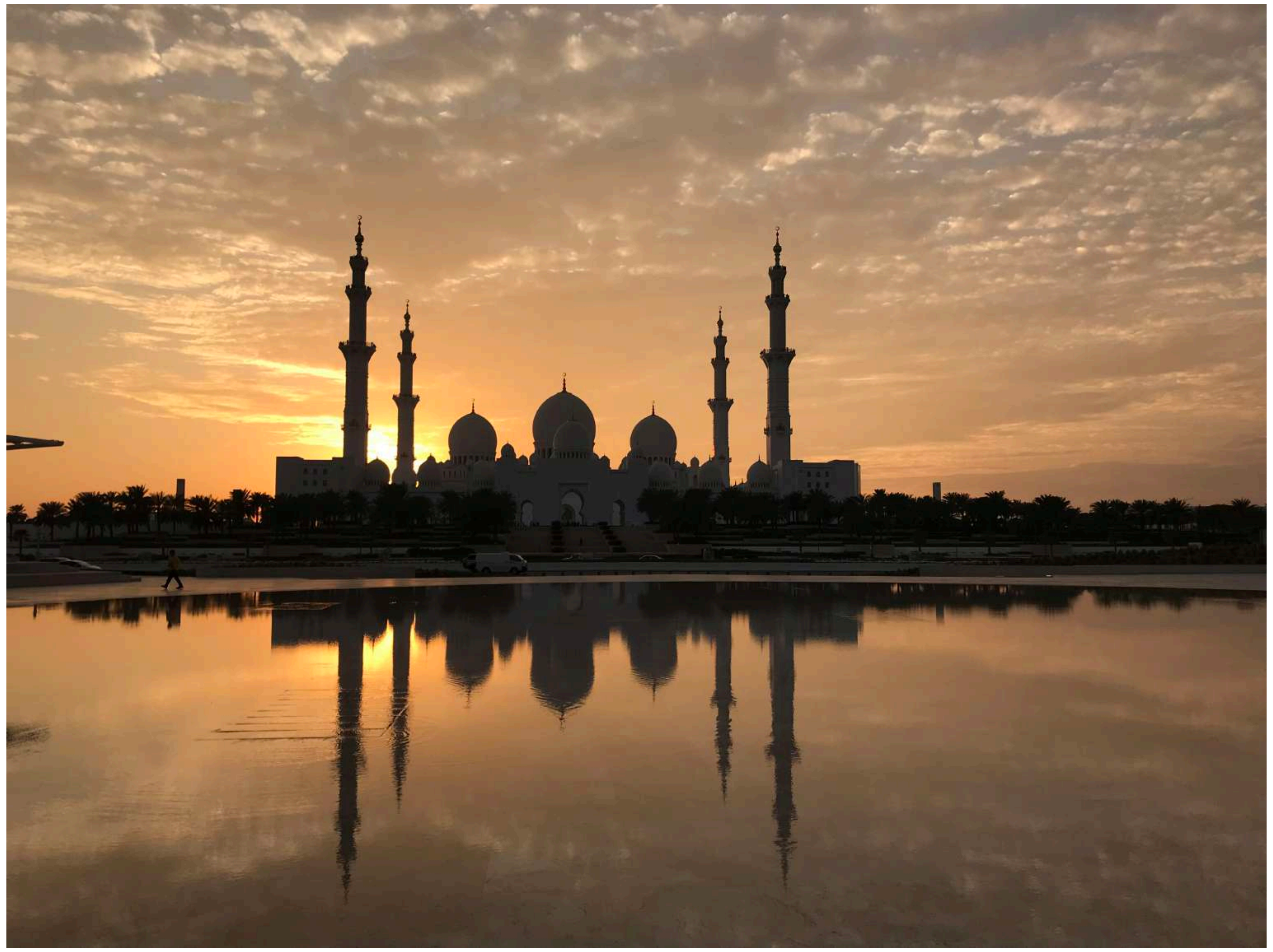
Self-Dual p-Form Gauge Theory & the Topology of the Graviton



Abu Dhabi, January 2024

Half Field Theory

- $q-1$-form gauge field $A, \quad F = dA$
- If $d = 2q$, and q odd: can impose SELF-DUALITY $\quad F = *F \quad$ halves d.o.f.
- Covariant action for SD theory?

