The Dynamics of Mathematical Reason

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Why are mathematics and the natural sciences typically treated so differently by analytic philosophers?

Interesting comparisons are possible

Kuhn, Lakatos, Polya,…

...MacIntyre, Collingwood, Cassirer, Shapere

One choice to make for a case study is the scale:

A research project, program, tradition, ...

It would be easy to box oneself into a tiny new-wave ‘practice-oriented’ corner of the small subdiscipline that is *the philosophy of mathematics*.

Looking for something more philosophically weighty to come out of a historical treatment of mathematics, several years ago I spotted a possibility to develop the ideas of Michael Friedman.

Michael Friedman

Michael Friedman

- Foundations of Space-Time Theories: Relativistic Physics and Philosophy of Science (1986)
- Kant and the Exact Sciences (1992)
- Reconsidering Logical Positivism (1999)
- A Parting of the Ways: Carnap, Cassirer, and Heidegger (2000)
- Dynamics of Reason (2001)
- Kant's Construction of Nature (2013)

What I call the dynamics of reason is an approach to the history and philosophy of science developed in response to Thomas Kuhn’s theory of scientific revolutions. Unlike many philosophical responses to Kuhn, however, my approach, like Kuhn’s, is essentially historical. Yet Kuhn’s historiography, from my point of view, is much too narrow. Whereas Kuhn focusses primarily on the development of the modern physical sciences from the Copernican revolution to Einsteinian relativity theory, I construct an historical narrative depicting the interplay between the development of the modern exact sciences from Newton to Einstein, on the one side, and the parallel development of modern scientific philosophy from Kant through logical empiricism, on the other. I use this narrative to support a neo-Kantian philosophical conception of the nature of the sciences in question—which, in particular, aims to give an account of the distinctive intersubjective rationality these sciences can justly claim. By contrast, <u>Kuhn’s picture led to philosophical challenges to this claim, I argue, precisely because he left out the parallel history of scientific philosophy</u>.

(Extending the Dynamics of Reason, 2011: 431)

I want to make clear how the neo-Kantian conception in question presents us with a fundamentally historicized version of scientific intersubjective rationality, so that the standards of objectivity in question are always local and contextual. Nevertheless, in spite of, and even because of, this necessary historicization, the way in which such standards change over time still preserves the trans-historical rationality of the entire process.

(Extending the Dynamics of Reason, 2011: 432)

How can a discipline replace its fundamental concepts from time to time and yet do so in a rational manner?

Friedman is hoping to provide a form of rational narrative for a special kind of intellectual enquiry – the modern exact sciences – which has shaped, and been shaped by, 'scientific philosophy'.

Friedman's schema:

One stage of mathematical physics

Mathematical language: Infinite 3D Euclidean space + calculus
Coordinating principles: Newton's Laws of motion
Empirical laws and regularities: Law of Gravitation, inertial mass = gravitational mass

replaced by another

Mathematical language: 4D-pseudo Riemannian manifold + tensor calculus
Coordinating principles: Invariance of speed of light, Einstein's equivalence principle, freefall as geodesic motion.
Empirical laws and regularities: Field equations, approximately flat time slices.

Contra Quine, we have a *historicized* a priori, theories play *constitutive* roles.

During a revolution, propositions may change their status:

•**Promotion:** A contingent fact of the Newtonian universe, that the inertial mass and the gravitational mass are the same, becomes a constitutive principle in the Einsteinian picture.

•**Demotion:** On the other hand, the constitutive lack of curvature of the Newtonian universe becomes an approximately true, but in places false, description of this universe.

Agreeing with Kuhn, while it is possible to reconstruct Newtonian physics as an empirical possibility in the new scheme, allowing us to reject it through observation, this is a radical reworking. There is a <u>retrospective</u> rationality, which reinterprets the former theory as a special case, but this gives no clue as to how the new framework emerged out of the old.

But Friedman wants a <u>prospective</u> rationality too. Radical incommensurability is wrong.

In the case of general relativity, the associated meta-scientific work was carried out by Helmholtz, Mach, and Poincaré, stretching the Kantian schematism in light of the transformations of geometry by Riemann, Lie and Klein, and in Helmholtz' case his own psychophysical research. Poincaré's meta-scientific work was conducted in the context of his conventionalist philosophy.

After the Einsteinian revolution, philosophers went to work trying to make sense of it.

