Identity and Substitutivity

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Since the publication of Frege's "Über Sinn und Bedeutung,"[1] there has been a good deal of discussion of something variously referred to as Leibniz' Law, Leibniz' Principle, Leibniz' Rule, or—in what one is led to suppose is a reference to the same thing—the Principle of Substitutivity. Much of the discussion has, I think, been interesting and valuable, but I think also that some of it has been marred by a failure to be perfectly clear what the law or principle in question is. Evidently it is something in connection with which it is somehow relevant to talk about 9 and the number of the planets, the Evening Star and the Morning Star, and Giorgione and Barbarelli. But it is not always sufficiently appreciated that whether and how these are relevant to Leibniz' Law depends upon which of several distinct propositions that Law is taken to be.

Let us begin at the beginning, namely, with the passage from Leibniz' writings to which the name 'Leibniz' Law' presumably alludes. In C. I. Lewis' translation this reads as follows:

> Two terms are the *same* if one can be substituted for the other without altering the truth of any statement. If we have A and B and A enters into some true proposition, and the substitution of B for A wherever it appears, results in a new proposition which is likewise true, and if this can be done for every such proposition,

[1] *Zeitschrift für Philosophie und philosophische Kritik,* vol. 100 (1892), pp. 25–50.

are the same, they can be substituted for one another as I have said.[2]

It is doubtful that Leibniz here succeeded in saying what he wanted to say. For one thing, the passage contains an unfortunate confusion of use and mention: words, or expressions, are substituted for one another and not, as Leibniz suggests, the things to which the words refer. For another, his use of the word 'proposition' appears to me to obscure an important distinction. Substitution is an operation performed upon sentences and yielding sentences as values; but, as Leibniz himself urged in other places,[3] it is what is expressed, or formulated, in sentences that is properly said to be true. Allowing, then, for these deficiencies of exposition, we may take Leibniz to have been enunciating the following:

(A) for all expressions α and β, $\ulcorner\alpha = \beta\urcorner$ expresses a true proposition if and only if, for all sentences S and S', if S' is like S save for containing an occurrence of β where S contains an occurrence of α, then S expresses a true proposition only if S' does also.

Even this does not have all the accuracy and precision one might hope for, but I think it will do for present purposes.

Let us agree to use the words 'substitution of β for α is truth preserving' to express the condition which, according to (A), is both necessary and sufficient for $\ulcorner\alpha = \beta\urcorner$ to express a true proposition. Then we may say that (A) is the conjunction of

(B) for all expressions α and β, $\ulcorner\alpha = \beta\urcorner$ expresses a true proposition if substitution of β for α is truth preserving

with

(C) for all expressions α and β, $\ulcorner\alpha = \beta\urcorner$ expresses a true proposition only if substitution of β for α is truth preserving.

Now it should be remarked at once that recent references to "the Principle of Substitutivity" are references to (C) rather than (A). Thus

[2] *A Survey of Symbolic Logic* (New York: Dover, 1960), p. 291.

[3] *E.g., New Essays concerning Human Understanding,* translated by A. G. Langley (LaSalle, Ill.: Open Court, 1949), pp. 450–451.

these words: "given a true statement of identity, one of its two terms may be substituted for the other in any true statement and the result will be true."[4] And, making allowances for what I should regard as an equivocal use of the word 'statement', this amounts to (C) rather than (A). Though historical purists will perhaps regret that (C) is sometimes referred to as "Leibniz' Law," it could hardly be claimed that departing in this way from Leibniz' formulation is of any great consequence: the logical relationships among (A), (B), and (C) are simply too transparent.

There is, however, another departure from Leibniz that is apt to seem a good deal more radical. Frequently what is put forward as "Leibniz' Law" is

(D) if $x = y$, then every property of x is a property of y.

