The Radiative Pion Decays.

(History, present situation and possible future researches)

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1 Introduction.

The study of phenomena's beyond Standard Model (SM) of electroweak interactions may give the start of new physics or to deepen our knowledge. The rare decays could be the instruments for decision such problems. There are a lot of theoretical and experimental works devoted to possible deviations from SM. In this short notes radiative decays of the lightest hadron - pion - are regarded. The pion beams may have very high intensity, so it possible to get results up to very low branching ratio. These problems must stimulate to work at the high intensity condition. The proposals CKM2(Fermilab) and OKA(IHEP)correspond this demand. For example, the beam of CKM2 experiment will have 100 MHz of pions. It gives possibilities to investigate rare decays of pions up to very low level of branching ratio. At middle pion energy $\sim 45GeV$ and length of decay volume $\sim 120m$ the number of decayed pions will be approximately equal 5%. Taking into account evaluations given in [1] it is possible to have 500 kevents of radiative pion decays. It is according to $\sim 10^{11} - 10^{12}$ level of Branching Ratio. So OKA and CKM2 experiment will give very valuable possibilities for studing ultra rare decays of pions.

2 Experimental Methods for Study of Rare Decays.

There are two experimental methods of measuring the dacays: the former deals with stopped pions , the later - with decays in flight. Typical layouts are shown on two figures: on fig.1 the layout PIBETA for stopped pions and on fig.2 the ISTRA setup for pion decays in flight. PIBETA and ISTRA setups had been described in details in works [5] and [10].

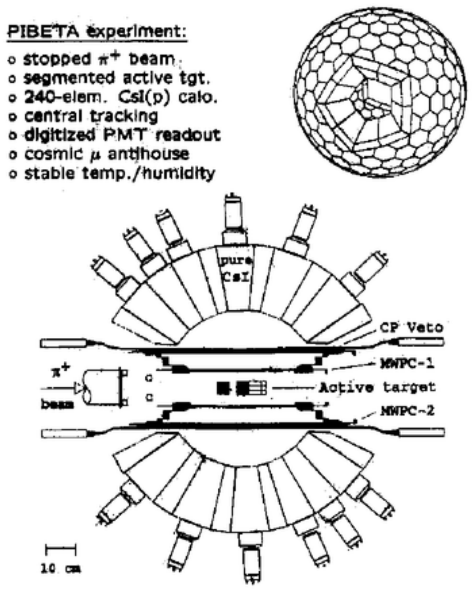

Figure 1: The layout of PIBETA setup

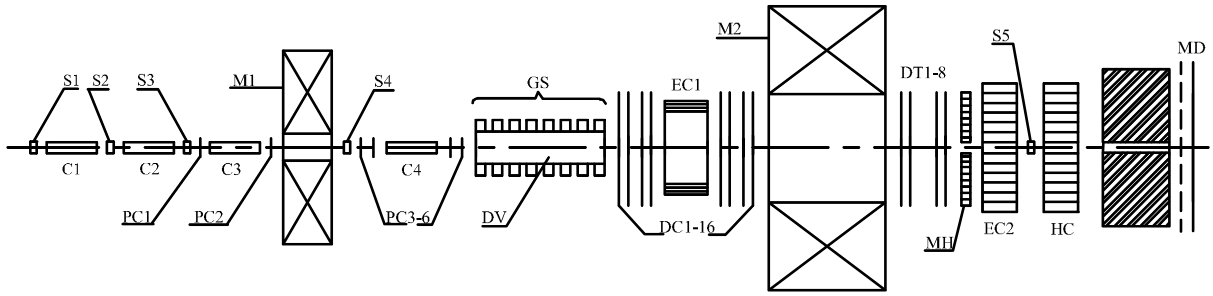

Figure 2: The layout of ISTRA-M setup: S1- S5 -scintillation counters; C1-C4 - Cherenkov gas counter; M1, M2 - beam and spectrometer magnets; PC1-PC6 - proportional chambers; DC1-DC16 - drift chambers; EC1 and EC2 - lead glass electromagnetic calorimeters; DT1-DT8 - drift tubes; MH - matrix hodoscope; HC - hadron calorimeter; MD - muon detector.

The experiment in flight have some advantages compare with stopped pion one. Owing to the high detection efficiency, the wide range of measured angles and energies of secondary particles, and the substantial suppression of the background from the inelastic interactions in the target and from cascade

decays.

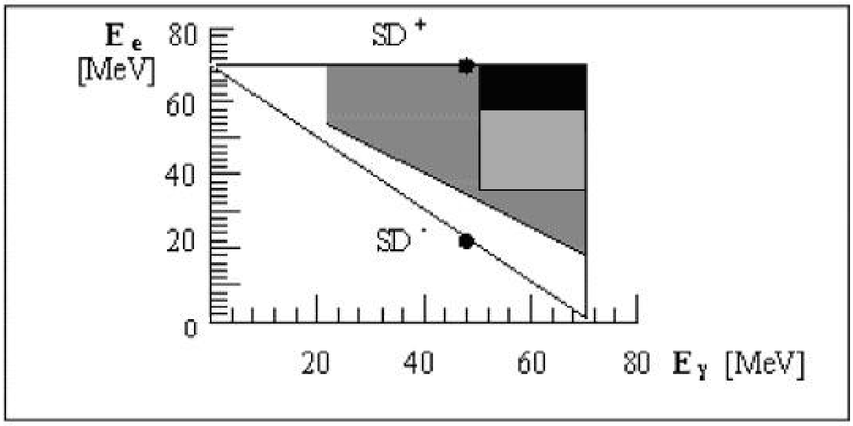

Figure 3: Kinematics regions of decay $\pi \rightarrow e\nu\gamma$ investigated at decay in flight(grey color, ISTRA setup) and in mainly stopped pions (black color). Light grey color(PIBETA).Black points correspond two maximum values of structure dependent terms of matrix elements.