In the process of doing so, Schlick, Reichenbach and Carnap were led to important innovations in philosophy.

So, any more of these episodes?

A similar account is given by Friedman of the Newtonian revolution.

• The invention of the calculus and its later development by Euler *et al*.; the meta-scientific spadework refining notions of motion, space, force, ..., being done by Galileo, Descartes, Leibniz and Newton himself.

• Then there is Kant at the other end of the revolution giving a philosophical shape to Newtonianism and separating philosophy from natural science in the process.

"...Kant's assimilation of Newton, refracted through Leibniz's complex set of ambitions in physics, metaphysics, politics, and theology, eventually led to the radical new idea of a purely moral religion." (442)

What of quantum mechanics? According to Friedman:

• Retrospective rationality was certainly achieved through a representation of the old science within the new.

• But while quantum mechanics has been hugely empirically successful, philosophical contributions have not been ‘timely’ (DR: 120-121).

• Ad hoc philosophical speculations of Wigner, Schroedinger, …

• Best prospect relates to Birkhoff and von Neumann's ideas on quantum logic.

However, as one logician writes:

“Among the magisterial mistakes of logic, one will first mention quantum logic, whose ridiculousness can only be ascribed to a feeling of superiority of the language – and ideas, even bad, as soon as they take a written form – over the physical world. Quantum logic is indeed a sort of punishment inflicted on nature, guilty of not yielding to the prejudices of logicians… just like Xerxes had the Hellespont – which had destroyed a boat bridge – whipped.” (Girard 2011, page xii)

“...the notorious quantum logic – an expression of the style <<popular democracy>>, where the role of the adjective is to negate the noun.” (Girard 2011, p. 369)

The Blind Spot: Lectures on Logic

This gesture towards quantum logic as the kind of change of the mathematico-logical basis to be expected should be treated with scepticism.

It seems to arise from too heavy a reliance on the 'set theory + classical first-order logic' framing of mathematics, requiring a change to the logic for his schema to work.

But what else could the next instantiation of the Friedmannian schema look like? What new mathematics would it involve?

It seems that we may not have to guess, since it may well have arrived.

Mathematical language: Differential cohomology

Coordinating principles: Quantum gauge field theory

Empirical laws and regularities: M-theory: The C-field 4-flux and 7-flux forms are subject to charge quantization in J-twisted Cohomotopy cohomology theory (U. Schreiber et al.)

Even without considering the physics, there’s important material to think through with regard to the development of the mathematics.

In particular, we might want to inspect the claim made by Friedman of its difference from physics:

In pure mathematics, however, there is a very clear sense in which an earlier conceptual framework (such as classical Euclidean geometry) is always translatable into a later one (such as the Riemannian theory of manifolds). In the case of coordinating principles in mathematical physics, however, the situation is quite different. To move to a new set of coordinating principles in a new constitutive framework (given by the principle of equivalence, for example): what counted as coordinating principles in the old framework now hold only (and approximately) as empirical laws, and the old constitutive framework, for precisely this reason, cannot be recovered as such. By embedding the old constitutive framework within a new expanded space of possibilities it has, at the same time, entirely lost its constitutive (possibly defining) role.

(DR, p. 99)

My objections in 2005 (<u>philsci archive</u>):

Ought we not tell of similar patterns of change in mathematics? Rather than speculative thoughts on quantum logic, why not point to actual, successful changes to the foundations of mathematics?

Don't we find a similar obstruction to perfect translatability, and similar shifts in constitutive function?

My idea at the time was to consider (higher) category theory:

"*A great deal of modern mathematics, by no means just algebraic topology, would quite literally be unthinkable without the language of categories, functors, and natural transformations introduced by Eilenberg and MacLane in their 1945 paper.*" (May 2000:11)

Since 2005 matters have come into sharper focus.

(Twisted, equivariant) differential cohomology finds its natural home in cohesive higher toposes.

The appropriate formal language here is *modal homotopy type theory*.

This represents a sufficiently radical change from the classical logic + ZFC picture.

Let’s look at some of the **mathematical** steps in the development of cohomology.