Here, notice, there is no talk of substitution, indeed no talk of expressions at all. We are given instead a necessary condition for an *object* x to be identical with an *object* y. And there would thus appear to be all the difference between (C) and (D) that there is between the world and discourse about it. Yet I think it is often supposed that (D) somehow comes to the same thing as (C), that (D) is only a "material mode" version of (C). So at any rate we might infer, given that either is apt to be called "Leibniz' Law." But is this view correct? Only if (D) implies (C). But *does* (D) imply (C)?

Let us agree to call (C) *the Principle of Substitutivity* and (D) *the Principle of Identity.* My question is, Does the Principle of Identity imply the Principle of Substitutivity? The question can be sharpened with the help of some further terminological conventions. Let S and S' be any sentences. I shall say that the pair (S, S') *is a counterexample to the Principle of Substitutivity* if and only if there are expressions α and β such that (1) $\ulcorner\alpha = \beta\urcorner$ expresses a true proposition, (2) S' is like S save for containing an occurrence of β where S contains an occurrence of α, (3) S expresses a true proposition, and (4) S' expresses a false proposition; and if, in addition, $\ulcorner S \cdot \sim S' \cdot \alpha = \beta\urcorner$ expresses a proposition from which the negation of the Principle of Identity follows, then (and only then) I shall say that the pair (S, S') *falsifies the Principle of Identity.* Now, the Principle of Substitutivity is false if and only if there is a counterexample to it, and the Principle of Identity implies the Principle of Substitutivity if and only if the falsity of the

[4] *From a Logical Point of View,* Second edition (New York and Evanston: Harper and Row, 1963), p. 139.

Principle of Substitutivity implies the falsity of the Principle of Identity. So to ask whether the Principle of Identity implies the Principle of Substitutivity is to ask whether from the proposition that there is a counterexample to the Principle of Substitutivity one can legitimately infer the falsity of the Principle of Identity. But surely such an inference would be legitimate only if any counterexample to the Principle of Substitutivity itself falsified the Principle of Identity. Thus we may appropriately ask, Does every counterexample to the Principle of Substitutivity falsify the Principle of Identity?

In discussing this question it is important to recognize once and for all that there *are* counterexamples to the Principle of Substitutivity. The Principle is simply false. Let S_1 and S_2 be, respectively, 'Giorgione was so-called because of his size' and 'Barbarelli was so-called because of his size'. These are alike, save that S_2 contains the name 'Barbarelli' where S_1 contains the name 'Giorgione', the sentence 'Giorgione = Barbarelli' expresses a true proposition; and S_1 expresses a true proposition and S_2 a false proposition. It follows that the pair (S_1, S_2) is a counterexample to the Principle of Substitutivity and hence that the Principle is false.[5]

Some respond to this by pointing out that the proposition expressed by S_1 is also expressed by the different sentence, "Giorgione was called 'Giorgione' because of his size," and that here substitution of 'Barbarelli' for the first occurrence of 'Giorgione' yields a sentence which, in contrast with S_2, expresses a true proposition. But the proper response to this is: true but irrelevant. For, however it may be with other pairs of sentences, the fact remains that the pair (S_1, S_2) is a counterexample. Again, it is sometimes said that the occurrence of 'Giorgione' in S_1 is not purely referential (not purely designative, oblique). But far from saving the Principle of Substitutivity, this only acknowledges that the pair (S_1, S_2) is indeed a counterexample to it. For we are also told that an occurrence of a name in a sentence counts as purely referential only if substitution for that occurrence of any and every co-designative expression preserves truth value. And, even if accompanied by an independent criterion of purely referential occurrence, this second response is really no more relevant than the first. For the Principle of Substitutivity, as formulated above, contains no qualifications; it purports to cover *all* occurrences of *all* expressions.