The experiments in flight allow to measure the energy and angle distributions in wide kinematical region. It gives more informative and reliable to measured experimental datas. As example, in fig.3 kinematics regions for decay $\pi \rightarrow e\nu\gamma$ are shown, which could be investigated at stopped pions and in flight.

The CKM2 experiment has all the advantages in flight methodic. It allows to get in the future very important and reliable experimental results.

In table 1 rare decays of the charged pions are given, reflecting possible of the main routs of investigations.

Table 1.Charged pion rare decays.

NN	**Decays**	**Branching ratio** $R = \Gamma_i/\Gamma_{all}$	**Existing inform.**	**Physical problem**
1.	$\pi \to e\nu\gamma$	$(1.61 \pm 0.23)10^{-7}$	$\gamma = F_A/F_V = +0.25 \pm 0.12$ $\theta_{e\gamma} > 135^o$ $\|F_V\| = 0.0259 \pm 0.0005$ from CVC and τ_{π^0} (SIN, 86), [2].; $\gamma = F_A/F_V = +0.52 \pm 0.06$ -2.48 ± 0.06 $\|F_V\| = 0.0259 \pm 0.0005$ from CVC and τ_{π^0} $\theta_{e\gamma} \approx 180^o$ (LAMPF,86),[3]; $\gamma = F_A/F_V = +0.41 \pm 0.23$ $\|F_V\| = 0.014 \pm 0.009$ $F_T = -(5.6 \pm 1.7)10^{-3}$ $0^o < \theta_{e\gamma} < 180^o$ (INR,90),[4]; $\gamma = F_A/F_V = 0.475 \pm 0.018$ $F_T = -(2.2 \pm 0.1)10^{-3}$ $\theta_{e\gamma} > 135^o$ (PSI,03),[5];	1.Measurement of γ and F_V. 2.Study of tensor term existence.
2.	$\pi \to \mu\nu\gamma$	$(2.0 \pm 0.24)10^{-4}$	Three works with low statistic [6]	Problem in the γ low-energy region.
3.	$\pi \to e\nu\pi^0$	$(1.038 \pm 0.004$ $\pm 0.007)10^{-8}$	Well studied. $V_{ud} = 0.9737$ [5]	Check of Stand.M. PDG02 $V_{ud} = 0.9734$
4.	$\pi \to e\nu e^+e^-$	$(3.2 \pm 0.5)10^{-9}$	$F_V = 0.023 \pm 0.014$ $F_A = 0.021 \pm 0.012$ $R = 0.059 \pm 0.008$ 98 ev. [7]	Study of form factors F and R
5.	$\pi \to \mu\nu e^+e^-$	$< 1.6 \cdot 10^{-6}$		Study of form factors F and R, Search T-vaolation

Most of all these processes have been investigated on the stopped pions.

3 Pion rare decays.

Radiation decay $\pi \to e\nu\gamma$ has been investigated in some works [2],[3],[4],[11],[12]. The main subject of studying was the definition of axial-vector F_A and vector formfactors F_V.

The amplitude of the radiative

$$\pi \to e\nu\gamma \tag{1}$$

decay is traditionally described in two terms corresponding to the inner bremsstrahlung (IB) and structure-dependent (SD) radiation. The IB contribution is closely connected with the $\pi \to e\nu$ decay and calculated by using the standard QED methods. The SD term is parameterized by two formfactors (F_V, F_A) that describe the vector (F_V) and the axial-vector(F_A) weak currents.The matrix element terms of decay (1) are given by:

$$M_{IB} = -i\frac{eG_F V_{ud}}{\sqrt{2}} f_\pi m_e \varepsilon^\mu \overline{e}[(k/kq - p/pq)^\mu + \frac{\sigma_{\mu\nu}q^\nu}{2kq}](1+\gamma^5)\nu_e \tag{2}$$

$$M_{SD} = \frac{eG_F V_{ud}}{\sqrt{2}M_\pi}\varepsilon^\mu[F_v e_{\mu\nu\rho\sigma}p^\rho q^\sigma + iF_A(pqg_{\mu\nu} - p_\mu g_\nu)]e\gamma^\nu(1+\gamma^5)\nu_e, \tag{3}$$

where V_{ud} – CKM matrix element, f_π = 131 MeV-const. pion decay, ε^μ-photon polarization vector, p, k, q – 4-momenta of pion, electron and photon; F_V and F_A are vector and axial-vector form factors: $F_{V,A}(t) = F_{V,A}(0)[1 + \Lambda_{V,A}t/{m_\pi}^2]$. The consideration of QCD interactions of ρ and a_1 mesons allows to calculate $\Lambda_V = {m_\pi}^2/{m_\rho}^2 = 0.033 \quad and \quad \Lambda_A = {m_\pi}^2/{m_{a1}}^2 = 0.017$, it is possible to treat $F_{V,A}$ independent from t. Accordingly to CVC, F_V is defined by π^0 life time:

$|F_V| = 1/\alpha\ [\sqrt{2/\pi\ m_{\pi 0}\ T_{\pi 0}}] = 0.0259 \pm 0.0005$

The value F_A-depends on the model and ranges in a wide region from $-3F_V$ to $1.4F_V$ [2] [3] [4] [5] . Usually ratio $\gamma = F_A/F_V$ is considered. The following kinematical variables are used: $x = 2E_\gamma/m_\pi \quad and \quad y = 2E_e/m_\pi$. It is also convenient to use variable $\lambda = (x + y - 1)/x = y sin^2(\theta_{e\gamma}/2)$.

The differential probability $\pi \quad \to e\nu\gamma$ decay is given by

$$\frac{dW_{\pi\to e\nu\gamma}}{dxdy} = \frac{\alpha W_{\pi\to e\nu}}{2\pi}[IB(x,y) + \left(\frac{F_V {m_\pi}^2}{2f_\pi m_e}\right)^2[(1+\gamma)^2 SD^+(x,y) + (1-\gamma)^2 SD^-(x,y)]], \tag{4}$$

where IB and $SD^\pm$ are known functions:

$$IB(x,y) = \frac{(1-y)[(1-x)^2+1]}{x(x+y-1)}; \qquad SD^+(x,y) = (1-x)^2(x+y-1);$$
$$SD^-(x,y) = (1-x)^2(1-y). \tag{5}$$

The first studies did not give possibility to divide two values γ [11],[12](See table 2). With the starting of meson factories and the in-flight experiment the problem was settled. But values of this parameter in mostly statistics provided works differ more than two standard deviations [3],[4](See table 2). Both these works were performed at stopped pions in narrow region of kinematics variables (angles between decaying e and γ is about 180^0). In work [1] investigations were done at angles region $0 - 180^0$. So reliability of definition γ was very high. Probability of decay at this was on three standard deviations less than from Standard Model of V-A weak interactions. Investigated effect could be explained enormously big tensor interaction, which destructively interfering with electromagnet interaction decreases possibility of decay. In works [4] and [13] analysis of existing experimental dates with tensor interactions was done. In table 2 the results of this analysis are shown. It can be seen that with taking into account the interference all values γ in limits of errors are agreeable.

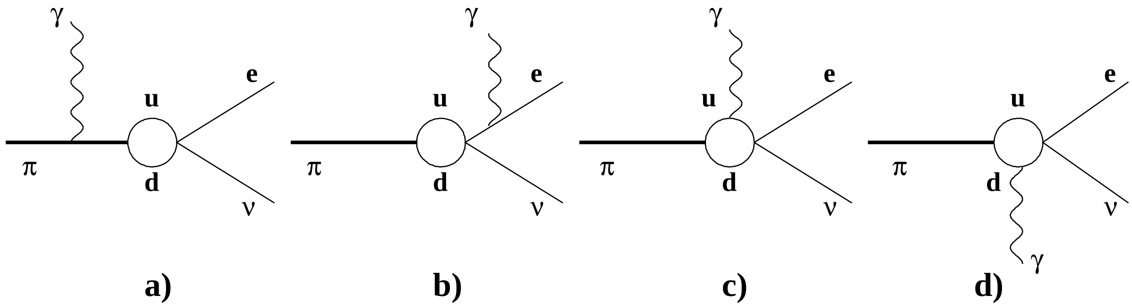

Figure 4: The contributions to the radiative decay $\pi \to e\nu\gamma$ in the framework quark model: a) and b) contain IB; c) and d) SD.

Table 2.Measured and corrected values of γ.

Experiment	$\gamma(F_T = 0)$	$\gamma^*(\mathbf{F_T} = -8.6 \cdot 10^{-3})$
CERN (P. Depommier et al.1963,[11])	0.26 ± 0.15 (-1.98 ± 0.15)	0.66 ± 0.15
Berkeley(A.Stetz et al.1978([12])	0.48 ± 0.12 (-2.42 ± 0.12)	0.60 ± 0.12
SIN (A.Bay et all.1986,[2])	0.52 ± 0.06	0.63 ± 0.06
LAMPF (L.Philonen et al.1986, [3])	0.22 ± 0.15	0.48 ± 0.15
INR (V.Bolotov et al.1990, [4])	0.41 ± 0.23	0.41 ± 0.23

*) Values γ were corrected with taking into account interference and kinematical region for each experiment.

Theoretical works exist in which the antisimmetrical tensor fields are regarded [14].

In range of simple quark model (fig.4) matrix element with tensor interaction could be written:

$$M_{\pi\to e\nu\gamma} = M_{IB} + M_{SD} + M_T; \quad (6)$$

The tensor interaction may be simulated by adding tensor radiation term to the structure dependent amplitude:

$$M_T = i(eG_F V_{ud}/\sqrt{2})\varepsilon^\mu q^\nu F_T u(p_e)\sigma_{\mu\nu}(1+\gamma^5)\nu(p_\nu) \quad (7)$$

The decay rate densities for the SD^- radiation and the interference term between the inner bremsstrahlung and the tensor radiation are similar, so destructive interference may reproduce the results of fit, giving $F_T = -(5.6 \pm 1.7) \cdot 10^{-3}$. This value does not contradict the listed constraints on a tensor coupling from nuclear beta decay as well as from muon decay (if universality is supposed). This result does not contradict the previous experiments carried out with stopped pions either [13].