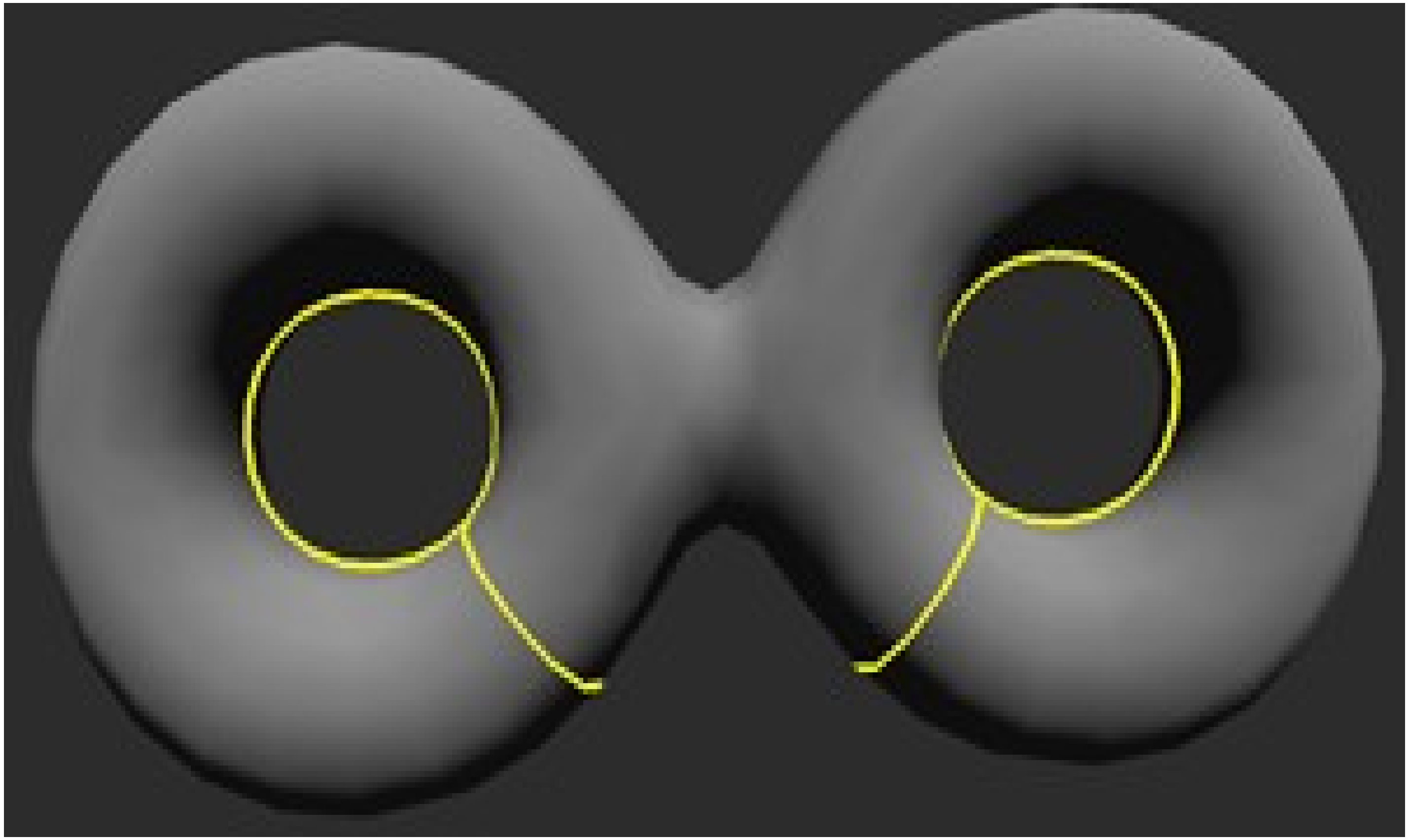
In **1930s**

- Constitutive language: algebra and topology, as set-theoretic.
- Theories: various defined homology and cohomology theories, associating algebraic entities to spaces
- Observations: some regularities found, e.g., (simpler) spaces give the same results for any theory, homotopy invariance.