The question remains, however, whether the pair (S_1, S_2) falsifies the Principle of Identity. If it does, then from the propositions expressed by S_1 and S_2 it must follow that Giorgione has some property that Barbarelli lacks. What could that property be? Evidently it is not

[5] *Cf.* Quine, *loc. cit.*

the property of being called 'Giorgione' because of one's size, since Giorgione and Barbarelli share that property. Nor will it do to say that it is the property of being so-called because of one's size, for this only invites the question, Being called *what* because of one's size? A more likely suggestion is that the property in question is that which a given object has if and only if the proposition that the object in question is so-called because of its size is a true proposition. Thus it might be suggested that if we let P be the property which a thing x has just in case the proposition that x is so-called because of its size is true, then, since the proposition that Giorgione was so-called because of his size is true, Giorgione has P, and, since the proposition that Barbarelli was so-called because of his size is false, Barbarelli lacks P; and from this, together with the identity of Giorgione with Barbarelli, it might be concluded that the pair (S_1, S_2) falsifies the Principle of Identity.

But the contention that there is such a property as P, possessed by Giorgione though not by Barbarelli, can be seen to be incoherent. The defender of P affirms

(1) Giorgione has P

and can scarcely deny

(2) Giorgione is called 'Barbarelli'.

From (1) and (2) it follows by existential generalization that

(3) there is someone called 'Barbarelli' and he has P;

and, by the proposed definition of P, this is equivalent to

(4) there is someone called 'Barbarelli' and the proposition that he is so-called because of his size is true.

But if we can make sense of (4) at all, we shall have to count it false: no one called 'Barbarelli' is so-called because of his size.

What is the advocate of P to say? He cannot object to the inference from (1) and (2) to (3). Existential generalization on (2) is surely permissible. And to contend that it is not permissible in the case of (1) is in effect to concede that there is no such property as P, for it is absurd to suggest that it is possible that Giorgione should have a certain property and yet that there should not be something that has that property. And, in any case, existential generalization on (1) is essential to the project of deducing the negation of the Principle of

Identity. For that was to be accomplished by arguing that from the propositions that Giorgione has P, that Barbarelli does not, and that Giorgione is identical with Barbarelli, it follows that *there is* something x and something y such that x has P, y does not, and yet x is identical with y.

Perhaps, then, the advocate of P will contend that the English sentence just now used to express (4) is simply not an accurate formulation of the proposition obtained by properly expanding (3) in accordance with the definition of P. He will point out that the expression 'so-called', as it occurs in that sentence, inevitably picks up 'Barbarelli' as antecedent and that accordingly the sentence is naturally read as expressing a proposition from which it follows that someone is called 'Barbarelli' because of his size. Of course, it is unlikely that there is an appropriate English sentence without this defect. So perhaps it will be suggested that we retain the sentence already used but assiduously avert our eyes from the reference back to 'Barbarelli'. Otherwise put, we shall perhaps be told that the proposition obtained by proper definitional expansion of (3) is one from which it follows that

(5) there is someone such that the proposition that he is so-called because of his size is true,

where now 'so-called' stands on its own, free from the misleading suggestions of a surrounding linguistic environment.

But obviously the expression 'so-called' is just the kind of expression that *cannot* thus stand on its own. To make sense of sentences in which it occurs, to determine what propositions they express, it is necessary to look to the environment—linguistic or otherwise—of the expression 'so-called'. And if this fails to reveal a referent, no proposition has as yet been formulated. It was, in part, the failure to recognize this that led to the proposed definition of P. According to that definition, a given object has P just in case the proposition that it is so-called because of its size is true. But how is this to be understood? If we take the expression 'so-called' to have a *fixed* referent—the name 'Giorgione', say—then P will not serve to falsify the Principle of Identity; and if we are to understand that the referent of 'so-called' changes with each difference in choice of name for the given object, then the definition presupposes what is false, namely, that there is such a thing as *the* proposition that the object in question is so-called because of its size.