Several works [14] were devoted to the study of possible deviation from SM in radiative pion decay. In one of them the involving of antisymmetric tensor fields into the standard electroweak theory allows to explain results of this work. It is evident that additional experimental and theoretical investigation of this problem should be carried out.

In work [15] the authors show that thereis region of phase space at large photon energies where the main physical bacground from muon decay is absent and that is optimal for searching a tensor interaction. In the analysis, it is convenient to describe the differential branching ratio as a function of the photon energy $x = 2E_\gamma/m_\pi$ and the variable $\lambda = (x + y - 1)/x = y\sin^2(\Theta_{e\gamma}/2)$. To formula (4) is added tensor interferentional terms:

$$\frac{dW_{\pi\to e\nu\gamma}}{dxdy} = \frac{\alpha W_{\pi\to e\nu}}{2\pi}\{IB(x,y) + \left(\frac{F_V {m_\pi}^2}{2f_\pi m_e}\right)^2 [(1+\gamma)^2 SD^+(x,y) + \tag{8}$$

$$+(1-\gamma)^2 SD^-(x,y)]\} + \frac{F_{T2}}{f_\pi m_e} T_1(x,\lambda) + \left(\frac{F_{T2}}{f_\pi m_e}\right)^2 T_2(x,\lambda).$$

Here all terms are independently on x and λ (see fig.5);

$$IB(x,y) = \frac{(1-\lambda)}{\lambda}\frac{(1-x)^2+1}{x}; \quad SD^+(x,\lambda) = \lambda^2 x^3(1-x); \tag{9}$$

$$SD^-(x,\lambda) = (1-\lambda)^2 x^3(1-x);\ T_1(x,\lambda) = (1-\lambda)x;\ T_2 = \lambda(1-\lambda)x^3.$$

From fig.5 one could see that the most suitable region searching a tensor interaction is $0.9 \le x \le 1.0$ and that large opening angles between electron and photon (the experiments with stopped pions) correspond to the suppression of tensor interaction.

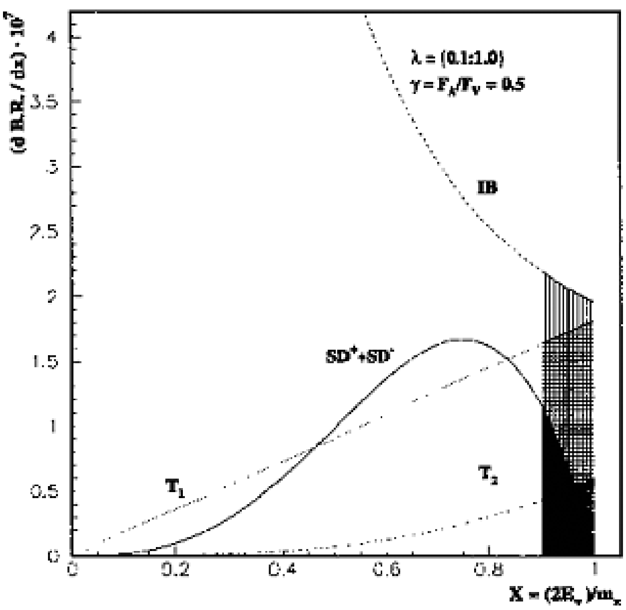

Figure 5: The dependences of the differential branching ratio as a function of the photon energy x.

In work [10] the possibilities of the study of the tensor interaction at ISTRA setup were investigated. Fig.6 illustrates the distribution of a kinematical variable $\boldsymbol{\lambda}$ in the case of the existence of tensor interaction and without it.

In fig.6 given: results of decay $\boldsymbol{\pi \to e\nu\gamma}$ events simulated by Monte-Carlo method received at setup OKA in high energy region of variable $\mathbf{0.9} \leq \boldsymbol{x} = \mathbf{2}\boldsymbol{E/m} \leq \mathbf{1.0}.$ It can be seen, that even at low statistics of events existence of tensor interaction could be defined

