{p}_i^2} + \frac{9}{4}\sigma \sum_{i=1}^{3} |\boldsymbol{r}_i - \boldsymbol{R}_{\rm cm}| + V_{\rm OGE}$$

Same parameters $(\sigma, \alpha_S, \gamma)$ as for two-gluon glueballs



$$\begin{aligned} V_{\rm OGE} \quad = \quad & -\frac{3}{2}\alpha_S \sum_{i<j=1}^{3} \left[\left(\frac{1}{4} + \frac{1}{3}\boldsymbol{S}_{ij}^2 \right) U(r_{ij}) - \frac{\pi}{\mu^2}\delta(\boldsymbol{r}_{ij}) \left(\beta + \frac{5}{6}\boldsymbol{S}_{ij}^2 \right) \right] \\ & -\frac{9\,\alpha_S}{4\mu^2} \sum_{i<j=1}^{3} \boldsymbol{L}_{ij} \cdot \boldsymbol{S}_{ij} \frac{1}{r_{ij}} \frac{d}{dr_{ij}} U(r_{ij}) \quad \text{with } U(r) = \frac{e^{-\mu r}}{r} \end{aligned}$$

We find the eigenvalue of this operator thanks to a Gaussian basis

The results

Gluons with spin $\rightarrow \boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$
Good results for 1^{--} and 3^{--} but higher 2^{--} $\leftarrow$ symmetry

Disagreement with lattice QCD for $PC = +-$
This model cannot explain the splitting ~ 2 GeV between 1^{+-} and 0^{+-}

$0^{+-}, 1^{+-}, 2^{+-}, 3^{+-}$ are $L = 1$ and degenerate in a model with spin

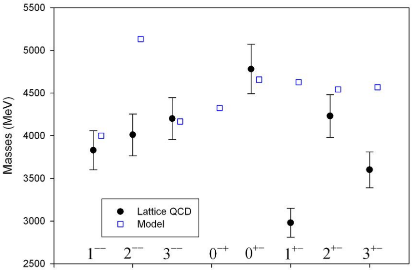
J^{PC}	(L, S)	J^{PC}	(L, S)
1^{--}	(0,1)	0^{+-}	(1,1)
2^{--}	(0,2)	1^{+-}	(1,1)
3^{--}	(0,3)	2^{+-}	(1,1)
0^{-+}	(1,1)	3^{+-}	(1,2)

Solution: Implementation of the helicity formalism for three transverse gluons.

$$d_{abc} A^a_\mu A^b_\nu A^c_\rho \qquad [(\mathbf{88})_{\mathbf{8}_s}\mathbf{8}]^{\mathbf{1}} \qquad C = -$$

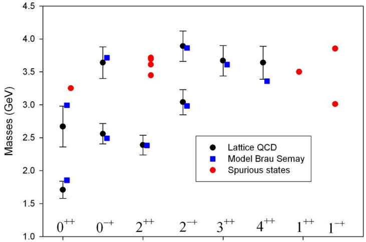
$$f_{abc} A^a_\mu A^b_\nu A^c_\rho \qquad [(\mathbf{88})_{\mathbf{8}_a}\mathbf{8}]^{\mathbf{1}} \qquad C = +$$

S	S_{int}	Symmetry	J^{PC}
0	0	1 A	0^{-+}
1	0, 1, 2	1 S, 2 MS	1^{--}
2	1, 2	2 MS	2^{--}
3	2	1 S	3^{--}



Helicity Formalism for two-gluon glueballs

Yang's theorem: $\rho \nrightarrow \gamma\gamma$
$\gamma\gamma \sim gg \to J \neq 1$
$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ cannot hold for relativistic system. $\boldsymbol{J}$ is the only relevant quantum number.
Solution: Helicity formalism for transverse gluons with helicity $s_i = 1$. Only two projection, $\lambda_i = \pm 1$.



Formalism to handle with massless particles [Jacob and Wick (1959)]
State J^{PC} in term of usual (L, S) states (with $\Lambda = \lambda_1 - \lambda_2$):

$$|J, M; \lambda_1, \lambda_2\rangle = \sum_{L,S} \left[\frac{2L+1}{2J+1}\right]^{1/2} \langle L0S\Lambda|\, J\Lambda\,\rangle \langle s_1\lambda_1 s_2 - \lambda_2|\, S\Lambda\,\rangle \left|{}^{2S+1}L_J\right\rangle .$$

Not eigenstates of the parity and of the permutation operator.

$$\begin{aligned} \mathtt{P}\,|J, M; \lambda_1, \lambda_2\rangle &= \eta_1\eta_2(-1)^J\,|J, M; -\lambda_1, -\lambda_2\rangle\,, \\ P_{12}\,|J, M; \lambda_1, \lambda_2\rangle &= (-1)^{J-2s_i}\,|J, M; \lambda_2, \lambda_1\rangle\,, \end{aligned}$$

Construction of states symmetric (bosons) and with a good parity → selection rules.

$$
\begin{aligned}
\left|S_+;(2k)^+\right\rangle &\Rightarrow 0^{++},2^{++},4^{++},\ldots\\
\left|S_-;(2k)^-\right\rangle &\Rightarrow 0^{-+},2^{-+},4^{-+},\ldots\\
\left|D_+;(2k+2)^+\right\rangle &\Rightarrow 2^{++},4^{++},\ldots\\
\left|D_-;(2k+3)^+\right\rangle &\Rightarrow 3^{++},5^{++},\ldots
\end{aligned}
$$