I suspect I have in a way been attacking a strawman. Perhaps no one would suppose that there is such a property as the alleged P or that the pair (S_1, S_2) falsifies the Principle of Identity. Nevertheless

the attack is not without point. It shows that not every counterexample to the Principle of Substitutivity is a counterexample to the Principle of Identity and therefore that the Principle of Identity does not imply the Principle of Substitutivity. And this, it seems to me, is something that ought to be recognized once and for all.

But of course, for all that has been said so far, it remains possible that *some* counterexamples to the Principle of Substitutivity *do* falsify the Principle of Identity and hence that the Principle of Identity is, like the Principle of Substitutivity, simply false. This view has had its proponents. One of them, the late E. J. Lemmon, wrote as follows:

> . . . '$x = y$' may be true, even though x has an attribute (for example, that of necessarily being x) which y has not got. Thus the morning star, though it *is* the evening star, has the attribute of being necessarily the morning star, which the evening star does not have. This . . . will be unpalatable to many, but I believe it to be a paradox of intensionality that should be accepted on a par with the paradoxes of infinity that we have now come to accept (for example, that a totality may be equinumerous with a proper part of itself). . . . The paradoxes of the infinite are paradoxical only because we normally think in terms of finite classes; this paradox of intensionality is paradoxical only because we normally think, with Leibniz, in extensional terms.[6]

Lemmon's alleged exception to the Principle of Identity at once suggests hosts of others. We can agree that whereas it is a necessary truth that 9 is greater than 7, it is only contingently true that the number of planets is greater than 7; and from this I suppose Lemmon and others of his persuasion would say it follows that 9 has a property the number of the planets lacks; and this in spite of the astronomical fact that 9 *is* the number of the planets. Again, though 9 is identical with 3^2, we may suppose Herbert knows that 9 is greater than 7 but is ignorant of the fact that 3^2 is greater than 7. And from this it will perhaps be concluded that although 9 has the property of being known by Herbert to be greater than 7, 3^2 does not.

Lemmon anticipated—correctly, I think—that many would find his position unpalatable. If y lacks a property x has, then to most people it will seem evident and undeniable that y cannot be the very same object as x. But what is one to say to those few who see the matter differently? I think it wise to concede at once that demonstration is out of the question. To *prove* there are no counterexamples to the Prin-

[6] "A Theory of Attributes Based on Modal Logic," *Acta Philosophica Fennica* (1963), p. 98.

ciple of Identity would require appeal to some more fundamental principle, and it is doubtful that any such is available. Still, there are strategies open to the Leibnizian. He may try to exhibit disturbing consequences of the negation of the Principle of Identity, hoping thereby to present considerations that will at least influence the intellect of the non-Leibnizian. He may try to show that one or another alleged counterexample is not really such. And he may seek to show that the non-Leibnizian is led to his position through bad arguments and intellectual confusions.

Demonstration of the nonequivalence of the Principle of Identity and the Principle of Substitutivity is itself an effort in this direction, for I suspect some have rejected the Principle of Identity only because they have confused it with the Principle of Substitutivity. In what follows I shall attempt further efforts, though of a quite limited nature. What I have to say concerns a single example; and although my discussion of it is somewhat detailed, I doubt that it is exhaustive.

Consider, then, the pair (S_3, S_4), where S_3 is the sentence '9 is necessarily greater than 7' and S_4 the sentence 'the number of planets is necessarily greater than 7'. Now of course my main concern is to determine whether this pair falsifies the Principle of Identity. But I think it will be of some value to attend first to the question whether it really is, as it is usually thought to be, a counterexample to the Principle of Substitutivity. There is a straightforward enough argument: from the premises

(6) S_3 expresses a true proposition if and only if '9 is greater than 7' expresses a necessary proposition,

(7) S_4 expresses a true proposition if and only if 'the number of planets is greater than 7' expresses a necessary proposition,

(8) '9 is greater than 7' expresses a necessary proposition,

and

(9) 'the number of planets is greater than 7' does not express a necessary proposition

it is inferred that

(10) S_3 expresses a true proposition, while S_4 expresses a false proposition;

and this coupled with

(11) '9 = the number of planets' expresses a true proposition

yields the desired conclusion.