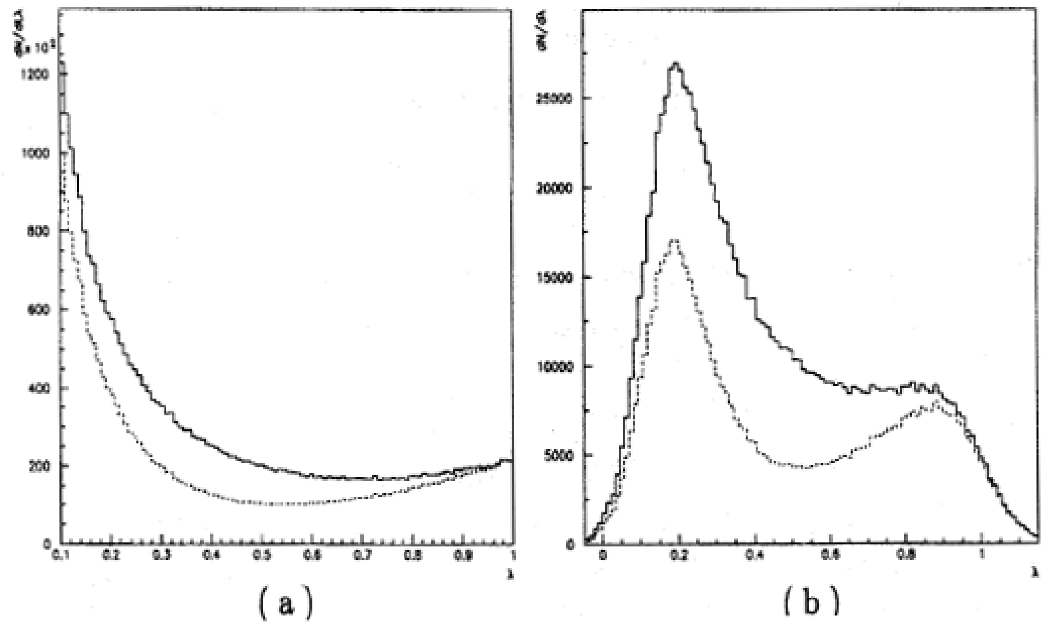

Figure 6: The distribution of the kinematical variable $\boldsymbol{\lambda}$ theoretical (a) and with experimental errors taken into account (b). Dashed line corresponds to the existence of the tensor interaction; solid line to absence of this interaction.

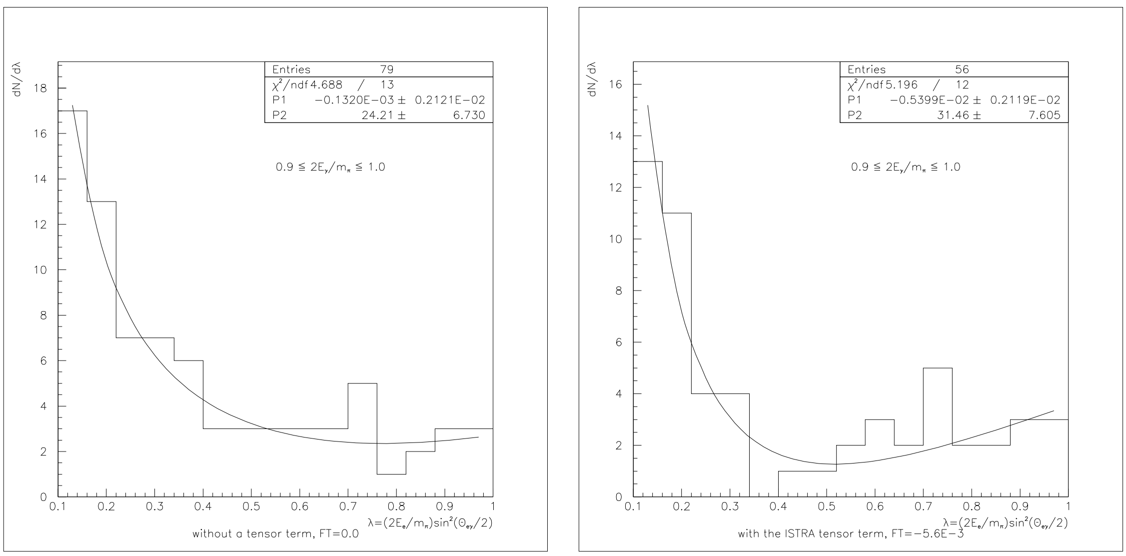

Figure 7: Distribution of radiative decay $\boldsymbol{\pi \to e\nu\gamma}$ events vs variable $\boldsymbol{\lambda}.$ Events are chosen for other variable $\boldsymbol{x}$ in high energy region of decaying $\boldsymbol{\gamma}(\mathbf{0.9} < \boldsymbol{x} < \mathbf{1.0}).$ Calculations were performed by Monte-Carlo method for one variant OKA setup.

Recently a report on preliminary results performed by the PIBETA Collaboration from PSI meson factory has appeared [5]. The fits were made in two-demensional kinematic space of $x = 2E_\gamma/m_\pi$ and $\lambda = (x + y - 1)/x = y\sin^2(\Theta_{e\gamma}/2)$ on a very large statistical material (60k events $\pi \rightarrow e\nu\gamma$ decays). Fitting experimental data requires $F_T \neq 0 (F_T \cong -0.0017 \pm 0.0001)$. Here is the quotation from PIBETA Annual Progress Report: " Thus, like the ISTRA data, our data appear to call for a destructive interference between the IB term and a small negative tensor amplitude."

The results from ISTRA and PIBETA setups don't solve the problem. That study should be continued.