Half Field Theory

- $q-1$-form gauge field $A, \quad F = dA$
- If $d = 2q$, and q odd: can impose SELF-DUALITY $\quad F = *F \quad$ halves d.o.f.
- Covariant action for SD theory?
- Sen's action: inspired by String Field Theory
- Quadratic: good for quantisation
- Generalises to allow Born-Infeld and Chern-Simons interactions

Sen's Theory

- Spacetime metric $g_{\mu\nu}$, Minkowski metric $\eta_{\mu\nu}$, Hodge duals $* = *_g\,, *_\eta$
- Fields in action couple to η, there is weird interaction term depending on $g_{\mu\nu}$
- TWO SD gauge fields A, C (constructed from fields appearing in action)

$$F = dA, F = * F \qquad\qquad G = dC, G = *_\eta G$$

- A: couples to space-time metric (and other physical fields)
- C: Couples to none of the physical fields: DECOUPLES

Sen's Theory

- $\eta_{\mu\nu}$ very restrictive: Most spacetimes don't admit Minkowski metric
- Coordinate independent?
- Strange symmetry: acts like diffeomorphisms on $g_{\mu\nu}$,A but $\eta_{\mu\nu}$,C invariant
- Would like coordinate independent theory that can be formulated on *any* spacetime

Non-Sen's Theory

- Replace $\eta_{\mu\nu}$ with metric $\bar{g}_{\mu\nu}$ $\quad *_\eta \to *_{\bar{g}} = \bar{*}$ CMH 2307.08748
- $F = dA, F = * F, \qquad G = dC, G = \bar{*}\, G$
- Hard bit: finding interaction term $f(g, \bar{g})$ and showing it gives required field equations
- Physical Sector $g_{\mu\nu}$, A + other physical fields, couple to each other
- Non-physical sector $\bar{g}_{\mu\nu}$, C couple to each other but not to any physical fields
- Gives desired physical sector plus shadow sector that decouples

The space with 2 metrics

- Spacetime with 2 metrics $\mathscr{M}(g, \bar{g})$
- Interesting bi-metric geometry, new structures, important in technical bits
- Action covariant, can be formulated on any spacetime
- $\bar{g}_{\mu\nu}$ can be a background metric or can be dynamical
- Similar "bi-metric structures" arise in massive gravity and interacting theory of 2 gravitons

de Rham, Gabadadze, Tolley   Hassan, Rosen

Doubled Geometry (after all)

- Two metrics: 2 kinds of "diffeomorphism" symmetries

$$\delta g_{\mu\nu} = 2\partial_{(\mu}\zeta_{\nu)} + \ldots, \quad \delta \bar{g}_{\mu\nu} = 2\partial_{(\mu}\chi_{\nu)} + \ldots$$

- Extends to symmetries of full theory
- The ζ_μ transformations act on Physical Sector $g_{\mu\nu}, A$ + other physical fields, do not act on Shadow Sector $\bar{g}_{\mu\nu}, C$
- The χ_μ transformations act on Shadow Sector, do not act on Physical Sector
- "Real diffeomorphisms" diagonal subgroup

Sen's Action

Fields: $q-1$form P, SD q-form Q, $Q = *_\eta Q$ $M(Q)_{\mu_1\ldots\mu_q} = \frac{1}{q!}M_{\mu_1\ldots\mu_q}^{\nu_1\ldots\nu_q}Q_{\nu_1\ldots\nu_q}$

$$S = \int\left(\frac{1}{2}dP \wedge *_\eta dP - 2Q \wedge dP - Q \wedge M(Q)\right)$$

Define: $G \equiv \frac{1}{2}(dP + *_\eta dP) + Q$ $G = *_\eta G$ $F \equiv Q + M(Q)$

Field equations imply: $dG = 0,$ $dF = 0$

Choose $M(Q)$ so that: $F = *F$

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New Action

Fields: $q-1$ form P, SD q-form Q, $Q = \bar{*}\, Q$ $\eta \to \bar{g}$, $*_\eta \to \bar{*}$

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Dependence on Metrics

Term in action $-\int Q \wedge M(Q)$ gives interaction between $Q, g, \bar{g}$

Action gives complicated theory of $P, Q, g, \bar{g}$

But gives simple theory of

$$G \equiv \frac{1}{2}(dP + \bar{*}dP) + Q \qquad\qquad F \equiv Q + M(Q)$$

with F interacting with g and G interacting with $\bar{g}$, but no interactions between the physical sector F, g and the shadow sector $G, \bar{g}$

2d Chiral Boson

Zweibein $\bar{e}^a_\mu$ for $\bar{g}$, $a, b = \pm$, $\bar{e}^\pm = 2^{-1/2}(\bar{e}^0 \pm \bar{e}^1)$, $\partial_a = \bar{e}^\mu_a \partial_\mu$

$$S = \int d^2x \sqrt{\bar{g}} \left(\partial_+ P \partial_- P + 2 Q_+ \partial_- P + M_{--} Q_+ Q_+ \right)$$

$$G_+ = \frac{1}{2}\partial_+ P + Q_+, \qquad F_+ = Q_+, \qquad F_- = M_{--} Q_+$$