The argument is clearly valid, and I shall suppose there is no doubt that (6), (8), (9), and (11) are true. Hence, if (7) is true, the conclusion will have to be granted. But is (7) true?

Those who think it is would perhaps invoke the following general principle:

(E) if α is any singular term and ϕ any predicate expression, $\ulcorner\alpha$ is necessarily $\phi\urcorner$ expresses a true proposition if and only if $\ulcorner\alpha$ is $\phi\urcorner$ expresses a necessary proposition.

And certainly *if* (E) is unexceptionable, (7) has to be counted true. But is (E) unexceptionable? Consider in this connection the sentence, S_5, 'the proposition at the top of page 210 of *Word and Object* is necessarily true'. Does this express a true proposition or not? Notice that, given (E), we can answer without knowing *what* proposition *is* at the top of page 210 of *Word and Object*—indeed, without knowing whether there is any proposition at all at the top of that page. For according to (E), S_5 expresses a true proposition if and only if the sentence, 'the proposition at the top of page 210 of *Word and Object* is true' expresses a necessary proposition. And clearly this last sentence does not express a necessary proposition; that is, the proposition

(12) the proposition at the top of page 210 of *Word and Object* is true

is not a necessary truth. But this shows that something is wrong with (E). Asked whether S_5 expresses a true proposition, we surely have some inclination to suppose that we cannot answer unless we *do* know what proposition appears at the top of page 210 of *Word and Object*. That is, it is altogether natural to take S_5 to express a proposition which is true if and only if

(13) there is exactly one proposition at the top of page 210 of *Word and Object*, which proposition is necessarily true.

And so understood, S_5 expresses a true proposition, for the proposition at the top of page 210 of *Word and Object* is the proposition that for every positive integer x, the class of positive integers less than or equal to x has x members, and this is necessarily true.

There is no need to insist that S_5 *has* to be read in such a way that it expresses a true proposition if and only if (13) is true. No doubt it can also be read in such a way that it expresses a true proposition if and only if (12) is necessary. But then S_5 will have to be counted ambiguous, and it is precisely this ambiguity that is not taken account of in (E).

Now, I think the same sort of ambiguity is present in S_4. No doubt that sentence can be so understood that it expresses a true proposition if and only if the sentence, 'the number of planets is greater than 7' expresses a necessary proposition. And, so understood, it does not express a true proposition since

(14) the number of planets is greater than 7

is not a necessary truth. But I should suppose that S_4 can just as easily be understood in such a way that it expresses a proposition which is true if and only if it is true that

(15) there is a unique number of planets, which number is necessarily greater than 7.

This is the way it would be understood by someone who supposed—what it is perfectly natural to suppose—that one cannot say whether S_4 expresses a true proposition unless one knows *which* number *is* the number of the planets. And understood in this way S_4 expresses a true proposition: There is a unique number of planets and it is necessarily greater than 7.

So, read in one way S_4 expresses a false proposition, and read in another, equally natural way it expresses a true proposition. Is there a similar ambiguity in S_3? I think there is, though I think it occasions no disparity in truth-value. S_3 can be understood *de dicto,* that is, as expressing the proposition that

(16) 9 is greater than 7

is a necessary truth. But it can also be understood *de re,* that is, as asserting of the number 9 that it is necessarily greater than 7. Under either interpretation it seems to me to express a true proposition.