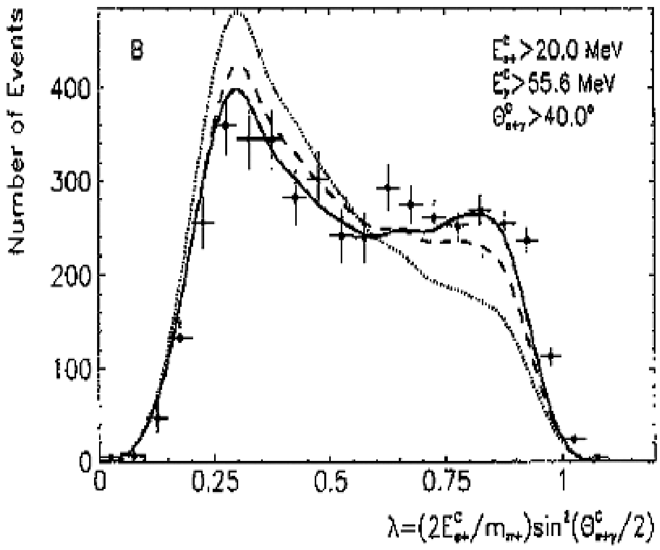

Figure 8: Measured spectrum of the kinematical variables λ in $\pi \rightarrow e\nu\gamma$ decay. Dotted curve: fit with F_V fixed by CVC hypothesis, and F_A taken from PDG 2002 compilation. Dashed curve: fit with F_V constrained by CVC, and F_A and F_T unconstrained. The resulting value for F_T is $-0.0017 \pm 0.0001.$ Preliminary results - work in progress.

If hypothesis electromagnetic neutrino momentum (DM) is true so interesting consequence for radiative decay $\pi \rightarrow e\nu\gamma$ is investigated [16]. If neutrino of Majorana fermion type exists, the amplitude of dipole momentum interfering with structure dependent part can cause distortion of energetic specters of the secondary electrons and photons (Fig.11). Assuming the neutrinos have masses and electric and magnetic dipole moments DM the addinional terms to (2), (3) of Standard Model amplitudes appear:

$$\overline{|M_{\pi\rightarrow e\nu\gamma}|^2} = \overline{M_{IB}^2} + \overline{|M_{CD}|^2} + \overline{|M_{DM}|^2} + 2Re\overline{(M^+{}_{IB}M_{DM})} + 2Re\overline{(M^+{}_{SM}M_{DM})}, \tag{10}$$

$$Re(M_{IB}^+ M_{DM}) \text{ and } Re(M_{SM}^+ M_{DM}) \sim Im\frac{\mu_{ji} + id_{ji}}{\mu_B} \text{ and } \triangle m_{ji}{}^2 = m_i{}^2 - m_j{}^2.$$

The conditions $Im(\mu_{ji} + id_{ji}) \neq 0$ and $\triangle m_{ji}{}^2 = m_i{}^2 - m_j{}^2 \neq 0$ may fulfill only for Majorana neutrinos.
In this case the outcoming photon can be attached to the neutrino leg (Fig.9). Imaginary part of decay amplitude (Fig.10) also may be nonzero if $Im(\mu_{ji} + id_{ji}) \neq 0.$

Hitherto the experimental limits on magnetic and the electric DM are [17],[18]:

$|p_{\nu(e)} + id_{\nu(e)}| < 1.5 \cdot 10^{-10}\mu_B, \quad |\mu_{\nu(\mu)} + id_{\nu(\mu)}| < 1.2 \cdot 10^{-9}\mu_B,$
$|\mu_{\nu(\tau)} + id_{\nu(\tau)}| < 4 \cdot 10^{-6}\mu_B$, where $\mu_B = e^2/2m_e$ is Bohr magneton.

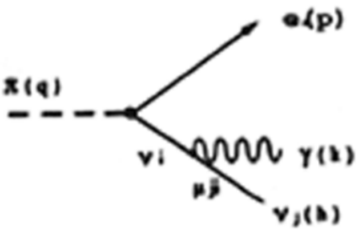

Figure 9: Feynman diagram for the process $\pi \to e\nu\gamma$ when the photon is emitted from the neutrino leg.

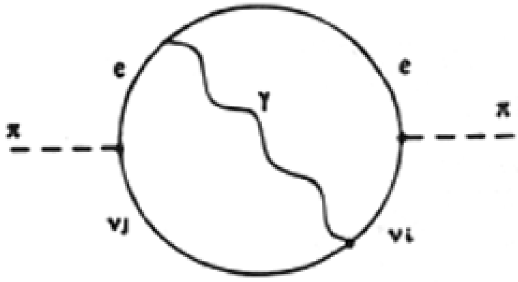

Figure 10: Loop diagram for the lowest-order contribution to the radiative pion decay through neutrino nondiagonal DM.

The distributions on fig.11 authors [16] have got assuming experimental limits from $\pi \to$ e ν_j decay [19], a mixing matrix element $U_{e3} = 10^{-2}$, a neutrino $m_3 = 5MeV$ and a neutrino magnetic DM $\mu_{13} = 4 \cdot 10^{-6}\mu_B$. To improve experimental limit on μ_{13} it is necessary to get the level of branching ratio of 10^{-12}. It is difficult but possible.