By **1952**

- Constitutive language: category theory
- Theories: axiomatised (co)homology, Eilenberg-Steenrod axioms, includes some observed properties from earlier as axioms.
- Observations: Čech 'homology' no longer a homology

Homology for measuring holes



Cohomology is richer

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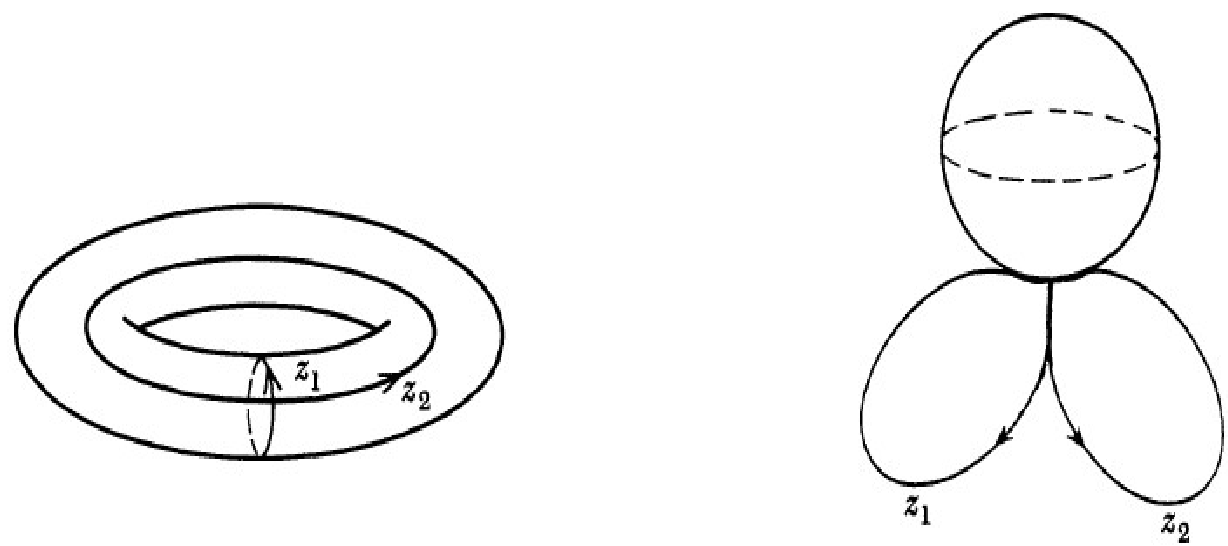

FIGURE 6

The torus and the "fake torus" consisting of two circles and a sphere joined at a point. The product $z_1 z_2$ on the torus gives the torus itself, but $z_1 z_2 = 0$ on the fake.

Eilenberg-Steenrod Axioms

A family of functors $H_n(\cdot)$ from the category of pairs of topological spaces and continuous maps, to the category of Abelian groups and group homomorphisms satisfies the Eilenberg-Steenrod axioms if the following conditions hold.

1. long exact sequence of a pair axiom. For every pair (X, A), there is a natural long exact sequence

$$\ldots \to H_n(A) \to H_n(X) \to H_n(X, A) \to H_{n-1}(A) \to \ldots,$$

where the map $H_n(A) \to H_n(X)$ is induced by the inclusion map $A \to X$ and $H_n(X) \to H_n(X, A)$ is induced by the inclusion map $(X, \phi) \to (X, A)$. The map $H_n(X, A) \to H_{n-1}(A)$ is called the boundary map.

2. homotopy axiom. If $f : (X, A) \to (Y, B)$ is homotopic to $g : (X, A) \to (Y, B)$, then their induced maps $f_* : H_n(X, A) \to H_n(Y, B)$ and $g_* : H_n(X, A) \to H_n(Y, B)$ are the same.

3. excision axiom. If X is a space with subspaces A and U such that the set closure of U is contained in the interior of A, then the inclusion map $(X \backslash U, A \backslash U) \to (X, A)$ induces an isomorphism $H_n(X\ U, A\ U) \to H_n(X, A)$.

4. dimension axiom. Let X be a single point space. $H_n(X) = 0$ unless $n = 0$, in which case $H_0(X) = G$ where G are some groups. The H_0 are called the coefficients of the homology theory $H(\cdot)$.

These are the axioms for a generalized homology theory. For a cohomology theory, instead of requiring that $H(\cdot)$ be a functor, it is required to be a co-functor (meaning the induced map points in the opposite direction). With that modification, the axioms are essentially the same (except that all the induced maps point backwards).

Category theory becomes a constitutive language allowing a definition of what it is to be a (co)homology theory.

Čech homology is no longer a homology theory.

Translation from the earlier phase is not perfect. (Cf. Strong homology.)

In the second half of the 20th century, the scope of cohomology expanded enormously.