What, then, is to be said of the pair (S_3, S_4)? Is it or is it not a counterexample to the Principle of Substitutivity? The fact is that in the formulation of that principle cases of sentential ambiguity were simply not anticipated. The principle was formulated under the useful

fiction that a sentence expresses at most one proposition. The fiction *is* a useful one. Let us preserve it by leaving the Principle of Substitutivity undisturbed and ruling that S_3 and S_4 are to be understood *de re,* while the new sentences S_6 'necessarily, 9 is greater than 7', and S_7, 'necessarily, the number of the planets is greater than 7' are to be understood *de dicto.* I suspect there is some sanction in English usage for these rulings, but whether there is or not is of little importance once the propositions in question have been distinguished. And thus we may say that whereas the pair (S_6, S_7) is a counterexample to the Principle of Substitutivity, the pair (S_3, S_4) is not.

But the question remains whether the pair (S_6, S_7) falsifies the Principle of Identity. If it does, then from

(17) necessarily, 9 is greater than 7,
(18) $9 =$ the number of planets,

and

(19) not (necessarily the number of planets is greater than 7)

it must follow that 9 has a property that the number of planets lacks. What might this property be? The quick answer is: the property of being necessarily greater than 7. But exactly what property is this? The question is urgent, for we might have supposed that the property of being necessarily greater than 7 is the property which, in S_3 and S_4 is *correctly* attributed to *both* 9 and the number of the planets; and what is presently needed is a property which in S_6 is correctly attributed to 9 but which in S_7 is *in*correctly attributed to the number of the planets. Perhaps we should ask how, in the light of (17) and (19), 9 is supposed to differ from the number of the planets. What is supposed to be true of 9 that is not true of the number of the planets? It might be suggested that in view of (17) it is true of 9 that necessarily it is greater than 7, while in view of (19) it is not true of the number of the planets that necessarily it is greater than 7. Given our conventions concerning the word 'necessarily', the suggestion comes to this: It is true of 9 that the proposition that it is greater than 7 is necessary, but it is not true of the number of planets that the proposition that *it* is greater than 7 is necessary. And so it will perhaps be suggested that if we define Q as the property which a thing x has if and only if the proposition that x is greater than 7 is necessary, then from (17) it will follow that 9 has Q and from (19) it will follow that the number of planets does not have Q.

The suggestion is worth some exploration. The advocate of Q

will of course agree that there is a unique number of planets. This is an immediate consequence of

(20) $(m)(m$ is a number of the planets iff $m = 9)$,

which is simply a fact of astronomy. Something, then, and one thing only, is a number of the planets. Does it have Q or not? This question, which I suppose certainly ought to have an answer, is bound to embarrass the advocate of Q. From (20) and

(21) 9 has Q

it follows that

(22) $(\exists n)((m)(m$ is a number of the planets iff $m = n)$ and n has $Q)$.

But equally, from the undeniable

(23) $(m)(m$ is a number of the planets iff $m =$ the number of planets)

and the non-Leibnizian's

(24) the number of planets lacks Q

it follows that

(25) $(\exists n)((m)(m$ is a number of the planets iff $m = n)$ and n lacks $Q)$.

The advocate of Q is thus committed to both (22) and (25): to the proposition that there is a unique number of planets and it has Q, and to the proposition that there is a unique number of planets and it lacks Q. But anyone who affirms both these is surely ill-equipped to answer the question whether, given that there is a unique number of planets, it has Q or not.

The point is not that (22) and (25) are incompatible. I think they are, but to invoke this would beg the question; for a contradiction follows from the conjunction of (22) and (25) only on the assumption of the Principle of Identity. Nor is the point that on Russell's theory of descriptions (22) is the expansion of

(26) the number of the planets has Q

and thus that the advocate of Q is committed to the very thing he wishes to deny. I suppose it is open to someone simply to reject Russell's theory. The point is rather this: If I am told that exactly one thing numbers the planets, I expect to be able to ask whether it—that number—has Q; and I expect my question to have a determinate answer. But no answer can be given by one who affirms both (22) and (25).