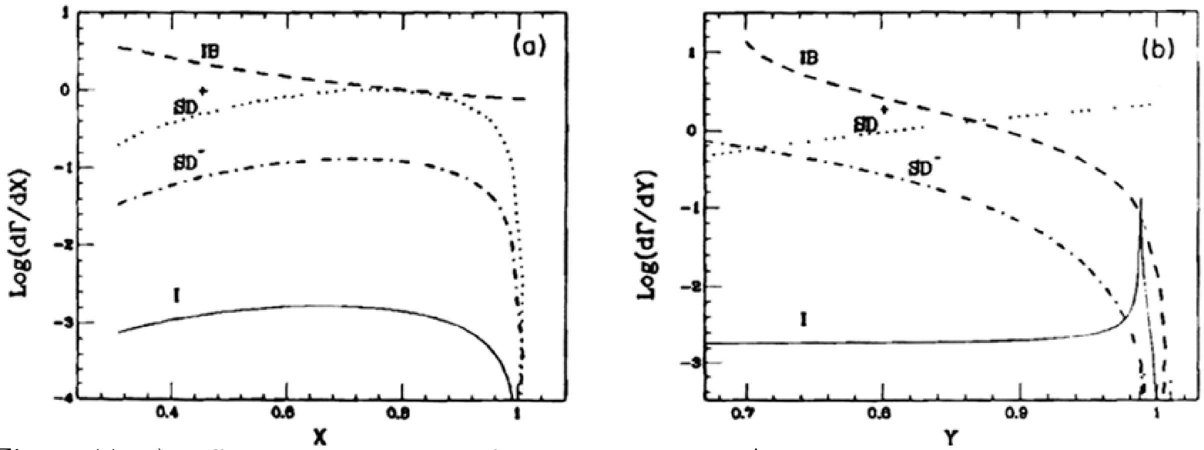

Figure 11: a) Differential probability of radiative decay $d\Gamma/dx$, integrated in region $1-0.8x \leq y \leq 1+r^2$, where $r = m_e/m_\pi$ Values $d\Gamma/dx$ are free. For interferencial SD-IB term absolute values are given. b)the same as a) for $d\Gamma/dx$ - in region $0.3 \leq x \leq 1-r^2$.

Probability of these decays is very low $< (1.4 - 5.1)10^{-15}$.

Checking T-invariance. The decays $\pi \to l\nu\gamma$, where $l = \mu, e$, may serve for testing T-violating interactions beyond the Standard Model.$\pi \to e\nu\gamma$ and $\pi \to e\nu e^+e^-$ must be real. In case of violation T-invariance they have imaginary part. At maximum of violation the value of imaginary part must be equal to real part. In this case T-odd correlations must appear. In case $\pi \to e\nu\gamma$ decay they are expressed by polarization vectors of positron (σ) and photon (ϵ) : $\overline{\sigma}(\overline{k} \times \overline{p})$ (A) $\overline{\epsilon}(\overline{k} \times \overline{p})$ (B). Correlation (A) is expressed only via transverse positron polarization P_T . In some spaces of phase volume P_T can get 67% [17].

In work [20] for these decays it was investigated the possibilities of measuring the transverse lepton polarization asymmetry:

$$P_T(x, y) = [d\Gamma(\overline{e}_T) - d\Gamma(-\overline{e}_T)]/[d\Gamma(\overline{e}_T) + d\Gamma(-\overline{e}_T)],$$

where polarization in direction $\overline{e}_T = [\overline{k} \times \overline{q}]/|\overline{k} \times \overline{q}|$ and k, q - are 4-momenta of lepton and photon. On figures 12 and 13 are shown the distributions of differential branching ratio of $\pi \to \mu\nu\gamma$ and $\pi \to e\nu\gamma$ decays over the Dalitz plot and the transverse lepton polarization asymmetry $\boldsymbol{P_T}$ as background from Standard Model interactions.It can see, that regions with large $\boldsymbol{P_T}$ overlap with regions large branching ratio of decays.

