Exercise 1

should be $-i$

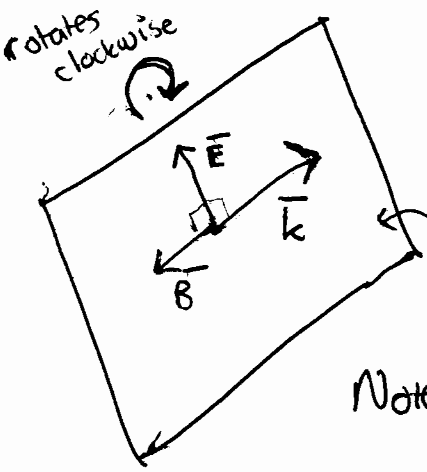
(Mistake in $\bar{k} \times \bar{E} = i\omega \bar{E}$) . Should read:

my notation

Let $\bar{k}$ be a vector in $\mathbb{R}^3$, and let $\omega = |\bar{k}|$. Fix $\bar{\mathcal{E}}_0 \in \mathbb{C}^3$ with $\bar{k} \cdot \bar{\mathcal{E}}_0 = 0$ and $\bar{k} \times \bar{\mathcal{E}}_0 = -i\omega \bar{\mathcal{E}}_0$. Show that

$$\bar{\mathcal{E}}(t, \bar{x}) = \bar{\mathcal{E}}_0 \cdot e^{-i(\omega t - \bar{k}\cdot\bar{x})}$$

satisfies the vacuum Maxwell equations.



~~[illegible]~~ negative helicity circularly polarized wave

plane perpendicular to $\bar{k}$.

Note: $\bar{E} \times \bar{B}$ points in direction of $\bar{k}$ (propagation).

$\bar{E}$ and $\bar{B}$ rotate clockwise according to an observer looking in the direction $(-\bar{k})$ [i.e. looking into the wave].

$$\bar{\mathcal{E}} = \bar{E} + i\bar{B}.$$

$$\begin{aligned}
\nabla \cdot \bar{\mathcal{E}} &= \partial_x \mathcal{E}_x + \partial_y \mathcal{E}_y + \partial_z \mathcal{E}_z \\
&= e^{-i(\dots)}\, i\left[k_x(\mathcal{E}_0)_x + k_y(\mathcal{E}_0)_y + k_z(\mathcal{E}_0)_z\right] \\
&= e^{-i(\dots)}\, i\left(\bar{k} \cdot \bar{\mathcal{E}}_0\right) = 0.
\end{aligned}$$

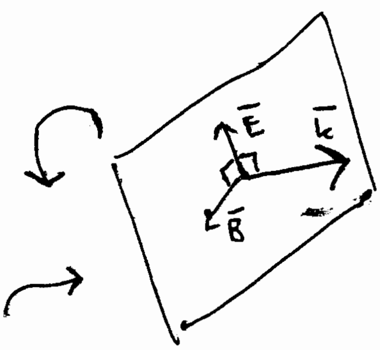
To calculate $\nabla \times \bar{\mathcal{E}}$, observe eg.

$$(\nabla \times \bar{\mathcal{E}})_x = \partial_y \mathcal{E}_z - \partial_z \mathcal{E}_y$$
$$= e^{-i(\cdots)} [i][k_y (\mathcal{E}_0)_z - k_z (\mathcal{E}_0)_y]$$
$$= e^{-i(\cdots)}\, i (\bar{k} \times \bar{\mathcal{E}}_0)_x$$

so
$$(\nabla \times \bar{\mathcal{E}}) = i e^{-i(\cdots)} \underbrace{\bar{k} \times \bar{\mathcal{E}}_0}_{=-i\omega \bar{\mathcal{E}}_0}$$
$$\overset{*}{=} e^{-i(\cdots)} \omega \bar{\mathcal{E}}_0$$
$$= \omega \bar{\mathcal{E}}$$
$$= i \frac{\partial \bar{\mathcal{E}}}{\partial t}.$$

Note: we could also have a circularly polarized wave of positive helicity:

$$\bar{\mathcal{E}} = \bar{\mathcal{E}}_0 \, e^{+i(\omega t - \bar{k}\cdot\bar{x})}$$

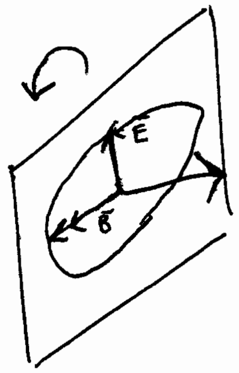


rotates counter clockwise.

Most general: a linear combination of positive helicity with negative helicity:

$$\bar{\mathcal{E}} = \bar{\mathcal{E}}_0^{+} e^{i(\cdots)} + \bar{\mathcal{E}}_0^{-} e^{i(\cdots)}$$

Now $\bar{E}$ and $\bar{B}$ fields rotate on an ellipse:



(that's because the equation

$$z = ae^{it} + be^{-it}$$

~~where a and~~ traces out an ellipse in the complex plane; i.e. two counterrotating circles of the same frequency but different radii trace out an ellipse.)