“The origins of cohomology theory are found in topology and algebra at the beginning of the last century but since then it has become a tool of nearly every branch of mathematics. It’s a way of life!” Ulrike Tillmann, Cohomology Theories

Cohomology can classify extensions and indicate obstructions. Local gluing of pieces may be globally obstructed.


AR CARE


AR CARE

A 2-cocycle allows us to extend single digit addition to double digits

TABLE 1. Carrying is required exactly when the ones digits sum to 10 or more.

z	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0	1	1
3	0	0	0	0	0	0	0	1	1	1
4	0	0	0	0	0	0	1	1	1	1
5	0	0	0	0	0	1	1	1	1	1
6	0	0	0	0	1	1	1	1	1	1
7	0	0	0	1	1	1	1	1	1	1
8	0	0	1	1	1	1	1	1	1	1
9	0	1	1	1	1	1	1	1	1	1

Applications in number theory, due to Weil and others.

Had there been a rational solution to

$$p^2 + q^2 = 1$$

not of the form

$$p = 2ab/(a^2 + b^2),\ q = (a^2 - b^2)/(a^2 + b^2),$$

there would be non-zero Galois cohomology where there can't be.

(Hilbert's Theorem 90)

In applications:

“The Condorcet paradox that individually consistent comparative rankings can lead to global inconsistencies is a favorite topic in voting theory. Its best explanation cohomology is less popular.” (Ghrist, Elementary Applied Topology)

“Cohomology is deeply routed (sic) in the following topics: Gauss’ surface theory, the Kirchhoff-Weyl theory of electrical networks, and Maxwell’s theory of electromagnetism. Cohomology lies at the heart of both modern differential topology and modern quantum field theory (the BRST approach).” (Zeidler)

The total mass of a classical system may be interpreted “as the cohomology class of a Galilean group one-cocycle and the obstruction to equivariance”. Mass “has a cohomological significance, it parametrizes the extensions of the Galileo group.” (Santiago Garcıa, hep-th/9306040)

Special Semester on Cohomology in Arithmetic, Fall 2020

Homological tools and ideas are pervasive in number theory. To defend this assertion, it suffices to evoke the role of **étale cohomology** in the study of the zeta functions of varieties over finite fields through the Weil conjectures, or the **cohomological approach to class field theory** formulated by Artin and Tate in the 1950's. The theory of motives, a manifestation of a **universal cohomology theory** attached to algebraic varieties, and the attendant **motivic cohomology** plays a central role in describing the special values of L-functions of varieties over number fields, via the conjectures of Deligne, Beilinson-Bloch, and Bloch-Kato. Much progress in the Langlands program exploits the fruitful connection between automorphic representations and the **cohomology** of associated Shimura varieties and more general arithmetic quotients of locally symmetric spaces. The study of special values of L-functions and the Langlands program, widely perceived as two fundamental yet separate strands of the subject in the early 1990's, were beautifully unified in Wiles' epoch-making proof of the Shimura-Taniyama conjecture, in which this conjecture was reduced to a special instance of the Bloch-Kato conjecture for the symmetric square motive of an elliptic curve. Recent years have seen great strides in our understanding of the **cohomology** of the arithmetic quotients arising in the study of automorphic representations, spurred in part by the desire to extend the range of applicability of the celebrated Taylor-Wiles method. This has led to new automorphy and potential automorphy results: most spectacularly, perhaps, for abelian surfaces, as well as elliptic curves over general CM fields

Etale cohomology, various versions of algebraic K-theory, the concept of ”arithmetic vs. geometric” cohomology theories, absolute Hodge cohomology, Hodge cohomology, Amitsur cohomology, archimedean cohomology, Andre-Quillen cohomology, Betti cohomology, Borel-Moore homology, cdh cohomology, Cech cohomology, Chow groups, arithmetic Chow groups, Arakelov Chow groups, group cohomology and continuous group cohomology, crystalline cohomology, crystalline Deligne cohomology, de Rham cohomology, Deligne cohomology, Deligne- Beilinson cohomology, smooth Deligne cohomology, Eichler cohomology, elliptic Bloch groups, equivariant Deligne cohomology, etale K-theory, etale motivic cohomology, flat cohomology, Fontaine-Messing cohomology, Friedlander- Suslin cohomology, Galois cohomology, Hyodo-Kato cohomology, Lawson homology, cohomology of Lie algebras, ”log” versions of Betti, de Rham, crystalline and etale cohomology, Milnor K-theory, Kato homology, Monsky-Washnitzer cohomology, morphic cohomology, motivic cohomology, nonabelian cohomology, Nisnevich cohomology, p-adic etale cohomology, parabolic cohomology, rigid cohomology, syntomic cohomology, rigid syntomic cohomology, relative log convergent cohomology, Rost’s cycle modules, singular cohomology of arithmetic schemes, Suslin homology, Tate cohomology, unramified cohomology, Weil-etale cohomology, Zariski cohomology, and various theories with compact support. Also, various notions of motives and of mixed motives, and various other kinds of algebraic cycle groups. In addition, many of the theories come with a choice of coefficients. One could also extend the list to theories occurring in other areas of mathematics, there would then be at least a few hundreds of them. (Andreas Holmstrom)