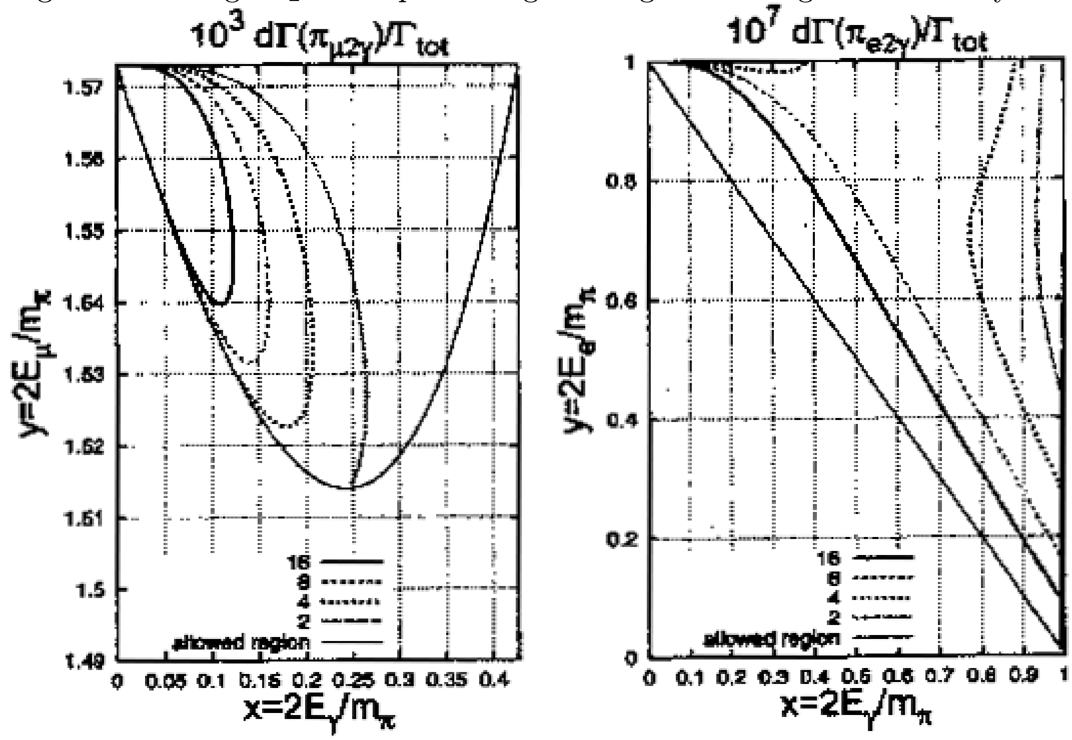

Figure 12: The distribution of differential branching ratio of $\boldsymbol{\pi_{\mu 2\gamma}}$ and $\boldsymbol{\pi_{e2\gamma}}$ decays over the Dalitz plot.

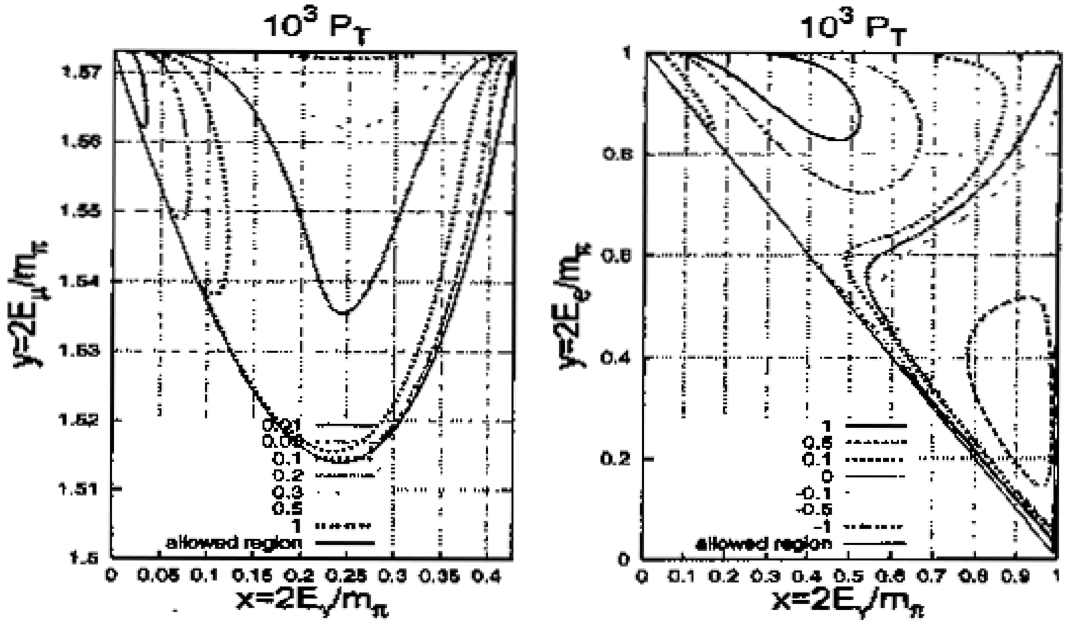

Figure 13: Transverse lepton polarization due to FSI in $\boldsymbol{\pi_{\mu 2\gamma}}$ and $\boldsymbol{\pi_{e2\gamma}}$ decays.

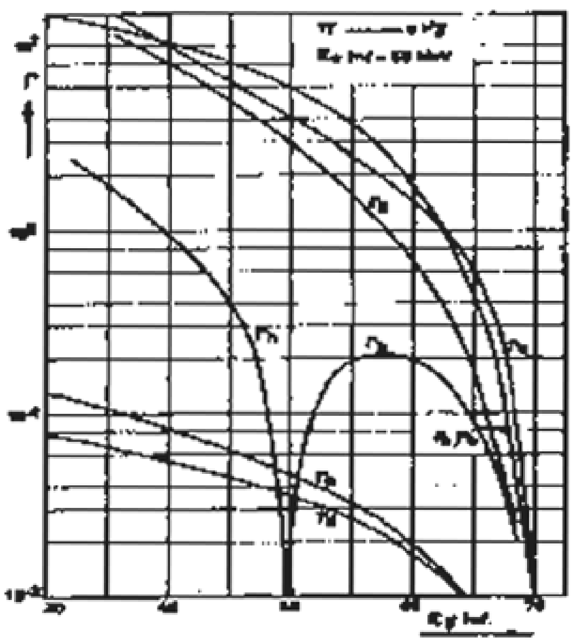

Figure 14: The absolute value of $\mathbf{\Gamma}_i$ the contributions to the total decay rate (i=a(SD),b(SD),c(IB), d and e(INT)) and to the total polarization (i=g,h)

The transverse lepton polarization asymmetry $\boldsymbol{P_T}$ is impossible to measure in flight experiments because the lepton energies to high.It is possible to study polarization in $\mathbf{\Gamma}_g$ and $\mathbf{\Gamma}_h$ contributions to the total polarization as function of photon energy (see Fig.14)[21].

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