I suspect it will be suggested that my words, 'There is a unique number of planets. Does it have Q?' amount to 'Does the number of planets have Q?' and that this is a question the advocate of Q is quite prepared to answer. After all, one of his claims is that the number of planets lacks Q. Now, I myself do not object to this rephrasing of my question. But I should like it noted that it is just the possibility of this sort of paraphrase that lends credence to Russell's theory—a theory that we have seen the advocate of Q must reject. And in any case, it seems to me that the question needs no paraphrase and that a friend of Q ought himself to find its original formulation perfectly intelligible. Recall that Q is supposed to be the property that an object has just in case the proposition that it is greater than 7 is a necessary truth. Well, there is an object—and one only—that numbers the planets. Can we not consider, then, the proposition that *it* is greater than 7? And should not reflection reveal whether this proposition is a necessary truth? I submit that reflection can reveal nothing better than *both* (22) and (25).

The difficulty originates in what seems to me to be an illegitimate form of definition. We are invited to speak of the property which an object x has if and only if the proposition that x is greater than 7 is a necessary truth. But it ought to be clear by now that it is simply a mistake to suppose that in the case of any given object there is such a thing as *the* proposition that it is greater than 7. Ever so many propositions will qualify as propositions that it, the object in question, is greater than 7. The point is obvious but often overlooked. There is an unfortunate temptation to suppose that it is possible to specify a function, in the mathematical sense, by stipulating that its domain is a particular well-defined class of objects and by stipulating further that, for any element x of that class, the value of the function for the argument x is the proposition that x is such-and-such—greater than 7, or whatever. But the fact is that these stipulations simply do not succeed in specifying a function. Suppose, for example, the domain of the alleged function is to be the class having 9 as sole member and suppose the value for x as argument is to be the proposition that x is greater than 7. What is the value for 9 as argument? If the proposition that 9 is greater than 7 qualifies, so too does the different proposition that the number of the planets is greater than 7; for the number of the planets is the only member of the class whose sole member is 9. Thus the alleged function is not single-valued and hence not properly a function at all.

Let me put the point another way. Consider the propositions

(27) 9 is greater than 7

and

(28) 8 is greater than 7.

In (27) it is said of 9 that it is greater than 7, and in (28) it is said of 8 that *it* is greater than 7. Thus (27) and (28) are alike in that in each it is said of something that it is greater than 7. But that of which this is said in (27) is not the same as that of which this is said in (28). This is how the propositions differ. It is what makes them *two*. In the light of this it is tempting to go on to suppose that (27) can be fully identified by saying that it is the proposition in which it is said of 9 that it is greater than 7: We specify the object concerning which something is said and specify further what is said of it. But the supposition that this succeeds in distinguishing (27) from all other propositions is not true. That of which in (27) something is said is the number 9, that is, the number of the planets; hence (27) has not yet been distinguished from the proposition that the number of the planets is greater than 7.

What strikes me as especially odd in the case of the definition of Q is that those who would use it to show the falsity of the Principle of Identity must implicitly recognize its illegitimacy. They speak, on the one hand, of *the* proposition that x is greater than 7, for arbitrary but unspecified choice of x; yet, on the other hand, it is crucial to their argument that for one and the same object x there be distinct propositions to the effect that x is greater than 7: After all, one such proposition is to be necessarily true, another only contingently so. Were there not such distinct propositions, it could hardly emerge that 9 has Q while something identical with it does not.

Of course, a really determined proponent of Q will not waver in the face of what I have been saying. He will insist that, given any object x, there *is* such a thing as *the* proposition that x is greater than 7. He will insist, in particular, that the necessary truth that 9 is greater than 7 really is identical with the contingent truth that the number of the planets is greater than 7. And he will see in this only another exception to the Principle of Identity. Now frankly this strikes me as a desperation move. But how is one to reply? To show that two things—propositions or any other things—really *are* two, nothing will suffice short of mentioning something true of one of them that is not true of the other. Perhaps in the end all that can be said is that the Principle of Identity is a self-evident truth.