Field equations give

$$G = \bar{*}\, G \qquad\qquad F = *\, F$$

if M chosen to be:

$$M_{--} = \frac{\mathscr{D}}{1 + \frac{1}{2}\mathscr{D} g^{\lambda\tau}\bar{g}_{\lambda\tau}} g^{++} \qquad \mathscr{D} = \frac{1}{2}[(\bar{g}^{\mu\nu}g_{\mu\nu})^2 - \bar{g}^{\mu\nu}g_{\nu\rho}\bar{g}^{\rho\sigma}g_{\sigma\mu}]$$

$$g^{++} = g^{\mu\nu}\bar{e}^+_\mu \bar{e}^+_\nu$$

The two metrics

- 2 kinds of mass/energy: physical mass and shadow mass
- Treat $g_{\mu\nu}$ as metric tensor field in usual way, giving physical gravitational field
- Conventional: take $\bar{g}_{\mu\nu}$ to be a 2nd metric tensor field, transition functions involve diffeomorphisms $\delta\bar{g}_{\mu\nu} = 2\partial_{(\mu}\xi_{\nu)} + \ldots$

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Diffeomorphism $\phi : x \to x' = \phi(x), \qquad g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = [\phi_* g]_{\mu\nu}(x')$

Infinitesimal: $x'^{\mu} = x^{\mu} - \xi^{\mu} + \ldots, \qquad g'_{\mu\nu}(x) = g_{\mu\nu}(x) + 2\partial_{(\mu}\xi_{\nu)} + \ldots$

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- Conventional: take $\bar{g}_{\mu\nu}$ to be a 2nd metric tensor field, transition functions involve diffeomorphisms $\delta\bar{g}_{\mu\nu} = 2\partial_{(\mu}\xi_{\nu)} + \ldots$
- Unconventional: take it to be a gauge field, allow spin-2 gauge transformations in transition functions: $\delta\bar{g}_{\mu\nu} = 2\partial_{(\mu}\xi_{\nu)} + 2\partial_{(\mu}\chi_{\nu)} + \ldots$

Conventional geometry

$\bar{g}$: conventional tensor on manifold $\mathscr{M}$

$$\bar{g} \in \Gamma(S_2)$$

$$S_2 = (T^* \otimes_{sym} T^*)\mathscr{M}$$

Un-Conventional geometry

Manifold $\mathscr{M}$

Atlas (U_i, ψ_i)

Open cover U_i

Symmetric tensors on each U_i

$\bar{g}_i \in \Gamma(S_i)$

$S_i = (T * \otimes_{sym} T^*)U_i$

On intersection $U_i \cap U_j$

Active diffeomorphism σ_{ij}

On triple intersection $U_i \cap U_j \cap U_k$

$\sigma_{ij}\sigma_{jk}\sigma_{ki} = 1$

Un-Conventional geometry

Manifold $\mathscr{M}$ Atlas (U_i, ψ_i) Open cover U_i

Symmetric tensors on each U_i

$$\bar{g}_i \in \Gamma(S_i) \qquad S_i = (T^* \otimes_{sym} T^*)U_i$$

On intersection $U_i \cap U_j$ Active diffeomorphism σ_{ij}

Transition functions: $\bar{g}_i = (\sigma_{ij})_* \, \bar{g}_j$

If σ_{ij} generated by vector field χ_{ij} $\bar{g}_i = \bar{g}_j + \mathscr{L}_{\chi_{ij}} \bar{g}_j + O(\chi_{ij}^2)$

Unconventional case

- Unconventional: take it to be a gauge field, allow spin-2 gauge transformations in transition functions: $\delta\bar{g}_{\mu\nu} = 2\partial_{(\mu}\xi_{\nu)} + 2\partial_{(\mu}\chi_{\nu)} + \ldots$
- Particular case: $\xi_\mu = -\chi_\mu$: $\qquad \delta\bar{g}_{\mu\nu} = 0$!
- e.g. $\bar{g}_{\mu\nu} = \eta_{\mu\nu}$!

Bi-Metric Geometry

Interpolating Structure $f_\mu{}^\nu$

$$g_{\mu\nu} = f_\mu{}^\rho f_\nu{}^\sigma \bar{g}_{\rho\sigma}$$

Generalisation of vielbein

Map on forms $\Phi : X \to \Phi(X)$

$$\Phi(X)_{\mu_1\ldots\mu_r} = f_{\mu_1}{}^{\alpha_1}\ldots f_{\mu_r}{}^{\alpha_r} X_{\alpha_1\ldots\alpha_r}$$

converts between the two Hodge duals for the two metrics

$$*\,\Phi(X) = \Phi(\bar{*}X)$$

maps $\bar{g}$-self-dual forms to g-self-dual forms

Conclusion

- Sen's action for chiral form fields generalised, OK for general spacetimes
- Extra shadow sector $\bar{g}_{\mu\nu}$, C which decouples from physical fields
- Shadow sector metric $\bar{g}_{\mu\nu}$ can be background or dynamical
- Good for quantum calculations
- Generalises to allow Born-Infeld and Chern-Simons interactions
- Physical form field A isn't a fundamental field, but constructed from P, Q, g, $\bar{g}$
- Bi-metric geometry, tensor gauge fields