Baez and Munian

Exercises 4-6

4. We simply restrict the charts to U, i.e.

$$V_\alpha = \underbrace{U_\alpha}_{\text{chart for } M} \cap U$$

which is open by definition of the subspace topology. Our chart maps are

$$\psi_\alpha = \phi_\alpha\big|_U \quad , \quad \psi_\alpha^{-1} = \phi_\alpha^{-1}\big|_{\phi_\alpha(U)}$$

which are continuous by definition of the subspace topology. All the transition functions will still be smooth, since

$$\psi_\beta \circ \psi_\alpha^{-1} = \phi_\beta|_U \circ \phi_\alpha^{-1}\big|_{\phi_\alpha(U)}$$

$$= \underbrace{\left(\phi_\beta \circ \phi_\alpha^{-1}\right)}_{\text{smooth.}}\Big|_{\phi_\alpha(U)}$$

5. We simply set our charts to be the cartesian products of the old ones:

$$\zeta_{\alpha,\beta} : U_\alpha \times V_\beta \longrightarrow \mathbb{R}^m \times \mathbb{R}^n$$

$$(x,y) \longmapsto (\phi_\alpha(x), \psi_\beta(y))$$

This collection of charts is clearly continuous and covers $M \times N$; its inverse is also continuous by definition of the topology on $M \times N$. Transition functions also clearly smooth.

6. We take our set of charts to be the disjoint union of the set of charts for M and N respectively. Clearly everything goes through since M and N never "interfere" with each other. That is, the only trouble that could arise is if there were to be new transition functions

$$\underset{\substack{\uparrow \\ \text{chart for } M}}{\phi_\alpha} \circ \underset{\substack{\nwarrow \\ \text{chart for } N}}{\psi_\beta^{-1}}$$

but these have empty domain since M and N are disjoint.