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1959

- The Brown representability theorem for generalized (Eilenberg-Steenrod) cohomology: Allows many new cohomologies, e.g., various cobordism theories (Thom), relating to all quarters of mathematics.

Then,

- Flourishing of cohomology theories, including sheaf cohomology. Understanding of generalized cohomology as (fully) abelian cohomology. Rise of nonabelian cohomology.

- Quillen and Brown develop abstract homotopy theory.

- Rise of topos theory and then $(\infty,1)$-toposes. Eilenberg-Steenrod axioms can be reformulated here very efficiently.

Rather remarkably, Urs Schreiber has proposed:

Slogan: Thousands of definitions of notions of cohomology and its variants are variants of just a single concept: an ∞-categorical hom-space in an $(\infty,1)$-topos.

Type theories and category theories

Simply typed λ-calculus	$\longleftrightarrow$	Cartesian closed category
Dependent type theory	$\longleftrightarrow$	Locally c.c. category
First-order predicate logic	$\longleftrightarrow$	Boolean/Heyting category
Geometric logic	$\longleftrightarrow$	Grothendieck topos
Higher-order logic	$\longleftrightarrow$	Elementary topos
??	$\longleftrightarrow$	Homotopical category

The 'internal language' of an $(\infty,1)$-topos is an <u>intensional</u> dependent type theory of the kind already studied by Martin-Löf:

- Voevodsky's Univalent Foundations
- homotopy type theory

- New constitutive language, homotopy type theory, embodying the 'equivalence principle'.

By **1952**

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- Observations: Cech ‘homology’ no longer a homology. (later reformed as Strong homology)

2010

- Constitutive language: $(\infty,1)$-topos theory/univalent type theory/homotopy type theory
- Theories: thousands of examples of cohomology are components of Hom-space in a $(\infty,1)$-topos; differential cohomology in cohesive $(\infty,1)$-topos.
- Observations: Demotions and promotions

Demotion and promotion (nLab)

'About the nPOV on cohomology

As we will see in the list of examples below, large numbers of examples of notions of cohomology do happen to have a natural interpretation in terms of connected components of hom-spaces in (∞,1)-categories. There are however some definitions of cohomology in the literature that do not fit this principle. But these tend to be <u>wrong</u> definitions, as illustrated by the following example.

In the literature there is a naive definition of Lie group cohomology and topological group cohomology, which is not interpretable in terms of hom-spaces in any natural (∞,1)-category. But later it was found by Segal and then independently by Brylinski that there is a refinement of this definition, which is better behaved. This refinement, it turns out, does have an interpretation in terms of homs in an (∞,1)-topos. This is described at group cohomology.'

- Set theory
- Every entity is a set, so can ask about membership everywhere.
- Observe that isomorphic structures behave the same.

- Category Theory
- Universal properties, adjunctions, isomorphism-invariance built in.
- Observe (co)limits don’t always work as wished. Need to work with operations defined up to homotopy.

- Homotopy Type Theory/Higher category theory
- Entities are the same if equivalent.

You can *retrospectively* understand why set theory was so successful from later systems, but you wouldn't be able to reconstruct it in all its unnecessary details. (*Set* as well-pointed boolean topos, free co-complete category on one object.)

We need a *prospective* understanding.

Back to physics

To HoTT/UF may be added 'modalities' for smoothness and supergeometry.

We have a new constitutive language, *modal homotopy type theory*, in which the concept of differential cohomology is best expressed.

It allows us to express (higher) gauge field theories in the same language.