States expressed in term of the usual basis (useful to compute matrix elements!).
Low-lying states:

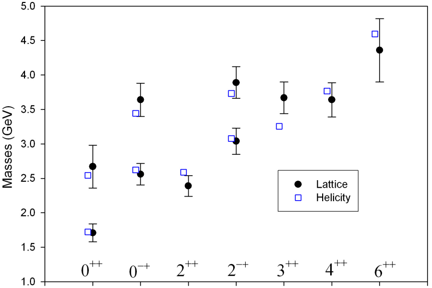
$$
\begin{aligned}
\left|S_+;0^+\right\rangle &= \sqrt{\frac{2}{3}}\left|{}^1S_0\right\rangle+\sqrt{\frac{1}{3}}\left|{}^5D_0\right\rangle,\\
\left|S_-;0^-\right\rangle &= \left|{}^3P_0\right\rangle,\\
\left|D_+;2^+\right\rangle &= \sqrt{\frac{2}{5}}\left|{}^5S_2\right\rangle+\sqrt{\frac{4}{7}}\left|{}^5D_2\right\rangle+\sqrt{\frac{1}{7}}\left|{}^5G_2\right\rangle,\\
\left|D_-;3^+\right\rangle &= \sqrt{\frac{5}{7}}\left|{}^5D_3\right\rangle+\sqrt{\frac{2}{7}}\left|{}^5G_3\right\rangle,\\
\left|S_-;2^-\right\rangle &= \sqrt{\frac{2}{5}}\left|{}^3P_2\right\rangle+\sqrt{\frac{3}{5}}\left|{}^3F_3\right\rangle.
\end{aligned}
$$

Same hierarchy as the lattice QCD
Application with a simple Cornell potential

$$H^0 = 2\sqrt{\boldsymbol{p}^2} + \frac{9}{4}\sigma r - 3\frac{\alpha_s}{r}.$$

Parameters:

$$\sigma = 0.185 \text{ GeV}^2 \qquad \alpha_s = 0.45$$



Addition of an instanton induced interaction to split the degeneracy between the 0^{++} and 0^{-+}

$$\Delta H_I = -P\,\mathcal{I}\,\delta_{J,0} \qquad \text{with } \mathcal{I} = 450 \text{ MeV}.$$

Instanton attractive in the scalar channel and repulsive in the pseudoscalar and equal in magnitude
Very good agreement without spin-dependent potential
Extension for three-body systems ?

Three-gluon glueballs with transverse gluons

$\mathcal{SU}(2) \times \mathcal{SU}(3)$ decomposition for two gluons $\rightarrow$ to the lowest J:

$$\square^{a} \otimes \square^{b} = \square\square^{(ab)} \oplus \bullet^{(ab)} \oplus \square^{[ab]}$$

Lowest J allowed for 3 gluons with helicity:

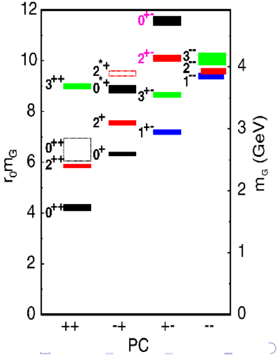
$$\square^{a} \otimes \square^{b} \otimes \square^{c} = \square\square\square^{(abc)} \oplus \square^{(abc)} \oplus \cdots \oplus \bullet^{[abc]}$$

Low-lying states are $J = 1$ and $J = 3$ with symmetric colour function and $J = 0$ with an antisymmetric colour function

Low-lying states are the $1^{\pm -}$, $3^{\pm -}$ and the $0^{\pm +}$

$J = 0^{P-}$ are not allowed for three-gluon glueballs
$\rightarrow 0^{+-}$ four transverse gluons

The result should be confirmed by a detailed analysis of the three-body helicity formalism
These developments are under construction



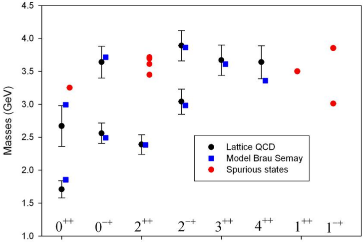
Conclusion

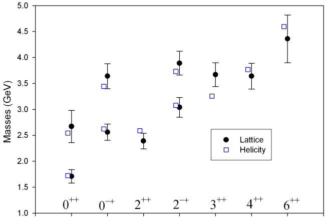
Model with spin-1 gluons reproduces the pure gauge spectrum but spin-dependent potentials should be added and the spectrum is plagued with unwanted states

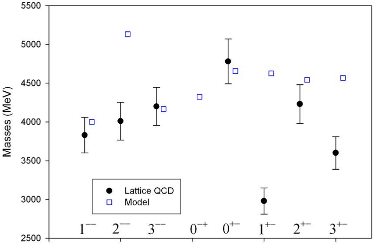
Three-gluon glueballs with massive gluon cannot reproduce the lattice data

With two transverse gluons, a simple linear+Coulomb potential reproduce the lattice QCD results.

Implementation of the helicity formalism for three transverse gluons should solve the hierarchy problem







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