Urs Schreiber, Differential cohomology in a cohesive topos

http://ncatlab.org/schreiber/show/differential+cohomology+in+a+cohesive+topos

“Fundamental physics is all controled by cohomology.” (Schreiber)

“Every $(\infty,1)$-topos comes with its intrinsic notion of cohomology. This encodes kinematics in physics.”

For every cohesive $(\infty,1)$-topos

“their intrinsic cohomology refines to differential cohomology in an $(\infty,1)$-topos classifying connections on ∞-bundles. This encodes dynamics in physics: a connection on a principal ∞-bundle is a gauge field which exerts forces. Such as:

the electromagnetic field, Yang-Mills field, the field of gravity, of supergravity, the Kalb-Ramond field, the supergravity C-field, the RR-field.”

(nLab, higher category theory and physics)

Principle of Equivalence (mathematics)

interpreted in a smooth setting takes the form of

General covariance

an aspect of which is

Principle of Equivalence (physics)

Mathematical language: Infinite 3D Euclidean space + calculus
Coordinating principles: Newton's Laws of motion
Empirical laws and regularities: Law of Gravitation, inertial mass = gravitational mass

Mathematical language: 4D-pseudo Riemannian manifold + tensor calculus
Coordinating principles: Invariance of speed of light, Einstein's equivalence principle, freefall as geodesic motion.
Empirical laws and regularities: Field equations, approximately flat time slices.

Lots to say about QM and QFT and gauge theory

Mathematical language: Modal HoTT, differential cohomology
Coordinating principles: Quantum gauge field theory
Empirical laws and regularities: String and M-theory

See how the new mathematical language bakes into the system what before was part of the physical coordinating principles – covariance.

Mathematical language: 4D-pseudo Riemannian manifold + tensor calculus
Coordinating principles: Invariance of speed of light, Einstein's equivalence principle, freefall as geodesic motion.
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Concluding questions

- Has philosophy in 20th century failed to support conceptual change in mathematics and mathematical physics? (Yes, we should have listened to Cassirer and Lautman.)

- Is there any good meta-scientific work going on now? (In the past, practitioners were often scientists.) Yes.

- Is Friedman's historical approach on the right track? What's better about it than other accounts (Quine, Lakatos,...)?

- Where does Friedman's own account feature in the course of his cycles?

Where next?

- There could be a wonderful case here of a Friedman-style seismic transformation.

- We would have to tell the stories of constructive type theory, category theory, quantum gauge field theory, and so much more.

- It may be possible to tap into good meta-scientific work going on now in mathematics, computer science, physics,...

- We (philosophers) need to make sense of these new languages.

- Perhaps we could expect the philosophical understanding of these languages to lead to something new for philosophy itself.

- (Please come tomorrow to my second talk on **modal homotopy type theory** to see if it does.)

Additional slides

Friedman himself later recognised part of the problem of downplaying the contribution of mathematics:

"Our problem, therefore, is not to characterize a purely abstract mapping between an uninterpreted formalism and sensory perceptions, but to understand the concrete historical process by which mathematical structures, physical theories of space, time, and motion, and mechanical constitutive principles organically evolve together so as to issue, successively, in increasingly sophisticated mathematical representations of experience." (2010, p. 698)

The difficulty arises when one accepts the sharp distinction, emphasized by Schlick, between an uninterpreted axiomatic system and intuitive perceptible experience, and one then views the constitutive principles in question (which, following Reichenbach, I called “coordinating principles” or “axioms of coordination”) as characterizing an abstract function or mapping associating the former with the latter. This picture is deeply problematic, I now believe, in at least two important respects: it assumes an overly simplified “formalistic” account of modern abstract mathematics, and, even worse, it portrays such abstract mathematics as being directly attached to intuitive perceptible experience at one fell swoop. (2010, pp. 697-8)

Our problem, therefore, is not to characterize a purely abstract mapping between an uninterpreted formalism and sensory perceptions, but to understand the concrete historical process by which mathematical structures, physical theories of space, time, and motion, and mechanical constitutive principles organically evolve together so as to issue, successively, in increasingly sophisticated mathematical representations of experience. (p. 698)

Michael Friedman (1993). “Remarks on the History of Science and the History of Philosophy.” In Horwich (ed.), *World Changes*, MIT Press, 37–54.

Mary Domski and Michael Dickson (eds.) (2010), *Discourse on a New Method: Reinvigorating the Marriage of History and Philosophy of Science*, Open Court Publishing, including Friedman (2010).

Michael Friedman (2010) *Synthetic History Reconsidered,* in Domski and Dickson (eds.) (2010), 571-813.

Michael Friedman (2011). 'Extending the Dynamics of Reason', *Erkenntnis* 75(3):431-444.

Schreiber et al.

In the limit of D=11 supergravity, the covariant phase space of M-theory must consist of torsion constraints in super torsion-free super-orbi $\mathbb{R}^{10,1|32}$-folds equipped with a suitable higher gauge field: the C-field. The first ingredient of a non-perturbative quantization of this phase space must be a choice of Dirac charge quantization condition for the C-field.

Hypothesis H: The C-field 4-flux and 7-flux forms in M-theory are subject to charge quantization in J-twisted Cohomotopy cohomology theory in that they are in the image of the non-abelian Chern character map from J-twisted Cohomotopy theory.

To return to the idea that philosophical interventions for quantum mechanics were untimely, perhaps this was because it was a deeper revolution, so harder to think out from an earlier starting point. Perhaps it could only be discovered initially in an ad hoc fashion.

...compared to this discovery that Newton's laws of motion were quite wrong in atoms, the theory of relativity was only a minor modification. (Feynman, QED, p. 5)

Why not see the delay in grounding QM as due to the need for full integration in quantum gauge field theory?

“It is noteworthy that already in this mathematical formulation of experimentally well-confirmed fundamental physics the seed of higher differential cohomology is hidden: Dirac had not only identified the electromagnetic field as a line bundle with connection, but he also correctly identified (rephrased in modern language) its underlying cohomological Chern class with the (physically hypothetical but formally inevitable) magnetic charge located in spacetime. But in order to make sense of this, he had to resort to removing the support of the magnetic charge density from the spacetime manifold, because Maxwell’s equations imply that at the support of any magnetic charge the 2-form representing the field strength of the electromagnetic field is in fact not closed and hence in particular not the curvature 2-form of an ordinary connection on an ordinary bundle.

In (Freed) this old argument was improved by refining the model for the electromagnetic field one more step: Dan Freed notices that the charge current 3-form is itself to be regarded as a curvature, but for a connection on a circle 2-bundle with connection – also called a bundle gerbe – , which is a cocycle in degree 3 ordinary differential cohomology. Accordingly, the electromagnetic field is fundamentally not quite a line bundle, but a twisted bundle with connection, with the twist being the magnetic charge 3-cocycle. Freed shows that this perspective is inevitable for understanding the quantum anomaly of the action functional for electromagnetism is the presence of magnetic charge.” (nLab)

Exacerbated by maths and physics failing to talk (1930s-1970s)

“I am acutely aware of the fact that the marriage between mathematics and physics, which was so enormously fruitful in past centuries, has recently ended in divorce”, Freeman Dyson, Missed Opportunities, 1972.

But joint work since then, often category theoretic, mirror symmetry, string duality (Kontsevich, Witten, etc.)

“The marriage between gauge theory and the geometry of fiber bundles from the sometime warring tribes of physics and mathematics is now over thirty years old. The marriage brokers were none other than Chern and Simons. The 1978 paper by Wu and Yang can be regarded as the announcement of this union. It has led to many wonderful offspring.” (K. Marathe, Topics in Physical Mathematics, 2010, p. xi).

“The love affair between math and physics has turned from a fling into a serious, committed relationship.” (Jeff Harvey, Strings 2011).

Cohomology in Physics

This survey is limited to the years before 2001 since there has been an explosion of cohomological applications in theoretical physics (even of K-theory) in the new century. Since 1931 but especially toward the end of the XXth century, there has been increased use of cohomological and more recently homotopy theoretical techniques in mathematical physics. (Jim Stasheff)

Cohomology plays a fundamental role in modern physics. (Zeidler, Quantum Field Theory, Volume 1, p. 14).

Cohomology is deeply routed (sic) in the following topics: Gauss' surface theory, the Kirchhoff-Weyl theory of electrical networks, and Maxwell's theory of electromagnetism. Cohomology lies at the heart of both modern differential topology and modern quantum field theory (the BRST approach). (Zeidler)

Fundamental physics is all controled by cohomology. (Schreiber)

Quantization requires *linear homotopy type theory*, a blend of *linear logic* and *homotopy type theory*.

Birkhoff-von Neumann quantum logic embeds into this.

Maybe Friedman was onto something...